

Project 16.3: Double Integrals

Objective

The objective of this project is to illustrate how Maple can be used to evaluate double integrals.

Narrative

If you have not already done so, read Section 16.1–16.3 of the text. In this project we illustrate how Maple can be used to evaluate double integrals.

Task

1. a) Type the command lines below into Maple in the order in which they are listed; they draw the rectangle R bounded by the lines $x = 1$, $x = 3$, $y = 2$, $y = 4$, and evaluate $\int_{y=2}^{y=4} \int_{x=1}^{x=3} x^2 y^3 e^{-xy} dx dy$.

```
> # Project 16.3: Double Integrals
> restart: with(plots):
> # Parts a and b
> setoptions(axes=normal,scaling=constrained):
> implicitplot({x=1,x=3,y=2,y=4},x=0..4,y=0..5,color=blue,thickness=2);
> f := (x,y) -> x^2*y^3*exp(-x*y);
> Int(Int(f(x,y),x=1..3),y=2..4) = int(int(f(x,y),x=1..3),y=2..4);
```

(Would you want to compute $\int_{y=2}^{y=4} \int_{x=1}^{x=3} x^2 y^3 e^{-xy} dx dy$ by hand? Probably not. This is one of the things that makes Maple practically quite useful!)

- b) Type the command line necessary to compute $\int_{x=1}^{x=3} \int_{y=2}^{y=4} x^2 y^3 e^{-xy} dy dx$. Observe that we get the same value for $\int \int_R x^2 y^3 e^{-xy} dA$ irrespective of the order of the limits. While this *always* happens when R is a rectangle it does *not* always happen when R is not a rectangle (as we will see below).

- c) Type the command lines below into Maple; they find the points of intersection of $y = 2x^2$ and $y = 1 + x^2$, draw the region R bounded by these two curves, and evaluate $\int \int_R (x + 2y) dA$.

```
> # Part c and d
> solve({y=2*x^2,y=1+x^2},{x,y});
> implicitplot({y=2*x^2,y=1+x^2},x=-1..1,y=0..3,thickness=2);
> f := (x,y) -> x+2*y;
> Int(Int(f(x,y),y=2*x^2..1+x^2),x=-1..1) = int(int(f(x,y),y=2*x^2..1+x^2),x=-1..1);
```

- d) Continue by typing the command line below into Maple; it evaluates $\int \int_R (x + 2y) dA$ where R is the region of part (c), *intentionally using the wrong order of integration*. Note that not only does our answer depend on the order of integration, but when we integrate in the wrong order our “answer” isn’t even a real number!

```
> Int(Int(f(x,y),x=-1..1),y=2*x^2..1+x^2) = int(int(f(x,y),x=-1..1),y=2*x^2..1+x^2);
```

2. a) Type the command lines necessary to find the points of intersection of $x = 1 - y^2$ and $x = y - y^3$, draw the region R bounded by these two curves, and evaluate $\int \int_R (x^2 - 4y) \, dA$. (*Hint*: Look at parts (c) and (d) of Task 1.)

At this point, make a hard-copy of your typed input and Maple's responses. Then, ...

- b) By hand, label the curves bounding R with their equations, and shade in R .

Comments

You can now easily do the exercises in Section 16.2 and 16.3 in the text using Maple!