MATH 164, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Project 10.5a*: The Logistic Equation

Objective

To investigate the logistic equation.

Narrative

If you have not already done so, read Section 10.5 of the text, and do Project 10.2a.

The logistic equation is used in the study of population dynamics to model the growth of a population that grows exponentially when it is small, and more slowly as it reaches the carrying capacity of its environment. (The carrying capacity of an environment is a limit to the population the environment can support because of the limited resources of the environment.) If $P = P(t)$ denotes the size of a population at time t, and L is the carrying capacity of the environment supporting this population, then the logistic equation is

$$\frac{dP}{dt} = kP(1 - \frac{P}{L})$$

where k is a positive constant.

Task

1. Using Maple, draw (in one graphic):

   a) the direction field associated to the logistic equation assuming that $k = 2$ and $L = 2$ for $t \in [0, 4]$ and $P \in [0, 4]$, and

   b) the five solutions corresponding to the initial conditions $P(0) = 0.01$, $P(0) = 0.25$, $P(0) = 1.5$, $P(0) = 4.0$, and $P(0) = 1000.0$. (If you think of L as being measured in units of 1 million, for example, then $L = 2$ means L is 2 million.)

   At this point, make a hard-copy of your typed input and Maple's responses. Then continue with the following steps.

2. On the graphic you produced for Task 1, label the coordinate axes, draw and label by hand the line whose equation is $P = L$, and label the curves corresponding to the five initial conditions. (Label the curve corresponding to $P(0) = 0.25$ by "$P(0) = 0.25$", for example.)

3. On the graphic you produced for Task 1, plot by hand the solution that passes through the point $P(2, 1)$.

4. In one or two sentences, justify — on the basis of population dynamics — the fact that if the initial level $P(0)$ of a population whose growth is logistic, is greater than the carrying capacity L of the environment then P *decreases* to L as t increases (rather than continuing to increase).