

Project 16.9a*: Transformations of the Plane

Objective

The objective of this project is to give you some experience with the geometry of some simple transformations of the plane.

Narrative

In this project we illustrate the geometry of some simple transformations of the plane by applying them to a simple test pattern.

Task

1. Type the following commands into Maple; they define the test pattern `TestPat`.

```
> # Project 16.9a*: Transformations of the Plane
> restart;
> with(plottools):with(plots):
> L1 := [t,0,t=0..1]: L2 := [1,t,t=0..1]:
> L3 := [1-t,1,t=0..1]: L4 := [0,1-t,t=0..1]:
> L5 := [t,1+t,t=0..0.5]: L6:= [t,2-t,t=0.5..1]:
> TestPat := plot({L1,L2,L3,L4,L5,L6},color=blue,scaling=constrained):
> display(TestPat);
```

2. Type the following commands into Maple; they define the transformation $T(x, y) \rightarrow (2x, 1.5y)$, and display the test pattern and its image under T .

```
> T := transform((x,y) -> [2*x,1.5*y]): display({TestPat,T(TestPat)});
```

3. Repeat Task 2 using the transformations:

- a) $T(x, y) \rightarrow (-x, y)$,
- b) $T(x, y) \rightarrow (x + 2, y + 1)$,
- c) $T(x, y) \rightarrow (\frac{\sqrt{2}}{2}(x - y), \frac{\sqrt{2}}{2}(x + y))$,
- d) $T(x, y) \rightarrow (x - y + 3, 2x + y)$,
- e) $T(x, y) \rightarrow (x + 2, x^2 + y)$.

At this time make a hard-copy of your typed input and Maple's responses. Then, ...

4. on each graphic you produced (on both the test pattern *and* the transformed test pattern), draw by hand at least a door — with a doorknob — and a window. (*Note:* The point of including the doorknob is to make sure there is some assymetry in your test pattern.)

Comments

We were careful to describe the segments of our test pattern by parametric curves. If, instead of doing this, we had plotted points and “connected the dots” then — as long as T takes lines to lines (as was the case with the first five transformations above) — we would have noticed no difference; however, there *would* have been a difference with a transformation such as the sixth transformation above since it does *not* take lines to lines.