

Project 4.3: Concavity and Inflection Points

Objective

To illustrate how Maple can be used to analyze the concavity and find the inflection points of a function.

Narrative

If you have not already done so, read Section 4.3 of the text. In particular, be sure to read Example 6, p. 245: below we use this example to illustrate how Maple can be used to analyze the concavity and find the inflection points of $f(x) = x^4 - 4x^3$.

Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference. Your lab report will be a hard copy of your typed input and Maple's responses (both text and graphics).

> # Project 4.3: Concavity and Inflection Points	
> restart;	Clear Maple's memory.
> f := x -> x^4-4*x^3;	Let $f(x) = x^4 - 4x^3$.
> f1 := D(f);	Let $f1$ denote the first derivative f' of f .
> f2 := D(f1);	Let $f2$ denote the second derivative f'' of f .
> ip:= fsolve(f2(x)=0,x);	Find all the possible inflection numbers of f by solving $f''(x) = 0$.
> print(ip[1],f(ip[1]));	What is the first possible inflection point?
> print(ip[2],f(ip[2]));	What is the second possible inflection point?
> plot({f(x),f2(x)},x=-1..4);	Plot the graphs of f and f'' over a y -range large enough to capture all relevant function behavior.

At this point, make a hard-copy of your typed input and Maple's responses. Then by hand ...

- b) label the graphs of f and f'' in the last plot,
- c) plot and label the inflection points of f , and
- d) highlight that part of the graph of f over which f is concave up, and that part of the graph of f'' over which the values of f'' are positive.

Comments

Remember that an inflection point is a point on the graph of a function at which the concavity changes (from concave up to concave down or from concave down to concave up), and that we can narrow our search for inflection points to a finite number of points by finding where $f''(x) = 0$ and where $f''(x)$ does not exist. On one hand, it is not until we have further information that we can say that one of these points is an inflection point. On the other hand, we restricted our attention to points at which $f''(x) = 0$ in the above example since $f(x)$ is a polynomial function so $f''(x)$ exists for all x .