MATH 261, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Project 15.8a: Constrained Optimization

Objective

The objective of this project is to illustrate some of the concepts behind constrained function optimization using Lagrange multipliers.

Narrative

If you have not already done so, read Section 15.8 in the text. In this section we learn that finding the extrema of the function $z = f(x, y)$ subject to the constraint $g(x, y) = 0$, can be reduced to solving the system of 3 equations

$$\nabla f(x, y) = \lambda \nabla g(x, y), \quad g(x, y) = 0$$

in the 3 unknowns x, y, and λ. In this project we consider the specific problem of finding the extrema of

$$f(x, y) = 3x^2 - xy + y^2 - 3x - 5y + 9$$

subject to the constraint (along the curve C whose equation is)

$$x^2 + y^2 = 1.$$

Task

a) Type the following lines into Maple.

```
> # Project 15.8a:  Constrained Optimization
> restart:  with(plots):
> plot0 := contourplot(3*x^2-x*y+y^2-3*x-5*y+9,x=-2..2,y=-2..2,color=red,contours=8):
> plot1 := implicitplot(x^2+y^2=1,x=-2..2,y=-2..2,color=blue):
> display({plot0,plot1});
```

At this time, make a hard-copy of your typed input and Maple's responses. Then, ...

b) By hand, carefully draw the level curves of f that correspond to the extreme values of f along C. Label each of the curves you have drawn by hand with the value of f along it. (To find the value of f along each curve, first *estimate* the coordinates of the point of intersection of each level curve with C, and f; all you need is an estimate: you do *not* have to solve the Lagrange multiplier system of equations! Then use Maple to substitute the coordinates of each point of intersection into f.)

c) Label the level curves on each side of each of the level curves you have drawn by hand with the value of f along it. (Use the same technique you used in part (b) to obtain these values.)

d) At each of the points where C meets these two level curves, draw and label the tangent line to C and a vector in the direction of the gradient ∇f of f. (Remember: The direction of the gradient is the direction of *steepest ascent*.)

Comments

There are often ways to approach constrained optimization problems other than Lagrange multipliers. For example, the problem addressed above can be reduced to a problem in the optimization of a single function of a single variable by observing that if we parametrize the curve $x^2 + y^2 = 1$ by $x(t) = \cos t, y(t) = \sin t, t \in [0, 2\pi]$ then, along the graph of $g(x, y) = 0$,

$$f = f(t) = \cos^2 t - \cos t \sin t + 3\sin^2 t - 5\cos t - 3\sin t + 9.$$