

1. seminar

Problem 1

Let $X \sim N(0, 1) \sim \Phi(x)$. Prove the formula $\Phi(-u) = 1 - \Phi(u)$

[Greek capital letter Φ stands for distribution function of standard normal random variable.]

Problem 2

Find the probability that the random variable $X \sim N(20, 16)$ takes the value less than 12 or greater than 28.

Problem 3

Let X_1, X_2 be independent random variables, $X_i \sim N(0, 1)$, $i = 1, 2$. Determine the distribution of transformed random variable $Y = 3 + X_1 - 2X_2$, further determine its parameters and finally determine its first quartile value (25% quantile).

Problem 4

The random variable $X_1 \sim N(19, 0.5^2)$ represents the exchange rate of dolar and $X_2 \sim N(32, 0.6^2)$ represents the exchange rate of euro. Correlation $R(X_1, X_2) = -0.8$. What is the probability that the value of currency basket $0.65X_1 + 0.35X_2$ will be greater than 24? Clues on how:

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim N_2\left(\begin{pmatrix} 19 \\ 32 \end{pmatrix}, \begin{pmatrix} 0.25 & -0.072 \\ -0.072 & 0.36 \end{pmatrix}\right)$$
$$0.65X_1 + 0.35X_2 = (0.65 \ 0.35) \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

Problem 5

Let X_1, X_2, X_3, X_4 be independent random variables, $X_i \sim N(0, 1)$,

$i = 1, 2, 3, 4$. Determine the distribution of transformed random variable $X = \frac{X_1\sqrt{3}}{\sqrt{X_2^2 + X_3^2 + X_4^2}}$?

Problem 6

Let $X \sim F(n_1, n_2)$. Determine the distribution of transformed random variable $Y = \frac{1}{X}$

Problem 7

Let $X \sim t(n)$. Determine the distribution of transformed random variable $Y = X^2$

Problem 8

Prove that

a) $u_\alpha = -u_{1-\alpha}$

b) $t_{\alpha(n)} = -t_{1-\alpha}(n)$

c) $F_\alpha(n_1, n_2) = \frac{1}{F_{1-\alpha}(n_2, n_1)}$

Problem 9

sbírka 10.1,10.2,10.3,10.5 - Looking up in probability tables