4. seminar

Problem 1
Independent laboratory measurements of particular constant μ are characterized by a random sample $X_1, \dots, X_n,\ E(X_i) = \mu,\ D(X_i) = \sigma^2,\ i = 1, \dots, n$. Consider statistics $M_n = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$ and $L_n = \frac{X_1+X_2}{2}$.

a) Prove that M_n and L_n are unbiased estimators of the constant μ.

b) Find out which of these estimators is better.

c) Prove that $\{M_n\}$ and $\{L_n\}$ make the sequence of asymptotically unbiased estimators of the parameter μ.

d) Prove that $\{M_n\}$ and $\{L_n\}$ make the sequence of consistent estimators of the parameter μ.

Problem 2
Let $X_{11}, \dots, X_{1n_1}$ and $X_{21}, \dots, X_{2n_2}$ be two independent random samples. The first sample follows the distribution with expected value μ_1 and variance σ^2, the second sample follows the distribution with expected value μ_2 and variance σ^2. Let $M_1,\ M_2$ denote sample means; let $S_1^2,\ S_2^2$ denote sample variances and $S_*^2 = \frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$ denote weighted mean of sample variances.

a) Prove that the statistic $M_1 - M_2$ is an unbiased estimator of the parametric function $\mu_1 - \mu_2$.

b) Prove that the statistic S_*^2 is an unbiased estimator of the parametric function σ^2.

Problem 3
Let $X_1, \dots, X_n$ be a random sample from the continuous uniform distribution $U(0, b)$, where $b > 0$ is an unknown parameter. The following statistics are defined:
$T_1 = X_1 + \frac{1}{2}X_2 + \frac{1}{3}X_3 + \frac{1}{6}X_4$ and $T_2 = \frac{1}{2}(X_1 + X_2 + X_3 + X_4)$.

a) Show that statistics T_1 and T_2 are unbiased estimators of the parameter b.

b) Decide which estimator is better.

c) Suggest any other estimator.
Let $X_{(n)} = max\{X_1, \dots, X_n\}$. Show that the statistic $T_3 = X_{(n)}$ is a consistent estimator of parameter b.

Problem 4
The plane speed was measured 5 times and the realization of sample mean was 870,3 m/sec. Find 95% confidence interval for parameter μ if the plane speed follows normal distribution with standard deviation $\sigma = 2.1 m/s$.