Project 15.7a*: Function Optimization

Objective

The objective of this project is to use Maple to find the extrema of a function of two variables.

Narrative

If you have not already done so, read Section 15.7 in the text.

In this project you will be asked to use Maple to find the extrema of $f(x,y) = (3x + 4y)/(x^2 + y^2 + 1)$. To do this, you will need to use the commands:

`f1 := D[1](f)` to define the function `f1` as $\partial f/\partial x$,

`f2 := D[2](f)` to define the function `f2` as $\partial f/\partial y$,

`f11 := D[1,1](f)` to define the function `f11` as $\partial^2 f/\partial x^2$,

`f12 := D[1,2](f)` to define the function `f12` as $\partial^2 f/\partial x \partial y$, and

`f22 := D[2,2](f)` to define the function `f22` as $\partial^2 f/\partial y^2$,

as well as the command `H := (x,y) -> f11(x,y)*f22(x,y)-f12(x,y)^2` to define the Hessian of $f(x,y)$. You will also need to use the `solve` command: `solve({g(x,y)=0,h(x,y)=0},{x,y})` solves the system of equations

$$g(x,y) = 0, \quad h(x,y) = 0$$

for x and y.

Task

If $f(x,y) = (3x + 4y)/(x^2 + y^2 + 1)$, use Maple to:

a) find the critical points of f, and

b) apply the second derivative test for functions of two variables to classify the values of f at these critical points as relative maxima, relative minima, or neither. (Your report should contain a statement — or statements — of the form, "$f($ something, something $)$ is a ＿＿＿＿." where (something, something) are the coordinates of a critical point, and you fill in the blank with "relative maximum", "relative minimum", or "saddle value".)

Finally, ..

c) Draw the graph of f, using an appropriate window size and point of view to confirm your findings.