Advanced Econometrics - Lecture 3

Instrumental Variable and GMM Estimator

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- The OLS Estimator: With Error Correlated Regressors
- Correlated Regressors: Some Cases
- Instrumental Variables (IV) Estimator: The Concept
- IV Estimator: The Method
- Calculation of the IV Estimator
- An Example
- The GIV Estimator
- The Generalized Method of Moments
- Some Tests

OLS Estimator

Linear model for y_t

$$y_t = x_t'\beta + \varepsilon_t,\ t = 1, \ldots, T$$

given observations x_{tk}, $k = 1, \ldots, K$, of the regressor variables and the error term ε_t

Properties of the OLS estimator $b = (\Sigma_t x_t x_t')^{-1} \Sigma_t x_t y_t$

1. OLS estimator b is **unbiased** if
   - (A1) $E\{\varepsilon\} = 0$
   - (A10) $E\{\varepsilon \mid X\} = 0$, i.e., X uninformative about $E\{\varepsilon_t\}$ for all t (ε is conditional mean independent of X)
     - (A2) [$\{x_t, t=1, \ldots, T\}$ and $\{\varepsilon_t, t=1, \ldots, T\}$ are independent] is stronger
     - (A8) [x_t and ε_t are independent for all t] is less strong
     - (A7) [$E\{x_t \varepsilon\} = 0$ for all t, no contemporary correlation] is even less strong than (A8)

OLS Estimator, cont'd

2. OLS estimator b is **consistent** for β if
   - (A8) x_t and ε_t are independent for all t
   - (A11) ε_t,~ IID(0,σ^2)
   - (A6) $(1/T)\Sigma_t x_t x_t'$ has as probability limit a nonsingular matrix Σ_{xx}

   (A8) can be substituted by (A7) [$E\{x_t \varepsilon\} = 0$ for all t, no contemporary correlation]
3. OLS estimator b is asymptotically normally distributed if (A6), (A8) and (A11) are true;
   - for large T, b follows approximately the **normal distribution**

     $b \sim_a N\{\beta, \sigma^2(\Sigma_t x_t x_t')^{-1}\}$

The Assumption (A7): E$\{x_t \varepsilon_t\}$ = 0 for all t

Implication of (A7): for all t, each of the regressors is uncorrelated with the current error term, no contemporary correlation

Stronger assumptions – (A2), (A8), (A10) – have same consequences

(A7) is required for unbiasedness and consistency of the OLS estimator

In reality, the assumption E$\{x_t \varepsilon_t\}$ = 0 is not always true

Examples of situations with E$\{x_t \varepsilon_t\} \neq 0$:

- Regression on the lagged dependent variable with autocorrelated error term
- Observations of a regressor with measurement errors
- Endogeneity of regressors
- Simultaneity

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Regressor with Measurement Error

$$y_t = \beta_1 + \beta_2 w_t + v_t$$

with white noise v_t, $V\{v_t\} = \sigma_v^2$, and $E\{v_t|w_t\} = 0$; conditional expectation of y_t given w_t : $E\{y_t|w_t\} = \beta_1 + \beta_2 w_t$

e.g., w_t is household income, y_t is household saving

Measurement process:

$$x_t = w_t + u_t$$

where u_t is (i) white noise with $V\{u_t\} = \sigma_u^2$, (ii) independent of v_t, and (iii) independent of w_t

The model to be analyzed is

$$y_t = \beta_1 + \beta_2 x_t + \varepsilon_t \quad \text{with } \varepsilon_t = v_t - \beta_2 u_t$$

- $E\{x_t\, \varepsilon_t\} = - \beta_2\, \sigma_u^2 \neq 0$: requirement for consistency is violated
- x_t and ε_t are negatively (positively) correlated if $\beta_2 > 0$ ($\beta_2 < 0$)

Measurement Error, cont'd

Inconsistency of b_2

$$\text{plim } b_2 = \beta_2 + E\{x_t\,\varepsilon_t\} / V\{x_t\}$$

$$= \beta \left(\; - \; \frac{\sigma}{\sigma \quad \sigma} \right)$$

β_2 is underestimated

Inconsistency of b_1

$$\text{plim } (b_1 - \beta_1) = - \text{ plim } (b_2 - \beta_2)\, E\{x_t\}$$

given $E\{x_t\} > 0$ for the reported income: β_1 is overestimated; inconsistency carries over

The model does not correspond to the conditional expectation of y_t given x_t:

$$E\{y_t|x_t\} = \beta_1 + \beta_2 x_t - \beta_2\, E\{u_t|x_t\}$$

Dynamic Regression

Allows to model dynamic effects of changes of *x* on *y*:

$$y_t = \beta_1 + \beta_2 x_t + \beta_3 y_{t-1} + \varepsilon_t$$

OLS estimators are consistent if $E\{x_t\, \varepsilon_t\} = 0$ and $E\{y_{t-1}\, \varepsilon_t\} = 0$

AR(1) model for ε_t:

$$\varepsilon_t = \rho\varepsilon_{t-1} + v_t$$

v_t white noise with σ_v^2

From $y_t = \beta_1 + \beta_2 x_t + \beta_3 y_{t-1} + \rho\varepsilon_{t-1} + v_t$ follows

$$E\{y_{t-1}\varepsilon_t\} = \beta_3\, E\{y_{t-2}\varepsilon_t\} + \rho^2\sigma_v^2(1 - \rho^2)^{-1}$$

which indicates that y_{t-1} is correlated with ε_t

OLS estimators not consistent

The model does not correspond to the conditional expectation of y_t given the regressors x_t and y_{t-1}:

$$E\{y_t|x_t,\, y_{t-1}\} = \beta_1 + \beta_2 x_t + \beta_3 y_{t-1} + E\{\varepsilon_t\, |x_t,\, y_{t-1}\}$$

Omission of a Relevant Regressor

OLS estimator is biased

Is OLS estimator consistent?

Example: Wage equation with x_{2i}: years of schooling and u_i: abilities (unobservable)

$$y_i = x_{1i}'\beta_1 + x_{2i}\beta_2 + u_i\gamma + v_i$$

Model for analysis:

$$y_i = x_i'\beta + \varepsilon_i \quad \text{with } \varepsilon_i = u_i\gamma + v_i$$

Given E{x_i v_i} = 0

$$\text{plim } b = \beta + \Sigma_{xx}^{-1} E\{x_i u_i\} \gamma$$

OLS estimator b are inconsistent if x_i and u_i are correlated ($\gamma \neq 0$); if higher abilities induce more years at school: estimator for β_2 might be misleading

Endogenous regressor: is correlated with error term; with errors uncorrelated regressors are called **exogenous**

Endogenous Regressors: Consequences

Model

$y_i = x_i'\beta + \varepsilon_i$ with white noise ε_i, $V\{\varepsilon_i\} = \sigma_\varepsilon^2$

or in matrix notation: $y = X\beta + \varepsilon$

Violation of (A7): $E\{X'\varepsilon\} \neq 0$

OLS estimator $b = \beta + (X'X)^{-1}X'\varepsilon$

- $E\{b\} \neq \beta$, b is biased; bias $E\{(X'X)^{-1}X'\varepsilon\}$ difficult to assess
- $\text{plim } b = \beta + \Sigma_{xx}^{-1} q$ with $q = \text{plim}(T^{-1}X'\varepsilon)$
  - For $q = 0$ (regressors and error terms are asymptotically uncorrelated), OLS estimators b are consistent also in case of non-exogenous regressors
  - For $q \neq 0$ (error terms and at least one regressor are asymptotically correlated): $\text{plim } b \neq \beta$, the OLS estimators b are not consistent

Simultaneity

The regressor x_t has an impact on y_t; at the same time y_t has an impact on x_t

Example: Consumption function

- x_t per capita income; y_t per capita consumption

$$y_t = \beta_1 + \beta_2 x_t + \varepsilon_i \qquad \text{(A)}$$

  β_2: marginal propensity to consume, $0 < \beta_2 < 1$
- z_t: per capita investment (exogenous, $E\{z_t \varepsilon_i\} = 0$)

$$x_t = y_t + z_t \qquad \text{(B)}$$

- Both y_t and x_t are endogenous: $E\{y_t \varepsilon_i\} = E\{x_t \varepsilon_i\} = \sigma_\varepsilon^2(1 - \beta_2)^{-1}$
- Equations (A) and (B) are the structural equations, the coefficients are behavioral parameters
- OLS estimator b_2 from (A) is inconsistent

$$\text{plim } b_2 = \beta_2 + \text{Cov}\{x_t \varepsilon_i\} / V\{x_t\} = \beta_2 + (1 - \beta_2)\, \sigma_\varepsilon^2(V\{z_t\} + \sigma_\varepsilon^2)^{-1}$$

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Example: Consumption Function

Data: annual differences of the logarithmic series PCR (*c*, Private Consumption) and PYR (*y*, Disposable Income of Households) from the AWM database (1970:1 to 2003:4)

Fitted model

$$\hat{c} = 0.011 + 0.718\, y$$

with $t = 15.55$, $R^2 = 0.65$, $DW = 0.50$

Attention! Income y_t in $c_t = \beta_1 + \beta_2 y_t + \varepsilon_t$ might be correlated with the errors: income is used for consumption and other expenditures

$$y_t = c_t + z_t$$

where z_t includes all income components besides consumption

Risk of inconsistency due to correlated y_t and ε_t

Consumption Function, cont'd

Alternative model: $c_t = \beta_1 + \beta_2 y_{t-1} + \varepsilon_t$

- y_{t-1} and ε_t are certainly uncorrelated
- No risk of inconsistency due to correlated y_t and ε_t
- y_{t-1} is certainly highly correlated with y_t, is almost as good as regressor as y_t

Fitted model:

$$\hat{c} = 0.012 + 0.660\ y_{-1}$$

with t = 12.86, R^2 = 0.56, DW = 0.79

Deterioration of t-statistic and R^2 are price for improvement of the estimator

IV Estimator: The Idea

Alternative to OLS estimator

- Avoids bias and inconsistency in case of endogenous regressors

Instrumental variable estimator (IV estimator):

Replace with error terms correlated regressors by regressors

- which are uncorrelated with the error terms
- which are (highly) correlated with the regressors that are to be replaced

and use OLS estimation

The hope is that the IV estimator is not biased and consistent or at least less than the OLS estimator

Price: Deteriorated model fit, e.g., *t*-statistic, R^2

IV Estimator: A Simple Case

The model

$y_t = \beta_1 + \beta_2 x_t + \varepsilon_t$ with endogenous regressor, $E\{x_t \varepsilon_t\} \neq 0$

the OLS estimator is inconsistent

Find an instrumental variable z_t satisfying

1. $E\{z_t \varepsilon_t\} = 0$, i.e., instrument is uncorrelated with error term (exogeneity)
2. $cov\{x_t, z_t\} \neq 0$, i.e., instrument is correlated with endogenous regressor and not linearly dependent of *x*'s (relevance)

Covariance of y_t with z_t

$$Cov\{y_t, z_t\} = \beta_2 \, Cov\{x_t, z_t\} + Cov\{\varepsilon_t, z_t\}$$

This gives

$$\beta_2 = Cov\{y_t, z_t\} \,/\, Cov\{x_t, z_t\}$$

IV Estimator: Simple Case, cont'd

The IV estimator is obtained by replacing the population covariances by the sample covariances

$$\hat{\beta} = \frac{\sum_t (z_t - \bar{z})(y_t - \bar{y})}{\sum_t (z_t - \bar{z})(x_t - \bar{x})}$$

Properties:

- The IV estimator is a consistent estimator for β_2 provided that the instruments are valid, i.e., they are exogenous and relevant
- Typically, it cannot not be shown that the IV estimator is unbiased; small sample properties are not known
- The IV estimator coincides with the OLS estimator if $z_t = x_t$

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IV Estimator: General Case

The model is

$y_t = x_t'\beta + \varepsilon_t$, ε_t white noise, $V\{\varepsilon_i\} = \sigma_\varepsilon^2$

with

$E\{\varepsilon_t x_t\} \neq 0$

at least one component x_k of x is correlated with the error term

The vector of instruments z_t (with the same dimension as x_t) fulfills

$E\{\varepsilon_t z_t\} = 0$

IV estimator based on the instruments z_t

$$\hat{\beta} = \left(\sum_t z_t x_t'\right)^{-1} \sum_t z_t y_t$$

IV Estimator: General Case, cont’d

The (asymptotic) covariance matrix of is given by

$$\hat{V}\{\hat{\beta}\} = \hat{\sigma}^2 \left(\sum_t x_t z_t' \left(\sum_t z_t z_t' \right)^{-1} \sum_t z_t x_t' \right)^{-1}$$

In the estimated covariance matrix, σ^2 is substituted by

$$\hat{\sigma}^2 = \frac{1}{T} \sum_t \left(y_t - x_t' \hat{\beta} \right)^2$$

The asymptotic distribution of IV estimators, given IID(0, σ_ε^2) error terms, leads to the approximate distribution

$$N\left(\beta, \hat{V}\{\hat{\beta}\}\right)$$

with the estimated covariance matrix

Derivation of IV Estimators

The model is

$$y_t = x_t'\beta + \varepsilon_t = x_{0t}'\beta_0 + \beta_K x_{Kt} + \varepsilon_t$$

with $x_{0t} = (x_{1t}, \ldots, x_{K-1,t})'$ containing the first K-1 components of x_t, and $E\{\varepsilon_t\, x_{0t}\} = 0$

K-the component is endogenous: $E\{\varepsilon_t\, x_{Kt}\} \neq 0$

The instrumental variable z_{Kt} fulfills

$$E\{\varepsilon_t\, z_{Kt}\} = 0$$

Moment conditions: K conditions to be satisfied by the coefficients, the K-th condition with z_{Kt} instead of x_{Kt}:

$$E\{\varepsilon_t\, x_{0t}\} = E\{(y_t - x_{0t}'\beta_0 - \beta_K x_{Kt})\, x_{0t}\} = 0 \quad (K\text{-1 conditions})$$

$$E\{\varepsilon_t\, z_t\} = E\{(y_t - x_{0t}'\beta_0 - \beta_K x_{Kt})\, z_{Kt}\} = 0$$

Number of conditions – and corresponding linear equations – equals the number of coefficients to be estimated

Derivation of IV Estimators, cont'd

The system of linear equations for the *K* coefficients β to be estimated can be uniquely solved for the coefficients β: the coefficients β are identified

To derive the IV estimators from the moment conditions, the expectations are replaced by sample averages

$$\frac{1}{T}\sum_t (y_t - x_t'\hat{\beta})\, x_{kt} = 0, \quad k = 1, \ldots, K-1$$

$$\frac{1}{T}\sum_t (y_t - x_t'\hat{\beta})\, z_{Kt} = 0$$

The solution of the linear equation system – with $z_t' = (x_{0t}', z_{Kt})$ – is

$$\hat{\beta} = \left(\sum_t z_t x_t'\right)^{-1} \sum_t z_t y_t$$

Identification requires that the *K*x*K* matrix $\Sigma_t z_t x_t'$ is finite and invertible; instrument *z* is relevant when this is fulfilled

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Calculation of IV Estimators

In matrix notation, the model is

$$y = X\beta + \varepsilon$$

The IV estimator is

$$\hat{\beta} = \sum \ldots^{-1} \sum \ldots = (\ldots)^{-1} \ldots$$

with z_t obtained from x_t by substituting values of the instrumental variable(s) for endogenous regressors

Calculation in two steps:

1. Regression of the explanatory variables $x_1, \ldots, x_K$ – including the endogenous ones – on the columns of Z gives fitted values

$$\hat{X} = (Z \ldots)^{-1} Z \ldots$$

2. Regression of y on the fitted explanatory variables gives

$$\hat{\beta} = (\ldots)^{-1} \hat{X} \ldots$$

Calculation of IV Estimators, cont'd

Remarks:

- The KxK matrix $Z'X = \Sigma_t z_t x_t'$ is required to be finite and invertible
- From

$$\hat{\beta} = (\hat{X}'\hat{X})^{-1}\hat{X}'y = (X'Z(Z'Z)^{-1}Z'X)^{-1}X'Z(Z'Z)^{-1}Z'y$$
$$= (Z'X)^{-1}Z'Z(X'Z)^{-1}X'Z(Z'Z)^{-1}Z'y = (Z'X)^{-1}Z'y$$

  it is obvious that the estimator obtained in the second step is the IV estimator
- However, the estimator obtained in the second step is more general; see below

Choice of Instrumental Variables

Instrumental variable are required to be

- exogenous, i.e., uncorrelated with the error terms
- Relevant, i.e., correlated with the endogenous regressors

Choice must be based on subject matter arguments, e.g., arguments from economic theory

Often not easy

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Example: Returns to Schooling

Wage equation with x_{2i}: years of schooling, and u_i: abilities (intelligence, family background, etc. ; unobservable)

$$y_i = x_{1i}'\beta_1 + x_{2i}\beta_2 + u_i\gamma + v_i$$

Empirically, more education implies higher income

Question: Is this effect causal?

- If yes, one year more at school increases wage by β_2
- Otherwise, abilities may cause higher income and also more years at school

Model for analysis:

$$y_i = x_i'\beta + \varepsilon_i \quad \text{with } \varepsilon_i = u_i\gamma + v_i$$

x_2 with $E\{x_{2i}u_i\} \neq 0$ is endogenous: OLS estimators b are inconsistent ; "ability bias"

Returns to Schooling: Data

- National Longitudinal Survey of Young Men (Card, 1995)
- Data from 3010 males, survey 1976
- Individual characteristics, incl. experience, race, region, family background etc.
- Human capital function

  $$\log(wage_i) = \beta_1 + \beta_2\, ed_i + \beta_3\, exp_i + \beta_3\, exp_i^2 + \varepsilon_i$$

  with *ed*: years of schooling, *exp*: years of experience
- Further explanatory variables: *black*: dummy for afro-american, *smsa*: dummy for living in metropolitan area, *south*: dummy for living in the south

Returns to Schooling: OLS Estimation

OLS estimated wage function : Output from GRETL

Model 2: OLS, using observations 1-3010
Dependent variable: l_WAGE76

	Koeffizient	Std.-fehler	t-Quotient	P-Wert
const	4.73366	0.0676026	70.02	0.0000 ***
ED76	0.0740090	0.00350544	21.11	2.28e-092 ***
EXP76	0.0835958	0.00664779	12.57	2.22e-035 ***
EXP762	-0.00224088	0.000317840	-7.050	2.21e-012 ***
BLACK	-0.189632	0.0176266	-10.76	1.64e-026 ***
SMSA76	0.161423	0.0155733	10.37	9.27e-025 ***
SOUTH76	-0.124862	0.0151182	-8.259	2.18e-016 ***

Mean dependent var	6.261832	S.D. dependent var	0.443798
Sum squared resid	420.4760	S.E. of regression	0.374191
R-squared	0.290505	Adjusted R-squared	0.289088
F(6, 3003)	204.9318	P-value(F)	1.5e-219
Log-likelihood	-1308.702	Akaike criterion	2631.403
Schwarz criterion	2673.471	Hannan-Quinn	2646.532

Returns to Schooling: Instrumental Variables

Instrumental variable

- Factors which affect schooling but is uncorrelated with error terms, in particular with unobserved abilities that are determining wage
- Costs of schooling, e.g., distance to school, number of siblings; parents' education; quarter of birth

General remarks:

- The choice of instruments that should be explained and motivated
- Models that explain endogenous regressors from exogenous regressors and instruments (Verbeek: reduced form) should show significant effect of the instruments
- Number of instruments can be larger than K

Returns to Schooling: Step 1 of IV Estimation

Generation of instruments: Output from GRETL

Model 3: OLS, using observations 1-3010
Dependent variable: ED76

	coefficient	std. error	t-ratio	p-value	
const	-1.81870	4.28974	-0.4240	0.6716	
AGE76	1.05881	0.300843	3.519	0.0004	***
sq_AGE76	-0.0187266	0.00522162	-3.586	0.0003	***
BLACK	-1.46842	0.115245	-12.74	2.96e-036	***
SMSA76	0.841142	0.105841	7.947	2.67e-015	***
SOUTH76	-0.429925	0.102575	-4.191	2.85e-05	***
NEARC4A	0.441082	0.0966588	4.563	5.24e-06	***

Mean dependent var	13.26346	S.D. dependent var	2.676913
Sum squared resid	18941.85	S.E. of regression	2.511502
R-squared	0.121520	Adjusted R-squared	0.119765
F(6, 3003)	69.23419	P-value(F)	5.49e-81
Log-likelihood	-7039.353	Akaike criterion	14092.71
Schwarz criterion	14134.77	Hannan-Quinn	14107.83

Returns to Schooling: Step 2 of IV Estimation

Generation of IV estimates: Output from GRETL

Model 4: OLS, using observations 1-3010
Dependent variable: l_WAGE76

	coefficient	std. error	t-ratio	p-value	
const	5.59706	0.377002	14.85	3.61e-048	***
ed_hat3	0.0294025	0.0288738	1.018	0.3086	
EXP76	0.0517741	0.00710229	7.290	3.95e-013	***
EXP762	-0.00209097	0.000353110	-5.922	3.55e-09	***
BLACK	-0.221608	0.0458602	-4.832	1.42e-06	***
SMSA76	0.172057	0.0317598	5.417	6.52e-08	***
SOUTH76	-0.135733	0.0215137	-6.309	3.22e-010	***

Mean dependent var	6.261832	S.D. dependent var	0.443798
Sum squared resid	482.7218	S.E. of regression	0.400932
R-squared	0.185474	Adjusted R-squared	0.183847
F(6, 3003)	113.9681	P-value(F)	6.5e-130
Log-likelihood	-1516.471	Akaike criterion	3046.943
Schwarz criterion	3089.011	Hannan-Quinn	3062.072

Returns to Schooling: TSLS Estimation

Generation of IV estimates: Output from GRETL

Model 8: TSLS, using observations 1-3010
Dependent variable: l_WAGE76
Instrumented: ED76 EXP76 EXP762
Instruments: const AGE76 sq_AGE76 BLACK SMSA76 SOUTH76 NEARC4A

	coefficient	std. error	t-ratio	p-value	
const	3.69771	0.495136	7.468	8.14e-014	***
ED76	0.164248	0.0419547	3.915	9.04e-05	***
EXP76	0.0445876	0.0255932	1.742	0.0815	*
EXP762	-0.000195255	0.00131101	-0.1489	0.8816	
BLACK	-0.0573333	0.0645713	-0.8879	0.3746	
SMSA76	0.0793715	0.0422150	1.880	0.0601	*
SOUTH76	-0.0836975	0.0261426	-3.202	0.0014	***

Mean dependent var	6.261832	S.D. dependent var	0.443798
Sum squared resid	577.9991	S.E. of regression	0.438718
R-squared	0.195884	Adjusted R-squared	0.194277
F(6, 3003)	126.2821	P-value(F)	8.9e-143

Returns to Schooling: Summary of Estimates

Estimated regression coefficients and *t*-statistics

1) The model differs from that used by Verbeek

	OLS	IV[1)]	TSLS	IV (M.V.)
ed76	0.0740	0.0294	0.1642	0.1329
	21.11	1.02	3.92	2.59
exp76	0.0836	0.0518	0.0446	0.0560
	12.75	7.29	1.74	2.15
exp762	-0.0022	-0.0021	-0.0002	-0.0008
	-7.05	-5.92	-0.15	-0.59
black	-0.1896	-0.2216	-0.0573	-0.1031
	-10.76	-4.83	-0.89	-1.33

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Arbitrary Number of Instruments

Moment conditions

$$E\{\varepsilon_i x_i\} = E\{(y_i - x_i'\beta) z_i\} = 0$$

one equation for each component of z_i

General case: R moment conditions

Substitution of expectations by sample averages gives

$$\frac{1}{N}\sum_i (y_i - x_i'\hat{\beta}) z_i = 0$$

1. $R < K$: infinite number of solutions, not enough instruments; unidentified model
2. $R = K$: one unique solution, the IV estimator; identified model

$$\hat{\beta} = \left(\sum_i z_i x_i'\right)^{-1} \sum_i z_i y_i$$

Generalized IV (GIV) Estimator

3. *R* > *K*: more instruments than necessary for identification; overidentified model

   Mimimizing the quadratic form in the sample moments

$$Q_N(\hat\beta) = \left(\frac{1}{N}\sum_i z_i(y_i - x_i'\hat\beta)\right)' W_N \left(\frac{1}{N}\sum_i z_i(y_i - x_i'\hat\beta)\right)$$

   with a *RxR* positive definite weighting matrix W_N gives the generalized instrumental variable (GIV) estimator

$$\hat\beta_{IV} = \left(X'Z(Z'Z)^{-1}Z'X\right)^{-1} X'Z(Z'Z)^{-1}Z'y = \left(\hat X'X\right)^{-1}\hat X'y$$

   "two stage least squares (TSLS) estimator"

The optimal weighting matrix correspond to the most efficient IV estimator

Instrumental Variables: Remarks

The instrumental variables are required to be

- exogenous, i.e., uncorrelated with the error terms
- relevant, i.e., correlated with the regressors that they are supposed to be instrumenting, not linear combinations of regressors

First step of the TSLS procedure: the (endogenous) variables are regressed on the instrumental variables (reduced form regression)

- The instruments for explaining x_i should be "sufficiently important"; check the *t*-statistics
- "Weak instruments": if instruments correlate only weakly with the endogenous regressor, the IV estimator may be biased, have large standard error, bad approximation to normal distribution

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- An Example
- The GIV Estimator
- The Generalized Method of Moments
- Some Tests

Generalized Method of Moments (GMM)

GMM generalizes the IV estimation concept to parameters of models which are not necessarily linear

The model is characterized by R moment conditions

$$E\{f(w_t, z_t, \theta)\} = 0$$

- f: R-vector function
- w_t: vector of observable variables, exogenous or endogenous
- z_t: vector of instrument variables
- θ: K-vector of unknown parameters

Example: For linear model $y_t = x_t'\beta + \varepsilon_t$, $w_t' = (y_t, x_t')$

GMM Estimator

Substitution of the moment conditions by sample equivalents:

$g_T(\theta) = (1/T)\ \Sigma_t\ f(w_t, z_t, \theta) = 0$

1. $R = K$: solve for θ to derive a unique consistent estimator
2. $R > K$: minimization wrt θ of the quadratic form

   $$Q_T(\theta) = g_T(\theta)'\ W_T\ g_T(\theta)$$

   with the positive definite weighting matrix W_T

   GMM estimator corresponds to the optimal weighting matrix

   $$W_T^{opt} = \{\Sigma_t f(w_t, z_t, \hat{\theta}) f(w_t, z_t, \hat{\theta})'\}^{-1}$$

   and is the most efficient estimator

   For a nonlinear $f(.)$, W_T depends of θ; iterative optimization algorithms

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Some Tests

For testing

- Exogeneity of regressors: Hausman-Wu test
- Suitability of variables to be used as instrumental variable: overidentifying restrictions or Sargan test

Hausman-Wu Test

For testing whether one or more regressors are endogenous (correlated with the error term)

Based on the assumption that the instruments are valid; i.e., given that E{ε_i z_i} = 0, E{$\varepsilon_i x_i$} = 0 can be tested

The idea of the test:

- Under the null, both the OLS and IV estimator are consistent; they should differ by sampling error only

Hausman-Wu Test, cont'd

Hausman–Wu is testing whether the residuals v_i from the reduced form equation of potentially endogenous regressors contribute to explaining

$$y_i = x_{1i}'\beta_1 + x_{2i}\beta_2 + v_i\gamma + \varepsilon_i$$

- the OLS estimators for β_1 and β_2 are the IV estimators
- $\gamma = 0$: x_{2i} is exogenous

For testing the null hypothesis:

- *t*-test of H_0: $\gamma = 0$
- *F*-test if more than 1 regressors are tested for exogeneity

Attention! Test has little power if instruments are weak or unsuitable

Sargan Test

For testing whether the instruments are valid

The validity of the instruments requires that all moment conditions are fulfilled; the *R* values of the sums

$$\frac{1}{N}\sum_i e_i z_i =$$

must be close to zero

Test statistic

$$\xi = N Q_N(\hat{\beta}) = \left(\sum_i e_i z_i\right)' \left(\hat{\sigma}^2 \sum_i z_i z_i'\right)^{-1} \left(\sum_i e_i z_i\right)$$

has under the null hypothesis an asymptotic Chi-squared distribution with *R-K* df

Sargan Test, cont'd

Remarks

- Only *R*-*K* of the *R* moment conditions are free on account of the first order conditions of the minimization problem
- The test is also called *overidentifying restrictions test*
- Rejection implies: the joint validity of all moment conditions and hence of all instruments is not acceptable
- The Sargan test gives no indication which instruments are invalid
- Test whether a subset of R-R_1 instruments is valid; R_1 (>*K*) instruments are out of doubt:
  - Calculate ξ for all *R* moment conditions
  - Calculate ξ_1 for the R_1 moment conditions
  - Under H_0, $\xi - \xi_1$ has a Chi-squared distribution with R-R_1 df

Exercise

1. Answer questions a. to e. of Exercise 5.2 of Verbeek.