Chapter 14

RESUSCITATING REAL BUSINESS CYCLES *

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Contents

Abstract

The Real Business Cycle (RBC) research program has grown spectacularly over the last decade, as its concepts and methods have diffused into mainstream macroeconomics. Yet, there is increasing skepticism that technology shocks are a major source of business fluctuations. This chapter exposits the basic RBC model and shows that it requires large technology shocks to produce realistic business cycles. While Solow residuals are sufficiently volatile, these imply frequent technological regress. Productivity studies permitting unobserved factor variation find much smaller technology shocks, suggesting the imminent demise of real business cycles. However, we show that greater factor variation also dramatically amplifies shocks: a RBC model with varying capacity utilization yields realistic business cycles from small, nonnegative changes in technology.

Keywords

1. Introduction

Business cycle research studies the causes and consequences of the recurrent expansions and contractions in aggregate economic activity that occur in most industrialized countries. Over the last century, exploration of real business cycles – the idea that economic fluctuations are caused primarily by real factors – has itself undergone periods of intense activity and relative dormancy. In the 1920s, real theories played a leading role: economists sought to use new microeconomic tools to learn about the aggregate consequences of shifts in demand and supply of goods and productive factors. However, the Great Depression of the 1930s had a dramatic effect on business cycle research. Economists began to believe that microeconomic theory was an inadequate basis for understanding business cycles. Real factors came to be less stressed, with greater weight given to monetary conditions and the psychology of households and firms. Government management of the economy came to be seen as not only desirable but essential.

The rise of Keynesian macroeconomics to a position of orthodoxy in aggregate economics meant that it took half a century for a revival of interest in equilibrium business cycle models. The breakdown in the performance of macroeconometric models in the 1970s and the associated rational expectations revolution pioneered by Lucas (1976) set the stage for a vigorous recovery, since the logic of rational expectations ultimately required general equilibrium analysis[1]. Kydland and Prescott (1982) and Long and Plosser (1983) first strikingly illustrated the promise of this approach, suggesting that one could build a successful business cycle model that involved market clearing, no monetary factors and no rationale for macroeconomic management. It is now perhaps hard to recall that this idea was met with surprise and disbelief.

By the end of the 1980s there was a central and controversial finding of real business cycle (RBC) research, as this line of work came to be called. Simple equilibrium models, when driven by shifts in total factor productivity measured using Solow's (1957) growth accounting approach, could generate time series with the same complex patterns of persistence, comovement and volatility as those of actual economies. Writing a survey of RBC research at that time, it was difficult to find sufficient material so we settled for expositing the basic model and forecasting future developments[2]. A decade later, our task in this chapter is substantially different: it is time to take stock of a decade of research, to assess criticisms, and to evaluate the health of the research program.

The first observation is that it has been a decade of spectacular growth: so many theoretical and empirical articles use the RBC approach that a full bibliography would

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[1] Sargent (1982).

[2] King, Plosser and Rebelo (1988a,b) surveyed this area when a single conference program could include most participants in the RBC research program.

likely exhaust the generous page constraint on our contribution to this volume.[3] Real business cycle analysis now occupies a major position in the core curriculum of nearly every graduate program.[4] At a recent NBER conference, a prominent Cambridge economist of the New Keynesian school described the RBC approach as the new orthodoxy of macroeconomics, without raising a challenge from the audience.

Continuing on the positive side of the ledger, the methods of the RBC research program are now commonly applied, being used in work in monetary economics, international economics, public finance, labor economics, asset pricing and so on. In contrast to early RBC studies, many of these model economies involve substantial market failure, so that government intervention is desirable. In others the business cycle is driven by shocks to the monetary sector or by exogenous shifts in beliefs. The dynamic stochastic general equilibrium model is firmly established as the laboratory in which modern macroeconomic analysis is conducted.

At the same time, there has been increasing concern about the mechanism at the core of standard RBC models: the idea that business cycles are driven mainly by large and cyclically volatile shocks to productivity, which in turn are well represented by Solow residuals as in the provocative study of Prescott (1986). A key difficulty is that typical estimates of Solow residuals imply a probability of technical regress on the order of 40%, which seems implausible to most economists. Recent studies have corrected the Solow residual for mismeasurement of inputs – notably, unobserved effort and capacity utilization – and inappropriate assumptions about market structure. These remeasurements have produced technology shocks with more plausible properties: notably, productivity growth is much less likely to be negative. In effect, these studies have caused productivity shocks to grow smaller and less cyclically volatile by introducing elements which respond sympathetically to economic activity [see, for example, Burnside, Eichenbaum and Rebelo (1996)].

Since the standard RBC model requires large and volatile productivity shocks, this remeasurement research is typically interpreted as indicating that our chapter should be a first draft of the obituary of the RBC research program. In fact, most of our survey does read like a chronicle of the life and death of RBC models. We begin in Section 2 by discussing the measurement of the business cycle as well as reviewing facts about growth and business cycles that have motivated the construction of aggregate models. We next turn in Section 3 to the basic neoclassical model of capital accumulation, as initially developed by Solow (1956) and others for the purpose of studying economic growth but now used more widely in the study of aggregate economic activity. We then celebrate the early victories of the RBC program in Section 4 and discuss early criticisms.

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[3] One valuable monitor of this ever-expanding literature is provided by Christian Zimmermann's web page (http://ideas.uqam.ca/QMRBC/index.html).

[4] One manifestation of the breadth of this intellectual impact is that Hall (1999) cites Berkeley's David Romer (1996) and Harvard's John Campbell (1994) for authoritative presentations of the basic RBC model.

There has been a substantial amount of research on real business cycles, but we organize our discussion around three main points in the next three sections. The central role of large and persistent productivity shocks in the basic model is discussed in Section 5. Important improvements to the basic RBC framework are highlighted in Section 6. The remeasurement of productivity shocks, which has fostered concern about the health of real business cycles, is reviewed in Section 7.

In Section 8 we argue that the incipient demise of the real business cycle is hardly as likely as suggested by conventional wisdom. In fact, rather than weaken the case for a real theory of the cycle, our view is that the recent remeasurement of productivity actually strengthens it. To make this point, Section 8 describes a very simple variant of the basic RBC model, but one that is different on two key dimensions. The economy has indivisible labor, which is one of the key improvements reviewed in Section 6, and has costly variation in capital utilization, which is one of the structural features that makes the standard Solow residual depart from productivity. There is a substantial remeasurement of productivity shocks mandated by this economy: when we do the necessary correction, the standard deviation of productivity growth drops to less than one-fifth of the standard deviation of the growth rate of the Solow residual. Productivity regress occurs in less than 1% of the post-war quarterly observations, even though the measured Solow residual shrinks 37% of the time. Yet these small shocks can generate empirically reasonable business cycles because our model features substantial amplification of productivity shocks: readily variable capital utilization and a highly elastic labor supply lead small changes in productivity to have major effects on macroeconomic activity. When we drive our model with such small measured productivity shocks, there is a remarkable coincidence between actual US business cycles and simulated time paths of output, consumption, investment, and labor input. The same structural features that lead the Solow residual to dramatically overstate productivity fluctuations also lead the economy to greatly amplify productivity shocks.

2. Stylized facts of aggregate activity

In the 1930s, Burns and Mitchell began to document the existence of a remarkable set of business cycle regularities. This research program culminated in their 1946 treatise on *Measuring Business Cycles*. Burns and Mitchell's arcane methodology led many economists to view their findings with skepticism [see e.g. Koopmans (1947)] and their methods fell into disuse[5]. But when Hodrick and Prescott (1980) employed modern time series tools to re-examine the empirical regularities of the business cycle, they

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[5] However, there is some recent interest in these methods. Watson (1994) uses the Burns and Mitchell methodology to contrast inter-war and post-war US business cycles. Simkins (1994) and King and Plosser (1994) show that RBC models produce artificial data that the Burns and Mitchell methods would recognize as having similar characteristics to US data.

found the Burns-Mitchell facts intact, lurking underneath almost half a century of accumulated dust. As stressed by Lucas (1977), the finding that "business cycles are all alike" suggested that the nature of macroeconomic fluctuations does not hinge on institutional factors or country-specific idiosyncrasies, so that one can hope to construct a unified theory of the business cycle.

2.1. Measuring business cycles with the HP filter

Most real quantities, such as US real national output in the top panel of Figure 1, grow through time. Hence, the statistical measurement of business cycles necessarily involves some way of making the series stationary, which is most commonly done by the removal of a secular trend. In their study of quarterly post-war US data, Hodrick and Prescott (1980) detrended their variables using a procedure now widely known as the HP filter. In essence, this method involves defining cyclical output y_t^c as current output y_t less a measure of trend output y_t^g, with trend output being a weighted average of past, current and future observations:

$$y_t^c = y_t - y_t^g = y_t - \sum_{j=-J}^{J} a_j y_{t-j}. \tag{2.1}$$

Figure 1 displays how cyclical output is constructed. In the first panel, the logarithm of current output is the more variable series and trend output is the smoother series. The HP cyclical component of output is the dotted line in the second panel, defined from the elements of the first panel as $y_t^c = y_t - y_t^g$ except that we have multiplied by 100 so that cyclical output is a percentage[6]. Aggregate output displays business cycles in that there are alternating periods of high and low output, but these episodes are of unequal duration and amplitude.

To see how the cyclical output measure produced by the HP filter compares with those from other detrending methods, we can look at the second and third panels of

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[6] The HP filter is derived by solving the following minimization problem:

$$\min_{\{y_t^g\}_{t=0}^{\infty}} \sum_{t=1}^{\infty} \left\{ (y_t - y_t^g)^2 + \lambda \left[(y_{t+1}^g - y_t^g) - (y_t^g - y_{t-1}^g) \right]^2 \right\}.$$

For quarterly data, the standard value chosen for the smoothing parameter λ is 1600. When $\lambda = \infty$ the solution to this problem is a linear trend, while with $\lambda = 0$ the trend coincides with the original series. In a finite sample context, the weights a_j in Equation (2.1) depend on the length of the sample, so that the text expression is a simplification. King and Rebelo (1993) discuss additional properties of this detrending procedure, including derivation of filter weights and frequency response functions for the case in which the sample is infinitely large. They establish that the HP filter has strong detrending properties, in the sense that it can make stationary series up through four orders of integration.

Output and HP Trend

HP and Band Pass (6,32) Cyclical Components

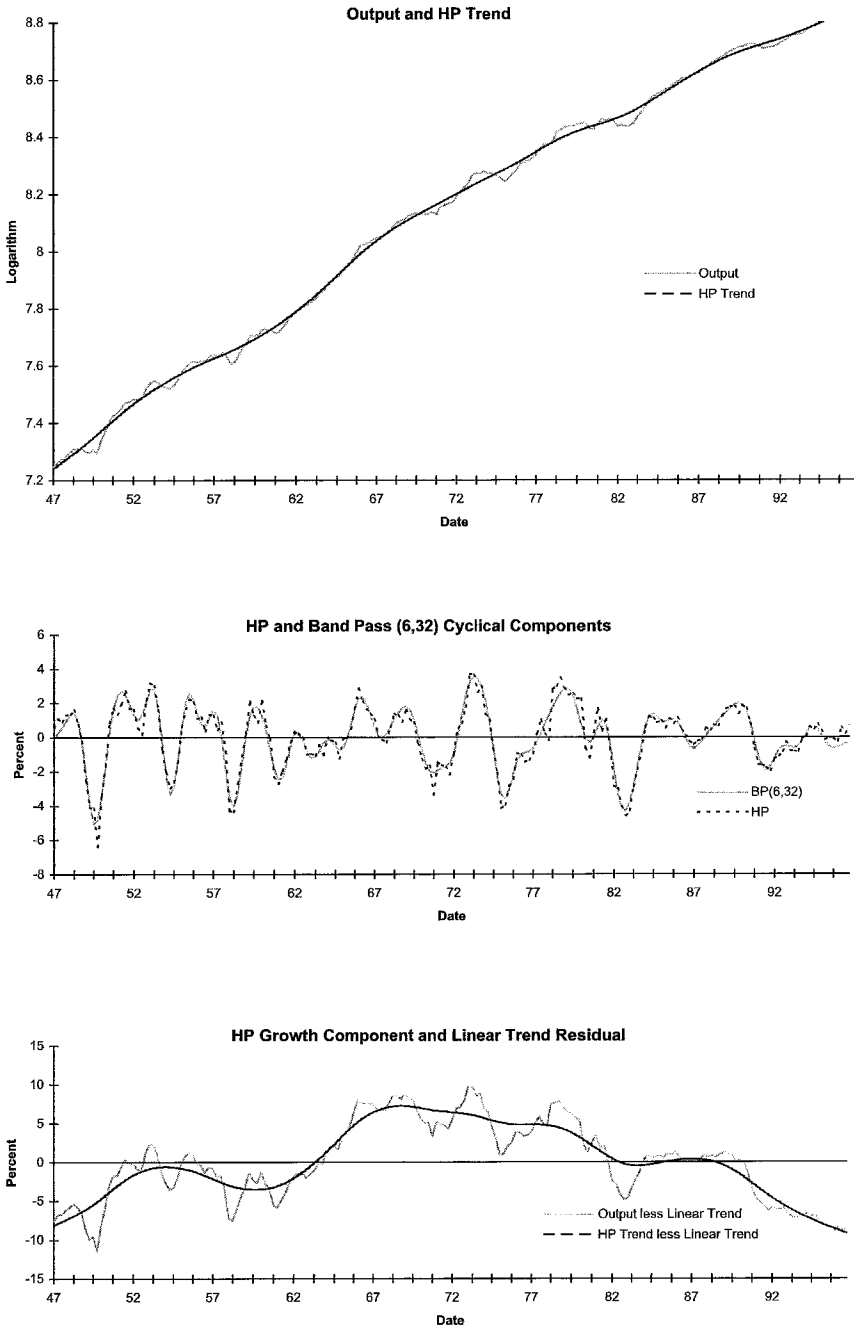**HP Growth Component and Linear Trend Residual**

Fig. 1. Trend and business cycle in US real output. Sample period is 1947:1–1996:4.

Figure 1. First, if we simply subtract a linear trend from the log of output, the resulting business cycle component would be the solid line in the third panel of Figure 1. This alternative measure of cyclical output is much more persistent than the HP measure: for example, output is high relative to its linear trend during most of the 1960s and 1970s. The dashed line in the third panel is the gap between the HP trend and a linear trend, thus indicating that the HP filter extracts much more low frequency information than a simple linear trend.

It is also useful to compare the HP cyclical component with the business cycle measures resulting from the band-pass (BP) filter procedure developed by Baxter and King (1995), since such measures are presented by Stock and Watson (1999) in their extensive compilation of business cycle facts elsewhere in this volume[7]. In the second panel of Figure 1, the HP measure of cyclical output is accompanied by a BP measure of cyclical output: this procedure makes the cyclical component mainly those parts of output with periodicities between 6 and 32 quarters. For series like output, which contain relatively little high frequency variation, Figure 1 shows that there is a minor difference between these alternative cyclical measures.

There has been some controversy about the suitability of the HP filter for business cycle research. Prescott (1986) notes that the HP filter resembles an approximate high-pass filter designed to eliminate stochastic components with periodicities greater than thirty-two quarters. Adopting that perspective, we are simply *defining* the business cycle in a fairly conventional way: it is those fluctuations in economic time series that have periodicity of eight years or less[8]. At the same time, the third panel of Figure 1 reminds us that there are slow-moving stochastic components of economic time series omitted by this definition, which may have substantial positive and normative significance.

2.2. Some stylized facts of US business cycles

Making some selections from the data set that Stock and Watson (1999) investigate more extensively, we apply the HP filter to produce cyclical components for key US macroeconomic variables[9]. Figures 2, 3 and 4 provide graphs of the HP business cycle components of major US aggregates. We use the cyclical component of output as a reference variable, placing it in each panel of each figure, so as to allow the reader to

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[7] Since an exact bandpass filter contains an infinite number of moving average terms, a practical bandpass filter cannot be produced exactly but involves approximations. Baxter and King (1995) derive the relevant formulas, imposing the constraint that the sum of the filterweights must be zero; they also compare the BP filter to several other detrending methods including the HP filter.

[8] Eight years corresponds to the longest reference cycle that Burns and Mitchell (1946) uncovered using very different methods. Stock and Watson (1999) adopt an alternative view of the interesting business cycle periodicities (six to twenty four quarters), but this is a difference of degree rather than kind.

[9] This data set covers the period 1947 (first quarter) to 1996 (fourth quarter).

Fig. 2. Cyclical components of US expenditures. Sample period is 1947:1–1996:4. All variables are detrended using the Hodrick–Prescott filter.

Fig. 3. Cyclical component of US factors of production. Sample period is 1947:1–1996:4. All variables are detrended using the Hodrick–Prescott filter.
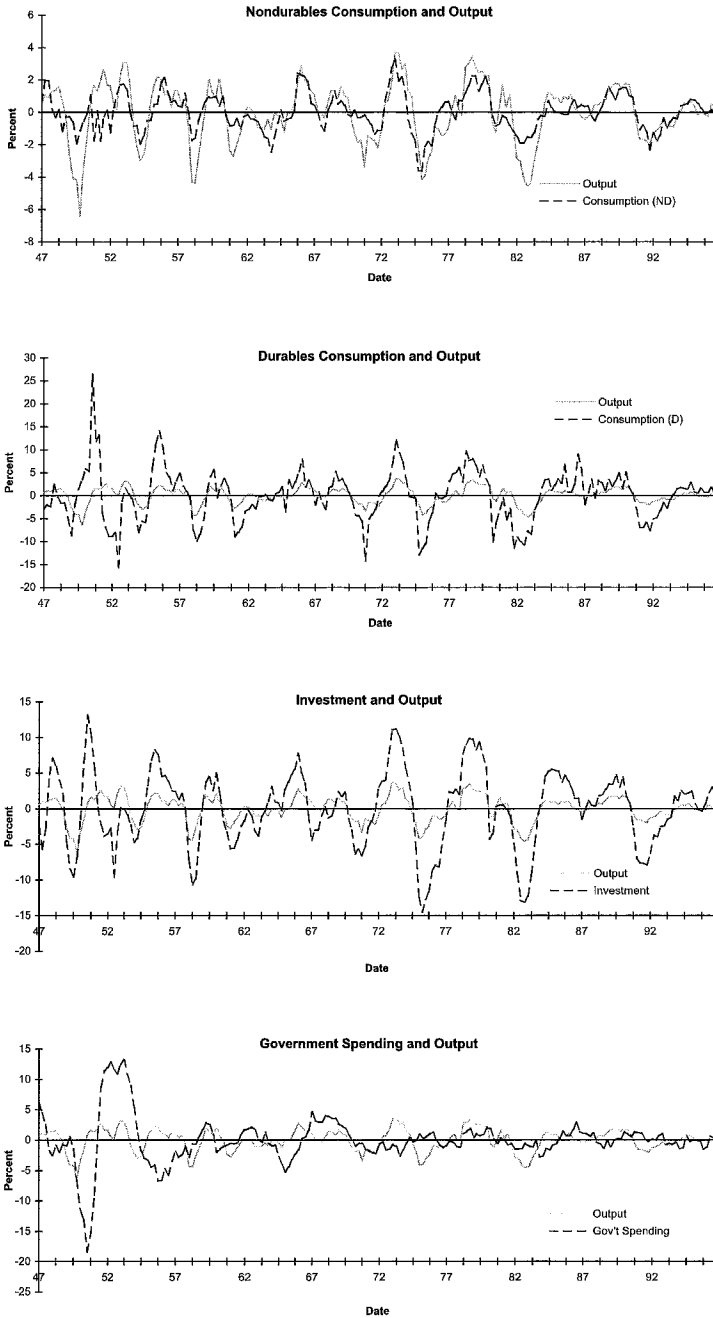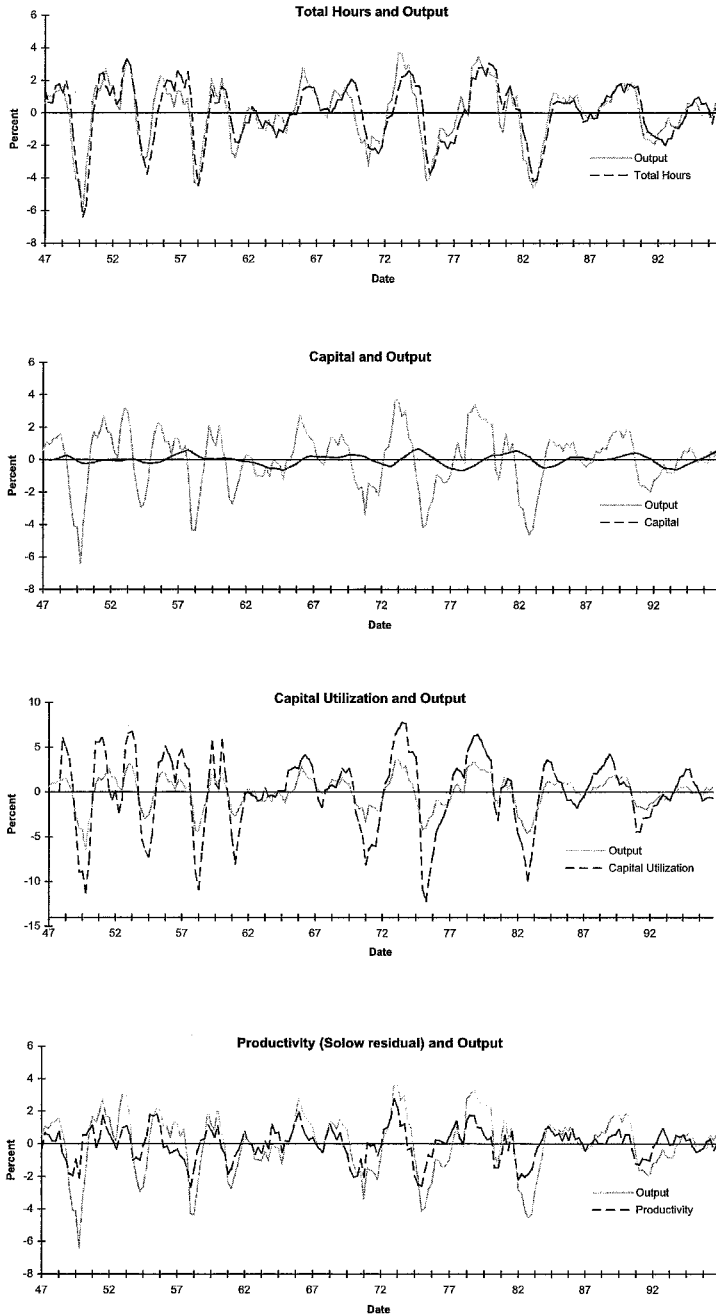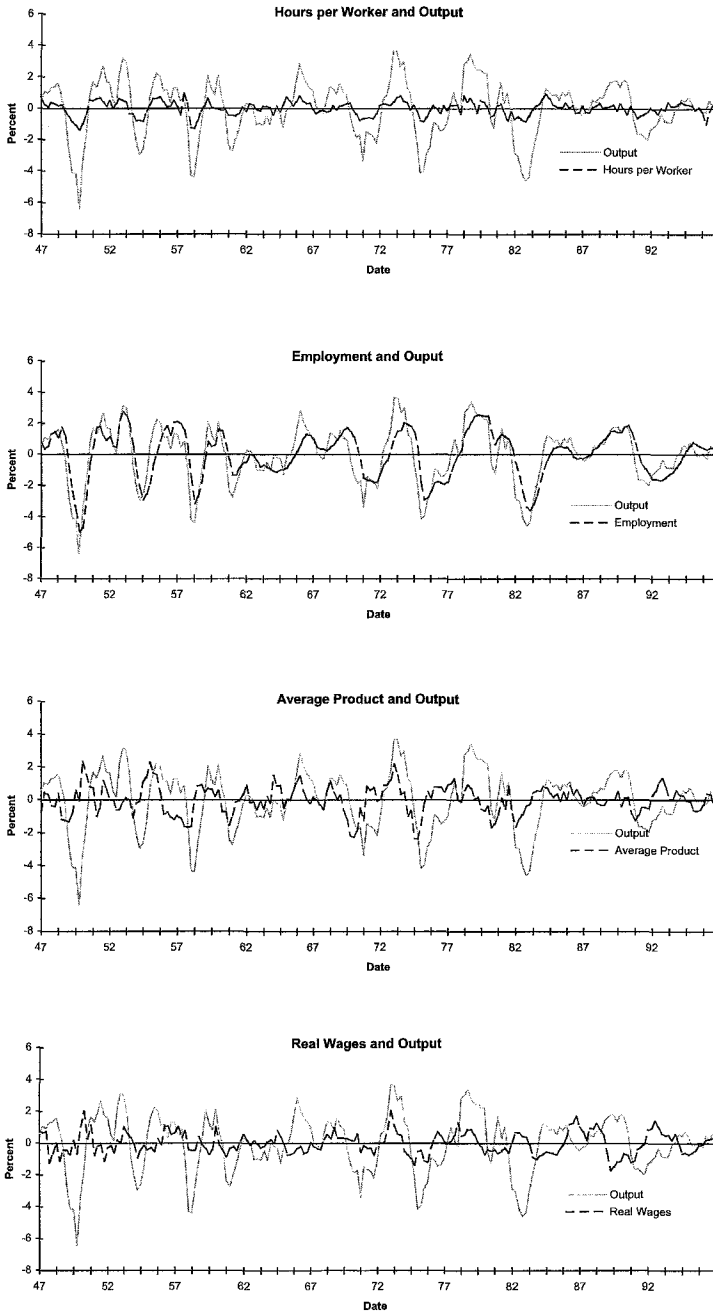
Fig. 4. Cyclical component of US labor market measures. Sample period is 1947:1–1996:4. All variables are detrended using the Hodrick–Prescott filter.

Table 1
Business cycle statistics for the US Economy

	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	−0.35
A	0.98	0.54	0.74	0.78

[a] All variables are in logarithms (with the exception of the real interest rate) and have been detrended with the HP filter. Data sources are described in Stock and Watson (1999), who created the real rate using VAR inflation expectations. Our notation in this table corresponds to that in the text, so that Y is per capita output, C is per capita consumption, I is per capita investment, N is per capita hours, w is the real wage (compensation per hour), r is the real interest rate, and A is total factor productivity.

easily gauge the relative volatility of the series in question and its comovement with output. Summary statistics for selected series are provided in Table 1 [10].

Volatility: Economists have long been interested in understanding the economic mechanisms that underlie the different volatilities of key macroeconomic aggregates. The facts are as follows, working sequentially within each figure and using the notation panel 2-1 to denote panel 1 of Figure 2 and so forth:
• Consumption of non-durables is less volatile than output (panel 2-1);
• Consumer durables purchases are more volatile than output (panel 2-2);
• Investment is three times more volatile than output (panel 2-3);
• Government expenditures are less volatile than output (panel 2-4);
• Total hours worked has about the same volatility as output (panel 3-1);
• Capital is much less volatile than output, but capital utilization in manufacturing is more volatile than output (panels 3-2 and 3-3) [11];
• Employment is as volatile as output, while hours per worker are much less volatile than output (panels 4-1 and 4-2), so that most of the cyclical variation in total hours worked stems from changes in employment;
• Labor productivity (output per man-hour) is less volatile than output (panel 4-3);

[10] For a discussion of open economy stylized facts see Baxter and Stockman (1989), Backus and Kehoe (1992) and Baxter (1995).
[11] This measure of capacity utilization, constructed by the Federal Reserve System, is subject to substantial measurement error, see Shapiro (1989).

• The real wage rate is much less volatile than output (panel 4-4).

Comovement: Figures 2 through 4 show that most macroeconomic series are procyclical, that is, they exhibit a positive contemporaneous correlation with output. The high degree of comovement between total hours worked and aggregate output, displayed in panel 3-1, is particularly striking. Three series are essentially acyclical – wages, government expenditures, and the capital stock – in the sense that their correlation with output is close to zero [12].

Persistence: All macroeconomic aggregates display substantial persistence; the first-order serial correlation for most detrended quarterly variables is on the order of 0.9. This high serial correlation is the reason why there is some predictability to the business cycle.

In presenting these business cycle facts, we are focusing on a small number of empirical features that have been extensively discussed in recent work on real business cycles. For example, in the interest of brevity, we have not discussed the lead-lag relations between our variables. In choosing the series to study, we have also left out nominal variables, whose cyclical behavior is at the heart of many controversies over the nature of business cycles [13]. However, we do report the cyclical behavior of a measure of the expected real rate of interest from Stock and Watson (1999) in Table 1. This real interest rate is constructed by subtracting a forecast of inflation from the nominal interest rate on US treasury bills. There is a negative correlation of the real interest rate with real output contemporaneously and, indeed, this negative relationship is even stronger between real output and lagged real interest rates. Many modern macroeconomic models, including real business cycle models, have difficulty matching this feature of business cycles [14].

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[12] The observation that the real wage is not tightly related to the business cycle goes back to Dunlop (1938) and Tarshis (1939) who stressed that this was at odds with Keynesian models. This finding is somewhat dependent on precisely how the real wage is constructed, depending on whether the numerator (the wage) includes various compensation items and on the index in the denominator (the price level). Two particular features of wage measurement that affect its cyclical behavior are as follows. First, firms pay for overtime hours in an expansion and layoff regular hours in a recession. Second, there is a cyclical composition bias in the labor force – lower quality workers are hired in expansions – which suggests that the real wage per efficiency unit of labor effort is procyclical.

[13] See Stock and Watson (1999, Sections 3.4, 3.6, and 4.1) for a discussion of literature and empirical results.

[14] King and Watson (1996) find this negative "leading indicator" relationship between the real interest rate and real activity, using BP filtered data. They also show that a number of modern macroeconomic models, including the basic RBC model, are unable to match this fact even when driven by complicated forcing processes that allow them to match most other features of the business cycle. However, while this result is provocative, it is important to stress that the behavior of this real interest rate involves assuming that the inflation forecasting equation is temporally stable and that agents know this forecasting structure in advance.
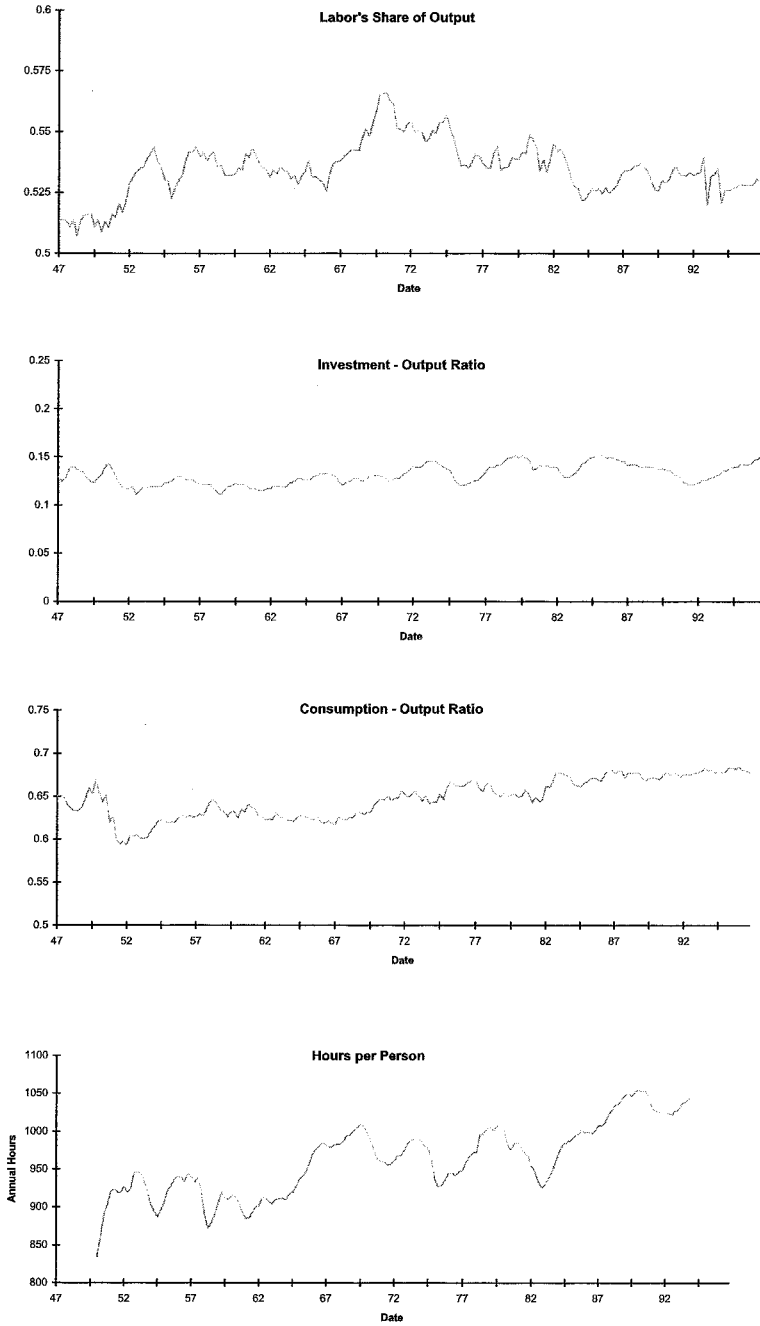
Fig. 5. Growth facts: great ratios and hours per person.

2.3. Some stylized facts of economic growth

While the US time series for many aggregates grow over time, there are many "great ratios" that appear to be relatively constant, suggesting that there are a small number of common forces which give rise to trend growth. As with the systematic patterns of business cycles, this finding is also consistent across many countries and time periods, suggesting that there may be a coherent theoretical explanation of its origin. These stylized facts of economic growth were uncovered as applied researchers such as Kuznets (1973) assembled long time series on economic growth. They are sometimes called the "Kaldor facts" of growth because Kaldor (1957) drew attention to them: in addition to the constancy of great ratios, he stressed that the growth process seemed to involve growth rates and interest rates that were stationary even though the level of economic aggregates were not.

The great ratios. Panels 1–3 of Figure 5 illustrate that the process of sustained growth appears to leave many of the shares of income components and output components relatively unaffected[15]. The ratios of investment to output and labor income to output appear to fluctuate around constant means. The ratio of consumption to output does increase from 1952 onwards, but there is nothing like the large trend that we saw in output in Figure 1. Stability of the great ratios implies that most series have a similar rate of growth, so that there is no deterministic trend in the ratios, and that factors causing permanent changes in the level of economic activity do so in a way that makes their effects proportional across series.

Labor and growth. During long-term economic growth, which most economists believe occurs mainly due to population growth and technical progress, measures of labor input per person are also relatively constant as documented in panel 4 of Figure 5. This relative constancy of hours per capita is remarkable given the rise in real wages that accompanies economic growth. Over our sample period, the real wage measure previously studied in panel 4 of Figure 4 grew at 1.76% per year, but there is little evidence of a trend in hours worked per person.

2.4. Implications of stylized facts

Some of the facts just described have been influential in shaping the views of economists about of how the economy operates. In terms of the business cycle facts, the high volatility of investment no doubt underlies Keynes' famous assertion that investors have "animal spirits". At the same time, the low cyclical volatility of capital is often taken to imply that one can safely abstract from movements in capital in constructing a theory of economic fluctuations. The remarkably high correlation

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[15] Klein and Kosobud (1961) produced an early test of the stability of the "great ratios" in the USA. More recently, King, Plosser, Stock and Watson (1991) drew attention to how the constancy of these great ratios was a cointegration implication about the logarithms of the variables, which they tested for the USA. Evidence on cointegration for other OECD countries is contained in Neusser (1991).

between hours worked and aggregate output has led some economists to believe that understanding the labor market is key to understanding business fluctuations. Finally, the relatively small variability of real wages and the lack of a close correspondence of wages with aggregate output, has led some economists to conclude that the wage rate is not an important allocative signal in the business cycle. The growth facts suggest the importance of building models that feature a common trend in most real aggregates.

3. The basic neoclassical model

In the 1950s and 1960s, aggregate economic activity was analyzed with two very different types of dynamic macroeconomic models. The trend components of aggregate economic activity were studied with "growth models" that stressed three sources of dynamics: population growth, productivity growth and capital formation. The business-cycle components were studied with Keynesian macroeconomic models, which stressed the interaction of consumption and investment but downplayed the importance of capital accumulation and productivity growth. While there were attempts to synthesize these developments towards the end of this period, the study of growth and business cycles most frequently involved very disparate models. Relative to this traditional macroeconomic approach, the real business cycle literature took a very different point of view. Its core is a neoclassical growth model of the form developed by Solow (1956), Cass (1965) and Koopmans (1965). It then follows Brock and Mirman (1972) in making this growth model stochastic, by positing that the production technology is buffeted by random aggregate shocks to productivity.

Our introduction to RBC analysis thus naturally begins by reviewing the "basic neoclassical model", which has implications for both growth and business cycles. In this section, we focus on the structure of the model, trace some of its implications for capital accumulation, and discuss Solow's work on productivity measurement. In the next section, we discuss its business cycle properties.

3.1. The structure

The basic neoclassical model is built on assumptions about preferences, endowments and technology that are designed to capture key features of growth and business cycles, while building a model economy that is readily amenable to economic analysis.

Preferences: The economy is populated by a large number of infinitely lived agents whose expected utility is defined as

$$E_0 \sum_{t=0}^{\infty} b^t u(C_t, L_t), \qquad b > 0, \tag{3.1}$$

where b denotes the discount factor, C_t represents consumption and L_t leisure. The symbol E_0 denotes the expectation of future values of C and L based on the information

available at time zero[16]. The infinite horizon assumption, which greatly simplifies the mathematical analysis of economic growth and business cycles, is usually justified by appealing to the presence of altruistic links across generations [Barro (1974)]. However, it can be viewed as an approximation to an economy with many long-lived agents[17]. In our exposition of this model, we treat the population as constant for simplicity, although we discuss the consequences of relaxing this assumption at various points below.

The momentary utility function $u(C_t, L_t)$ in Equation (3.1) is assumed concave and obeys regularity conditions discussed in the Appendix. It implies a preference for smooth profiles of consumption and leisure. It also implies a willingness to substitute across time if interest rates and wage rates imply differing costs of consumption and leisure at different dates. Thus, the neoclassical model imbeds a form of the permanent income hypothesis of Friedman (1957).

Endowments: The fundamental endowment that individuals have is their time, which can be split between work (N_t) and leisure activities (L_t). Normalizing the total amount of time in each period to one, the time constraint is:

$$N_t + L_t = 1. \tag{3.2}$$

We abstract from other endowments of resources since the production from unimproved land and from nonrenewable resources is a small fraction of output in most developed countries.

Technology: The output of the economy is assumed to depend on a production function that combines labor and capital inputs. To capture the upward trend in output per capita that is shown in Figure 1, the basic neoclassical model incorporates secular improvement in factor productivity. In particular, output (Y_t) depends on the amounts of capital (K_t) and labor (N_t) according to a constant returns to scale production function which satisfies regularity conditions discussed in the Appendix.

$$Y_t = A_t F(K_t, N_t X_t), \tag{3.3}$$

where A_t is a random "productivity shock" variable, whose law of motion will be described further below, and X_t represents the deterministic component of productivity. This latter component of productivity is assumed to expand at a constant rate,

$$X_{t+1} = \gamma X_t, \qquad \gamma > 1. \tag{3.4}$$

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[16] To simplify, we adopt a dating convention that does not distinguish between "planning time" for the individual and "calendar time" for the economy. Alternative presentations that emphasize this distinction would write the objective as $E_t \sum_{j=0}^{\infty} b^j u(C_{t+j}, L_{t+j})$, where t is calendar time and j is planning time.

[17] Rios-Rull (1994) finds that an overlapping generations model calibrated to the age structure of the US population has business cycle properties that are similar to an infinite horizon model.

The output of the economy can be used for consumption or investment (I_t) so that an additional resource constraint is:

$$Y_t = C_t + I_t. \tag{3.5}$$

This equation corresponds to the basic national income accounting identity for a closed economy with no government. The stock of capital evolves according to:

$$K_{t+1} = I_t + (1 - \delta)K_t, \tag{3.6}$$

where δ is the rate of depreciation. This formula coincides with the one used in practice to estimate the stock of capital according to the "perpetual inventory method"[18].

The form of the production function (3.3) is motivated by the growth facts and was widely employed in growth models after Phelps (1966) showed that steady-state growth – a situation in which all variables grow at a constant rate – required that the deterministic component of technology be expressible in labor augmenting form in economies with Equations (3.5) and (3.6)[19]. In fact, in the feasible steady states of this model consumption, investment, output and capital all grow at the *same* rate – the rate of trend technical progress – so that the great ratios are stationary.

Initial conditions: The economy starts out with a capital stock $K_0 > 0$. It also begins with a level of the technology trend $X_0 > 0$, which we set equal to unity for convenience, and an initial productivity shock $A_0 > 0$.

3.2. Steady-state growth and transforming the economy

Our assumptions on the production side of the model ensure that a steady-state path is feasible in the face of the trend productivity expansion in X_t. However, additional assumptions are necessary to make such a steady state desirable. In the standard fixed labor version of the basic neoclassical model momentary utility has to take the form

$$u(C) = \frac{1}{1 - \sigma}[C^{1-\sigma} - 1], \tag{3.7}$$

where $\sigma > 0$. This utility function insures that the marginal rate of substitution between consumption at dates t and $t + 1$ depends only on the growth rate of consumption.

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[18] In practice the perpetual inventory method allows the depreciation rate to vary through time according to empirical measures of economic depreciation schedules. Ambler and Paquet (1994) study a RBC model with depreciation shocks.

[19] Three types of technical progress frequently discussed in the literature can be represented in a general production function:

$$Y_t = X_t^H F(K_t X_t^K, N_t X_t).$$

The variable X_t^H represents total factor augmenting (Hicks-neutral) technical progress, X_t^K capital augmenting technical progress, and X_t labor augmenting (Harrod-neutral) technical progress. When the production function is Cobb–Douglas these different forms of technical progress are interchangeable and, hence, they are all consistent with balanced growth. For all other production functions, the only form of technical progress consistent with steady-state growth is labor augmenting.

In the basic neoclassical model of growth and business cycles, which features endogenous labor supply, a steady state also requires that hours per person be invariant to the level of productivity. King, Plosser and Rebelo (1988a,b) show that the momentary utility function must be expressible as

$$u(c, L) = \frac{1}{1 - \sigma}\{[Cv(L)]^{1-\sigma} - 1\}, \tag{3.8}$$

which also implies exactly offsetting income and substitution effects of wage changes on labor supply[20]. The function $v(\cdot)$ satisfies regularity conditions discussed in the Appendix.

When these restrictions are imposed, it is possible to transform the economy – so that steady-state growth is eliminated – by scaling all of the trending variables by the initial level of X. Using lower case letters to denote these ratios, for example $y_t = Y_t/X_t$, we can then write the optimal growth problem as maximizing the transformed utility function:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \tag{3.9}$$

with $\beta = b\gamma^{1-\sigma}$ being a modified discount factor satisfying $0 < \beta < 1$. Utility is maximized subject to the transformed constraints:

$$N_t = 1 - L_t, \tag{3.10}$$

$$y_t = A_t F(k_t, N_t), \tag{3.11}$$

$$y_t = c_t + i_t, \tag{3.12}$$

$$\gamma k_{t+1} = i_t + (1 - \delta)k_t. \tag{3.13}$$

Relative to an economy in which there is no growth due to X, this transformed economy involves an altered discount factor and a slight modification of the capital accumulation equation. Given this close correspondence, RBC analyses sometimes omit growth all together or simply start with the transformed economy[21].

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[20] That is, suppose that Equation (3.8) is maximized subject to the static budget constraint $C \leqslant w(1 - L)$. The equality of the real wage with the marginal rate of substitution between leisure and consumption implies

$$w = \frac{CDv(L)}{v(L)} = \frac{[w(1 - L)]Dv(L)}{v(L)},$$

with the latter equality following from eliminating consumption using the budget constraint. Changes in w have no effect on the optimal level of leisure and labor supply.

[21] By leaving out population growth, we have essentially proceeded in this manner. However, since productivity is labor-augmenting, we can reinterpret the stationary transformation as one that involves dividing through by both the population and the productivity of labor. Under this interpretation, γ is the growth rate of population and productivity.

Constraints (3.11), (3.12), and (3.13) can be summarized by the equation

$$c_t + \gamma k_{t+1} = A_t F(k_t, N_t) + (1 - \delta) k_t. \tag{3.14}$$

3.3. Optimal capital accumulation

The optimal path of capital accumulation can be obtained by choosing sequences for consumption $\{c_t\}_{t=0}^{\infty}$, leisure $\{L_t\}_{t=0}^{\infty}$, labor $\{N_t\}_{t=0}^{\infty}$ and the capital stock $\{k_{t+1}\}_{t=0}^{\infty}$ to maximize Equation (3.9) subject to conditions (3.10) and (3.14). For this purpose, we form the "Lagrangian"

$$
\begin{aligned}
L = &\sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \\
&+ \sum_{t=0}^{\infty} \beta^t \lambda_t [A_t F(k_t, N_t) + (1 - \delta) k_t - c_t - \gamma k_{t+1}] \\
&+ \sum_{t=0}^{\infty} \beta^t \omega_t [1 - L_t - N_t].
\end{aligned}
$$

The first-order conditions include

$$c_t \quad : \ D_1 u(c_t, L_t) = \lambda_t, \tag{3.15}$$

$$L_t \quad : \ D_2 u(c_t, L_t) = \omega_t, \tag{3.16}$$

$$N_t \quad : \ \lambda_t A_t D_2 F(k_t, N_t) = \omega_t, \tag{3.17}$$

$$k_{t+1} : \ \beta \lambda_{t+1} [A_{t+1} D_1 F(k_{t+1}, N_{t+1}) + 1 - \delta] = \gamma \lambda_t, \tag{3.18}$$

where we use the notation $D_i u(c, L)$ to denote the partial derivative of the function $u(c, L)$ with respect to its ith argument. The first pair of these efficiency conditions dictate that the marginal utility of consumption be set equal to its shadow price [associated with the constraint (3.14)] and that the marginal utility of leisure be set equal to its shadow price (associated with the time constraint $N_t + L_t = 1$). The second pair of efficiency conditions dictate that the utility value of goods produced with a marginal unit of work (the marginal value product $\lambda_t A_t D_2 F(k_t, N_t)$) equal its utility denominated cost (ω_t) and that the present value of the future product of capital ($\beta \lambda_{t+1}[A_{t+1} D_1 F(k_{t+1}, N_{t+1}) + 1 - \delta]$) equal its current utility cost ($\gamma \lambda_t$). An optimal consumption, leisure, work and capital plan – sequences $\{c_t\}_{t=0}^{\infty}$, $\{L_t\}_{t=0}^{\infty}$, $\{N_t\}_{t=0}^{\infty}$, and $\{k_t\}_{t=0}^{\infty}$ – satisfies these first-order conditions, the original constraints, the initial condition requirement on k_t and the transversality condition, $\lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = 0$.

Optimal capital accumulation in the basic neoclassical model is a "general equilibrium" phenomenon in three ways. First, the choices of consumption, labor and capital accumulation are interdependent at each point in time and across time:

a solution for optimal capital accumulation involves specifying sequences for all three of these variables. Second, the requirement that the optimal decisions must respect the resource constraints of the economy is signalled by the shadow prices (ω_t and λ_t). An optimal capital accumulation plan thus also involves specification of sequences of these prices. Third, if these shadow prices were market prices for individual households, then they would similarly signal these agents to supply and demand the optimal quantities. That is, there is an equivalence between the optimal quantities chosen by the social planner and those in a dynamic competitive general equilibrium in the basic neoclassical model. We will thus move between optimal and market outcomes in our discussion of the basic model as it seems useful in the next several sections. We will return later to discuss how work on real business cycles relates to other developments in dynamic stochastic general equilibrium modeling.

3.4. The nature of the steady state

There is a unique stationary state that occurs in the transformed economy when $A_t = A$ for all t. The first-order conditions can be used to describe this stationary state in a recursive manner. First, the capital accumulation efficiency condition implies that

$$AD_1F(k,N) = (r + \delta), \tag{3.19}$$

where $r = \frac{\gamma}{\beta} - 1$ is the stationary state real interest rate and $r + \delta$ is the stationary state rental price of capital. Given that the production function is constant returns-to-scale, the marginal product of capital depends on the capital–labor ratio $\frac{k}{N}$ rather than on the levels of the factors. Accordingly, Equation (3.19) determines the capital–labor ratio as a function of productivity and the rental rate. Second, given this capital–labor ratio and the level of A, the marginal product of labor is also determined, since $AD_2F(k,N) = AD_2F(\frac{k}{N},1)$. Thus, there is a real wage rate $w = \omega/\lambda$ that is determined independently of the total quantity of labor:

$$w = \frac{\omega}{\lambda} = AD_2F\left(\frac{k}{N},1\right).$$

Third, there are unique levels of work, consumption and the shadow price of consumption that satisfy the remaining equations.

We know that the variables in the original, untransformed economy are related to those of the transformed economy by a simple scaling procedure, $Y_t = y_tX_t$, etc. Hence, if the transformed economy is in a stationary state, then the original economy will be in a steady state with many variables – including, consumption, capital output and real wages – growing at the same rate. Other variables will be constant, notably work effort and the real interest rate.
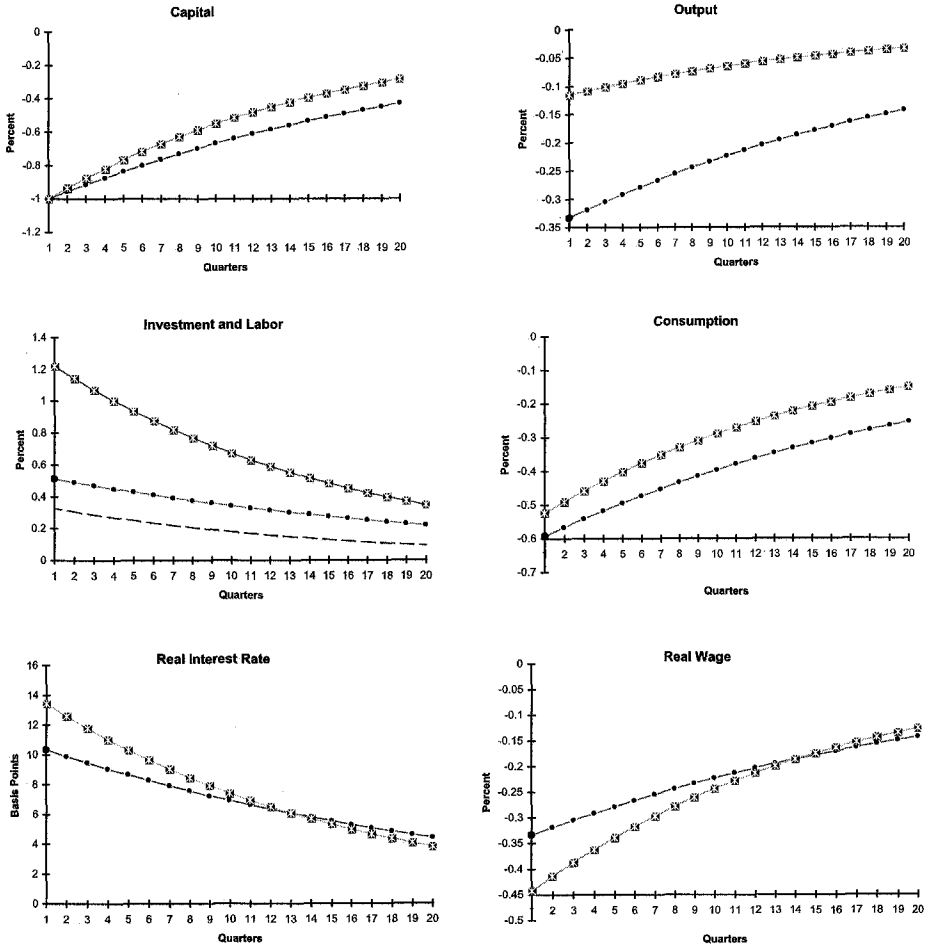
Fig. 6. Transitional dynamics: Basic RBC model (stars); fixed labor model (circles); dashes in mid left-hand panel represent labor response.

3.5. Transitional dynamics

Transitional dynamics arise whenever the intitial capital stock is different from its steady-state value[22]. For stationary versions of the fixed labor model, Cass (1965) and Koopmans (1965) established that it is *always* optimal for the economy's capital stock to move monotonically toward the stationary level from *any* positive initial level of

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[22] In the transformed economy, this is the movement from an initial level of capital k_0 to the stationary level k.

capital. Working to establish this stability theorem, Cass and Koopmans were hampered by the absence of an explicit solution to the model, which stemmed from the presence of many interdependent choices for consumption and capital at different dates. Thus, they were able to establish the stability property, rather than ascertaining how the pace of the transition process depended on the underlying preferences and technology.

More precise descriptions required the detailed specification of preferences and technology, with near steady-state linear approximations sometimes being used to evaluate the nature of the global transitional dynamics. Figure 6 illustrates the nature of these local transitional dynamics of capital, as well as the related movements in output, investment and consumption. There are two sets of paths in each of the panels: the 'o' path describes the fixed-labor model used by Cass and Koopmans and the '*' path describes the variable-labor model used in RBC analysis. In these panels, all variables are displayed as percentage deviations from their corresponding stationary values. For both models, the economy is assumed to start off with a capital stock that is one percent lower than its stationary value, so that both of the paths in the upper left panel have an entry of −1 at the first date.

Looking first at the 'o' paths that describe the transitional dynamics of the Cass–Koopmans model, we see that capital accumulates through time toward its stationary level, i.e., the (negative) deviation from the stationary level becomes smaller in magnitude through time. Since capital is low relative to the steady state, the second panel shows that output is also low relative to the stationary state. Capital is built up through time by individuals postponing consumption. In a market economy, a high real rate of return is the allocative signal that makes the postponement of consumption occur, with the rate of return given by $r_t = [AD_1F(k_{t+1}, N) - \delta]$ in the fixed labor economy. Using the preference specification (3.7), in fact, one can show that the growth rate of consumption is given by

$$\log\left(\frac{c_{t+1}}{c_t}\right) = -\frac{1}{\sigma}\log\left(\frac{\lambda_{t+1}}{\lambda_t}\right) \approx \frac{1}{\sigma}[r_t - r], \tag{3.20}$$

so that a high real interest rate fosters consumption growth along the transition path[23]. The initial level of consumption is set by the wealth of individuals or, equivalently, so that the efficient path of consumption will ultimately be at the stationary level. In general, as the economy saves to accumulate the new higher stationary level of capital, net investment $i_t - \delta k_t$ is positive along the transition path. In this figure gross investment is also higher than its stationary level during the process of capital accumulation.

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[23] In fact, the value of σ used in constructing Figure 6 is one, so that consumption growth and the rate of return are equal. The time unit of these graphs is a quarter of a year, however, and the interest rate is expressed at an annual rate, so that the slope of the consumption path is one fourth of the annualized interest rate shown in the next to last panel. A basis point is one hundredth of a percentage point, so that an annual interest rate that is 0.10 percentage points above the steady-state level causes consumption to grow from about −0.600 to about −0.575 in the second panel.

When work effort is endogenous, as in the '$*$' path of Figure 6, these transitional dynamics change significantly, although there is still a period of capital accumulation toward the stationary level. The extra margin matters for the speed of the transition. When capital is initially low individuals work harder and produce more output that is used for capital accumulation. This extra effort occurs despite the fact that the real wage ($w_t = AD_1F(k_t, N_t)$) is low relative to its stationary level. Again, the allocative signal is a high rate of return that makes it desirable to forgo leisure (as well as consumption) during the transition process. In fact, there is a subtle general equilibrium channel that enhances the magnitude of the effect on the rate of return. With more future effort anticipated, the rate of return $r_t = [AD_1F(k_{t+1}, N_{t+1}) - \delta]$ is higher than in the fixed labor case, because additional effort raises the marginal product of capital. Further, this higher rate of return stimulates current work effort. On net, variable effort raises the speed of transition and mitigates the effects of initially low capital on output and consumption, while enhancing the investment response.

3.6. The (Un)importance of capital formation

The capital accumulation mechanism at the heart of the basic neoclassical model is sometimes viewed as relatively unimportant for growth and business cycles. In the growth area, Solow followed his (fixed saving rate) growth model (1956) with a celebrated demonstration that productivity was much more important to economic growth than was capital formation. It is easiest to exposit this result if we assume that the production function is Cobb–Douglas:

$$Y_t = A_t K_t^{1-\alpha}(N_t X_t)^{\alpha}, \tag{3.21}$$

with the parameter α being measurable as labor's share of national output (an idea which we discuss more below) and thus being constrained to be between zero and one[24]. Then, using time series for output, labor and capital, we can compute the Solow residual as:

$$\log SR_t = \log Y_t - \alpha \log N_t - (1 - \alpha) \log K_t. \tag{3.22}$$

Abstracting from measurement error in outputs or inputs, the Solow residual can be used to uncover the economy's underlying productivity process,

$$\log SR_t = \log(A_t) + \alpha \log(X_t). \tag{3.23}$$

To evaluate the importance of capital accumulation to economic growth, Solow (1957) looked at how the average growth rate of output per unit of labor input

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[24] Our depiction of Solow's (1957) procedure is impressionistic rather than literal. Solow worked in changes rather than levels and incorporated time varying, rather than constant factor shares. Moreover, in ways which anticipate recent developments, he also sought to correct for changes in the utilization of capital.

(the average of $\log(Y_t/N_t) - \log(Y_{t-1}/N_{t-1})$) was divided between growth in productivity (the average of $\log(SR_t) - \log(SR_{t-1})$) and growth in capital per unit of labor input (the average of $\log(K_t/N_t) - \log(K_{t-1}/N_{t-1})$). The result was a surprising one and must also have been disappointing in view of his just completed work on the dynamics of capital accumulation [Solow (1956)]: capital accumulation accounted for only one eighth of the total, with the remainder attributable to growth in productivity. Thus, the transitional dynamics of capital formation turned out to be not too important for understanding economic growth.

Moreover, the transitional dynamics of Figure 6 do not display the positive comovement of output, consumption, investment and work effort that take place during business cycles. Labor and investment are higher than in the steady state when capital is low while consumption and output are below the steady state. Further, consumption is much more responsive to a low capital stock than either labor or output, which is inconsistent with the evidence on relative volatilities reviewed earlier.

Sometimes these results are interpreted as indicating that one should construct macroeconomic models which abstract from capital and growth, since the introduction of these features complicates the analysis without helping to understand business cycle dynamics. However, real business cycle analysis suggests that this conclusion is unwarranted: the process of investment and capital accumulation can be very important for how the economy responds to shocks.

3.7. Constructing dynamic stochastic models

In this section, we have concentrated on describing the steady state and the transitional dynamics of the basic neoclassical model, as an example of the type of dynamic general equilibrium model now used in RBC analysis and other areas of macroeconomics. There is now a rich tool kit for studying the theoretical properties of stochastic equilibrium in these models, such as the advances described by Stokey, Lucas and Prescott (1989). A systematic analysis of the Brock and Mirman (1972) stochastic growth model, modified to include variable labor supply as above, calls for the application of these methods. We review these developments in the Appendix, using the basic RBC framework to highlight two important issues. First, we characterize the optimal decision rules for consumption, capital, output, investment and labor using dynamic programming. Second, we demonstrate that the outcomes of the optimal growth model are the same as the outcomes of a dynamic stochastic general equilibrium model, in which firms and workers trade goods and factors in competitive markets. This equivalence requires that firms and workers have rational expectations about future economic conditions. Another notable result for this stochastic model, first established by Brock and Mirman (1972), is that the stationary state is replaced by a stationary distribution, in which the economy fluctuates in response to shocks.

We have also discussed the local transitional dynamics of the basic neoclassical model illustrated in Figure 6. The development of real business cycle models and dynamic stochastic general equilibrium theory has also heightened interest in methods

for solving and simulating dynamic equilibrium models. In this survey, we rely on now-standard loglinear approximation methods for solving the various real business cycle models that we construct[25]. These methods have been shown to be highly accurate for the basic RBC model[26]. The application of these methods contains essentially two steps. First, it is necessary for us to specify the utility function, the production function, the depreciation rate and so forth so that we can solve for the steady state of the model economy, working much as we did in Section 3.4. Second, we take loglinear approximations to the resource constraints (3.10)–(3.13) and the efficiency conditions (3.15)–(3.18). We then assume that these approximate equations hold in expected value – a certainty equivalence assumption – and solve the resulting expectational linear difference equation system. This yields a system of linear difference equations forced by random shocks, from which moments and simulations can be easily computed.

4. The Real Business Cycle shock

The stark simplicity of the analysis of Prescott (1986) provided a dramatic demonstration of the empirical power of RBC models. His results were surprising because the neoclassical model – even with stochastic productivity – was widely viewed as suitable for long-run analysis, but not for the study of business cycles. In particular, Prescott (1986) showed that a simulated version of the basic neoclassical model could generate business cycle statistics like those in Table 1 when driven by productivity shocks. To make these shocks "realistic", Prescott required that they have the same statistical properties as the actual residuals from an aggregate production function computed using the method of Solow (1957). Building on this idea, Plosser (1989) showed that empirical Solow residuals constructed from post-war US data produced model time series for macroeconomic activity that appeared visually close to the actual business cycle fluctuations on a period-by-period basis. In this section, we display these moment and time-path results as an introduction to real business cycles.

4.1. The driving process

The crucial assumption in RBC analysis is that the stochastic component of productivity can be extracted from the empirical Solow residual using Equation (3.23), $\log(SR_t) = \log(A_t) + \alpha \log(X_t)$. Then, assuming that $\log(A_t)$ follows an AR(1) process,

$$\log(A_t) = \rho \log(A_{t-1}) + \varepsilon_t, \tag{4.1}$$

and exploiting the fact that $\log(X_t) = \log(X_{t-1}) + \log(\gamma)$, it is possible to estimate the stochastic process for productivity and, in particular, the persistence parameter ρ and

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[25] There are versions of the basic RBC model that can be solved analytically but require restrictive assumptions on preferences and technology. See, e.g., Radner (1966), Long and Plosser (1983), Devereux, Gregory and Smith (1992), and Rebelo and Xie (1999).

[26] See, e.g., Danthine, Donaldson and Mehra (1989), Christiano (1990), and Dotsey and Mao (1992).

the standard deviation of the innovation ε_t. To do this, one fits a linear trend to $\log SR_t$ in order to compute γ. Then, one uses the residuals from this regression to estimate ρ and standard deviation of ε_t. For our quarterly data set, the resulting point estimates are 0.979 for ρ and .0072 for the standard deviation of ε_t. The high estimated value of ρ reflects the substantial serial correlation in panel 4 of Figure 3, where the variable described as productivity is the Solow residual.

4.2. Calibrating and solving the model

The work of Kydland and Prescott (1982) and Long and Plosser (1983) illustrated the value of exploring the workings of stochastic dynamic models by using a "reasonable" set of parameter values. Following the methodological recommendations of Lucas (1980) in his influential "Methods and Problems in Business Cycle Theory," Kydland and Prescott relied on microeconomic empirical studies and on the long-run properties of the economy to choose parameter values. To explore the operation of their multiple sector business cycle model, Long and Plosser (1983) drew parameters from input-output tables for the US economy. This new approach, which came to be known as "calibration" has at times been controversial. This partly reflects the fact that most research in macroeconomics followed one of two other routes prior to the rational expectations revolution[27]. First, many authors explored qualitative features of theoretical models and compared them informally with empirical evidence. Second, many researchers formally estimated and tested models.

To see how the calibration approach works, let us apply it to our basic neoclassical model. There are broadly two parts of calibration. One must begin by choosing functional forms which imply that certain parameters are important and then one must assign parameter values.

The great ratios in the steady state: From our discussion in Section 3.4 above, we know that the production side of the model determines nearly everything about the steady state. We choose the discount factor so that the steady-state real interest rate coincides with the average return to capital in the economy. This is 6.5% per annum, if we equate it with the average return on the Standard and Poor 500 Index over the period 1948–1986. Since we are interested in a quarterly model, we choose the discount factor b so that the quarterly real interest rate is 0.065/4. In the Cobb–Douglas production function (3.21), there are three parameters α, X and A. We set α equal to

[27] This calibration approach is commonly associated solely with the RBC program, but it was also used in the early 1980s by researchers studying the effects of nominal contracting on economic fluctuation, such as Blanchard and Taylor. The calibration approach had been previously used in other areas of economics, such as public finance and international trade, which employed complicated, though static, general equilibrium models. Calibration is now routine in a wide range of macroeconomic areas, although it was controversial in the late 1980s because of Kydland and Prescott's (1991) insistence that it should be used instead of standard econometric methodology. A nontechnical review of the interaction between the quantitative theory approach and econometrics is provided by King (1995).

two-thirds, which is a standard value for the long run US labor income share[28]. Both X_0 (the initial level of technical progress) and the mean value of A are parameters which affect only the scale of the economy, and hence can be normalized to one. The growth rate of technical progress is chosen to coincide with the average growth rate of per capita output in the USA during the post war period (1.6% per annum), which implies a quarterly gross growth rate of technical progress of $\gamma = 1.004$[29]. The rate of depreciation is chosen to be 10% per annum ($\delta = 0.025$)[30]. Taking all of this information together, we can solve for the capital–labor ratio k/N, using the requirement that $r + \delta = AD_1F(k, N)$. In the Cobb–Douglas case we obtain:

$$\frac{k}{N} = \left[\frac{(1 - \alpha)A}{r + \delta} \right]^{\frac{1}{\alpha}}.$$

In turn, this implies that the steady-state value of the capital–output ratio is $k/y = (k/N)/(y/N)$ since the average product of labor $y/N = A(k/N)^{1-\alpha}$. We also thus compute the steady-state ratios $i/y = (\gamma - 1 + \delta)(k/y)$ and $c/y = 1 - (i/y)$, as well as the steady-state real wage rate $w = \alpha A(k/N)^{1-\alpha}$.

Parameterizing utility: The constant elasticity class of utility functions (3.8) is motivated by having steady-state growth in productivity lead to steady-state growth in consumption and a constant average level of hours per person. In our discussion of this basic model, we use the momentary utility function

$$u(c_t, L_t) = \log(c_t) + \frac{\theta}{1 - \eta}(L_t^{1-\eta} - 1), \tag{4.2}$$

Once we specify the parameter which governs the labor supply elasticity (η) we choose θ to match steady state N, which is about 20% of the available time in the USA in the postwar period[31]. The studies of Prescott (1986) and Plosser (1989) used the "log–log" case, which makes utility $u(C_t, L_t) = \log(C_t) + \theta \log(L_t)$, motivating this form by arguing that a range of microeconomic and asset-pricing evidence suggests a coefficient of risk aversion of $\sigma = 1$[32]. We begin with this case (in which $\eta = 1$) and then consider some alternative values in Section 6 below.

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[28] This is higher than the value of $\alpha = 0.58$ used in King, Plosser and Rebelo (1988a,b). This number is somewhat sensitive to the treatment of the government sector and of proprietor's income. See Cooley and Prescott (1995) for a discussion.

[29] Many calibration studies ignore growth all together, as we ignore growth in population. Incorporating population growth would raise the appropriate value of γ to 1.008.

[30] Here we use a conventional value for δ, but there is some evidence that it should be lower. The ratio of capital consumption allowances to the capital stock (excluding consumer durables and government capital) for the USA in the post war period takes values of the order of 6 percent [see Stokey and Rebelo (1995, Appendix B)]. The average investment share is very sensitive to γ and δ, but the near steady-state dynamics are not.

[31] That is, the condition $w = \frac{D_2u(c,L)}{D_1u(c,L)} = \frac{\theta c}{L^\eta}$ implies that $\alpha = \frac{wN}{y} = \frac{\theta N}{L^\eta}\frac{c}{y}$, which can be solved for θ.

[32] There is substantial uncertainty about σ, which tends to be estimated with a very large standard error, see Kocherlakota (1996).

Table 2
Calibration of baseline model

σ	b	θ	η	γ	α	δ	ρ	σ_ε
1	0.984	3.48	1	1.004	0.667	0.025	0.979	0.0072

Collecting these results, we have Table 2 listing the parameters that are used in the baseline model in the remainder of this section.

Loglinearizing the model economy: The next step in solving the model is to approximate its various equations, which is most frequently done so as to produce log-linear relations. Sometimes, as with the utility specification (4.2), it happens that the relations of interest are exactly log-linear to start. For example, the consumption efficiency condition (3.15) is $\lambda_t = D_1 u(c_t, L_t) = (c_t)^{-1}$ and the leisure efficiency condition (3.16) is $\omega_t = \lambda_t w_t = D_2 u(c_t, L_t) = L_t^{-\eta}$, so that:

$$-\hat{c}_t = \widehat{\lambda}_t, \tag{4.3}$$
$$-\eta\widehat{L}_t = (\widehat{\lambda}_t + \widehat{w}_t). \tag{4.4}$$

In these expressions, a circumflex ("hat") over a variable represents proportionate deviations of that variable from its steady-state level, $\hat{c}_t = \log(c_t/c)$, etc.

The Cobb–Douglas production function also implies that the efficiency condition (3.17) – which can be rewritten as an equality between the real wage rate and the marginal product of labor – is exactly loglinear,

$$\widehat{w}_t = \hat{A}_t + (1 - \alpha)(\hat{k}_t - \widehat{N}_t), \tag{4.5}$$

i.e., the real wage is raised by productivity and by increases in the capital labor ratio.

Other equations of the model are not exactly log-linear and so must be approximated. The time constraint is $N_t + L_t = 1$ so that small changes in labor and leisure satisfy $dN_t + dL_t = 0$ and thus $N\frac{dN_t}{N} + L\frac{dL_t}{L} = 1$. Since $\log(\frac{N_t}{N}) \approx \frac{dN_t}{N}$, we conclude[33]:

$$(N)\widehat{N}_t + (L)\widehat{L}_t = 0. \tag{4.6}$$

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[33] An alternative derivation of this and other results involves assuming that the behavioral equation depends on $\widehat{N}_t = \log(N_t/N)$ etc. and then taking a linear approximation in the hatted variables. For example, the time constraint is

$$N \exp(\widehat{N}_t) + L \exp(\widehat{L}_t) = 1,$$

so that it is approximately Equation (4.6), given that a first-order Taylor series approximation to $\exp(\widehat{N}_t)$ about $\widehat{N}_t = 0$ is $\widehat{N}_t$.

Since the constraint on uses of goods takes the form $c_t + i_t = A_t k_t^{(1-\alpha)} N_t^{\alpha}$ so that a mixture of the two methods used above yields

$$\left(\frac{c}{y}\right) \hat{c}_t + \left(\frac{i}{y}\right) \hat{i}_t = \hat{y}_t = \hat{A}_t + \alpha \widehat{N}_t + (1 - \alpha) \hat{k}_t. \tag{4.7}$$

Other equations such as (3.6) and (3.18), which contain variables at different dates, can be similarly approximated. The result is a loglinear dynamic system that can be solved numerically.

Interpreting aspects of the model economy: One benefit of this solution strategy is that the researcher may be able to interpret certain aspects of the model economy prior to obtaining its numerical solution. For one example, Equation (4.5) can be interpreted as a description of "labor demand", so as to discuss the influence of productivity, the real wage rate and the stock of capital on the quantity of labor. For another, combining Equations (4.4) and (4.6), we arrive at a "labor supply schedule" that relates the quantity of labor to the real wage and the shadow price of goods, i.e.,

$$\widehat{N}_t = \frac{L}{\eta N} (\widehat{\lambda}_t + \widehat{w}_t),$$

so that a higher value of η lowers the labor supply elasticity. Individually, these equations describing "labor supply" and "labor demand" can be used to evaluate the consistency of the macroeconomic model with microeconomic evidence. Taken together, they provide an explanation of how the quantity of labor and the real wage rate respond to variations in productivity, the capital stock, and the shadow price.

4.3. Business cycle moments

One way of evaluating the predictions of the basic RBC model is to compare moments that summarize the actual experience of an economy with similar moments from the model. On the basis of such a moment comparison, Prescott (1986) argued that the basic RBC model predicts the observed "large fluctuations in output and employment" and, more specifically, that "standard theory ... correctly predicts the amplitude of ... fluctuations, their serial correlation properties, and the fact that investment is about six times as volatile as consumption."

Table 3 reports summary statistics on HP cyclical components of key variables for simulations of the basic neoclassical model driven by productivity shocks. These statistics are comparable to those reported in Table 1 for the US economy.

Volatility of output and its components: Productivity shocks produce a model economy that is nearly as volatile as the actual US economy. More specifically, comparing the ratio of model and empirical standard deviations, Kydland and Prescott (1991) have argued that the real business cycle model explains the dominant part

Table 3
Business cycle statistics for basic RBC model[a,b]

	Standard deviation	Relative standard deviation	First-order autocorrelation	Contemporaneous correlation with output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

[a] All variables have been logged (with the exception of the real interest rate) and detrended with the HP filter.
[b] The moments in this table are population moments computed from the solution of the model. Prescott (1986) produced multiple simulations, each with the same number of observations available in the data, and reported the average HP-filtered moments across these simulations.

of business cycles[34]. For the numbers in Tables 1 and 3, the Kydland–Prescott variance ratio is $0.77 = (1.39/1.81)^2$, suggesting that the RBC model explains 77% of business fluctuations. Using a variation on the basic model which introduces costs of moving labor out of the business sector, Kydland and Prescott (1991) argued that "technology shocks account for 70 percent of business cycle fluctuations". Using a slightly different version of the model Prescott (1986) had previously attributed 75% of output fluctuations to productivity shocks.

The real business cycle model is consistent with the observed large variability of investment relative to output, as indicated by the relative standard deviations reported in the second columns of Tables 1 and 3. In particular, investment is about three times more volatile than output in both the actual economy (where the ratio of standard deviations is 5.30/1.81=2.93) and the model economy (where the ratio of standard deviations is 2.95). Consumption is substantially smoother than output in both the model and actual economies. In our basic model, however, consumption is only about one-third as volatile as output while it is over two thirds as volatile as output in the US economy. We return to discussion of this feature of the economy in Section 6 below.

Persistence and comovement with output. Business cycles are persistently high or low levels of economic activity: one measure of this persistence is the first-order serial correlation coefficient. Table 3 shows that the persistence generated by the basic model

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[34] See Eichenbaum (1991) for a criticism of this interpretation of the variance ratio.

is high, but weaker than in the data (see Table 1). The relative standard deviations also provide a measure of the limited extent to which the basic RBC model amplifies productivity shocks: in terms of its business cycle behavior, output is 1.48 times as volatile as productivity.

Business cycles also involve substantial comovement of aggregate output with inputs (such as labor) and the components of output (such as consumption and investment). Accordingly, Table 3 reports the contemporaneous correlation of output with the other four measures. All of these correlations are quite high, indicating the basic RBC model captures the general pattern of comovement in the data. However, the empirical correlations of output with labor, investment and productivity are substantially smaller than their model counterparts.

From this battery of statistics, we can see that the RBC model produces a surprisingly good account of US economic activity. However, there are also evident discrepancies. Notably, consumption and labor input in the basic model are each much less volatile than in the data. Further, the basic RBC model produces a strongly procyclical real wage and real interest rate, which does not accord well with the US experience summarized in Table 1.

4.3.1. Simulations of US business cycles

Figure 7 depicts US data together time series generated by simulating the model with the innovations to the actual US Solow residual. On the basis of results similar to those in this figure, Plosser (1989) argued that "the simple (RBC) model appears to replicate a significant portion of the behavior of the economy during recessions and during other periods"[35]. Indeed, looking at the first panel of Figure 7, it is clear that the basic RBC model gives quite a good account of the quarter-to-quarter variation in the output time series. The correlation between these series is 0.79; the model also works well in all major recession and expansion episodes.

Turning to the individual components of output, the performance of the RBC model is also surprisingly good for such a simple model. Consumption in the model and the data are strongly positively associated (the contemporaneous correlation is 0.76), although the model's series in the bottom panel of Figure 7 is much less volatile than the actual experience, as suggested by the previous discussion of moments above.

Investment in the model and the data also move together in the third panel of Figure 7, although model investment appears to lead actual investment by one to two quarters. One measure of this lead is that the contemporaneous correlation of model and actual investment is 0.63 and the correlation between actual investment and past model investment is 0.73 at one lag and 0.69 at two lags. While the volatility of labor is

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[35] This section uses a model that is essentially the same as that in Plosser (1989), but our simulated time series are slightly different due to (i) differences in data; and (ii) differences in filtering. Plosser's use of the first-difference filter emphasizes higher frequency components of time series relative to our use of the HP filter.
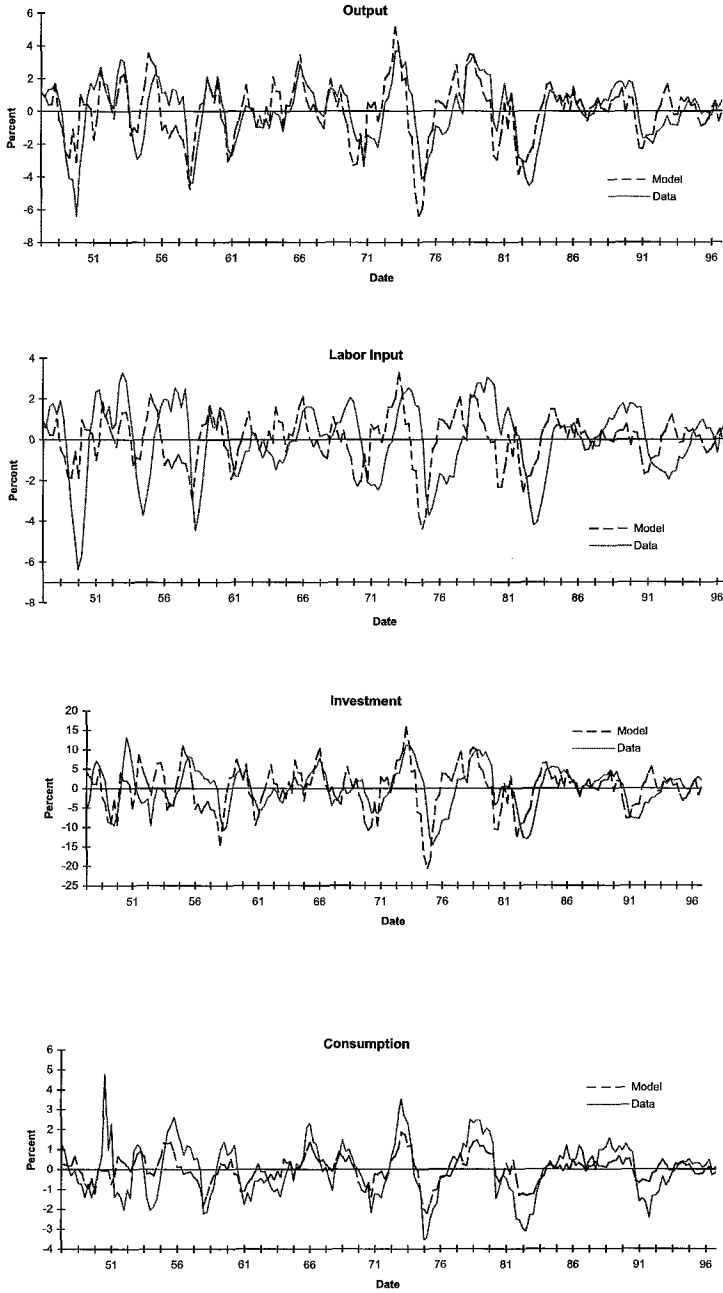
Fig. 7. Basic model: simulated business cycles. Sample period is 1947:2–1996:4. All variables are detrended using the Hodrick–Prescott filter.

broadly similar in the data and in the model, there is much less of a period-to-period correspondence between labor and output in the simulations in the second panel of Figure 7 than there is in the US economy.

4.4. The importance of capital accumulation

The process of capital accumulation is central to business cycles in the RBC framework. To highlight this importance, consider the effect of a positive productivity shock under the assumption that investment is zero at all dates. Then, higher productivity would raise the level of the production function and the marginal product of labor schedule, with each increasing proportionately. From the standpoint of the representative individual, this would work just like a secular rise in the real wage rate, with exactly offsetting wealth and substitution effects on labor. Thus, hours per worker would be invariant to productivity, with consumption moving one-for-one with output[36]. By contrast, in the RBC framework, investment increases in response to a positive productivity shock, i.e., the representative household optimally saves some fraction of the higher current output. Thus, it is also efficient to lower consumption and raise work effort relative to the fixed investment case, producing the cyclical comovements that we see in the actual economy.

However, the introduction of capital as a factor of production in the basic model tends to make it difficult to match the behavior of output and labor input that we saw in the first panel of Figure 3, where labor is nearly as volatile as output. Fundamentally, this reflects the fact that capital is not particularly variable over the business cycle (see the second panel of Figure 3). More specifically, the Solow decomposition indicates that $\hat{y}_t = \hat{A}_t + \alpha\widehat{N}_t + (1 - \alpha)\hat{k}_t$, so that we expect a one percent change in labor to produce an $\alpha = \frac{2}{3}$ per cent change in output. It also works to mitigate the volatility of labor input, since it makes the marginal product of labor decline with the quantity of labor, given that the capital stock is essentially fixed over the business cycle. That is, despite substantial business cycle changes in investment, these do not have a large effect on the capital stock.

4.5. Early successes and criticisms

The results of Table 3 and Figure 7 illustrate why the last decade has witnessed an explosion of research on real business cycles. The basic RBC model holds the promise of being a coherent framework that integrates growth and business cycles. At the same time, there were clear areas in which the model needed to be improved, so that there

[36] With a total factor productivity shock, this exact offset requires that there be a Cobb–Douglas production function, although it holds for any production function if the shock is labor augmenting. While it clearly holds if there is no capital, it also holds if capital is present, as may readily be verified using the line of argument in footnote 20. A key part of this more general result is that capital income increases with the productivity shock.

clearly was important additional work to done. Prescott (1986) summarizes moment implications as indicating that "the match between theory and observation is excellent, but far from perfect". Plosser (1989) summarizes the model simulations as indicating that "the whole idea that such a simple model with no government, no market failures of any kind, rational expectations, no adjustment costs could replicate actual experience this well is very surprising". Rogoff (1986) warns of the potential power of the RBC model: "The ... real business cycle results ... are certainly productive. It has been said that a brilliant theory is one which at first seems ridiculous and later seems obvious. There are many that feel that (RBC) research has passed the first test. But they should recognize the definite possibility that it may someday pass the second test as well."

One notable part of the RBC program is its insistence on the construction of dynamic stochastic general equilibrium models, which is now the accepted approach to macroeconomic analysis across a wide range of research areas and perspectives. Even those who are skeptical of the central role of productivity shocks have accepted the idea that "the basic methodological approach ... (is) ... relevant to models in which monetary disturbances play a greater role" as Rogoff (1986) forecasted that they would.

But it is the other component of the RBC approach that was immediately controversial and remains so to this day: that technology shocks are the dominant source of fluctuations. The striking performance of the basic RBC model drew a strong critical reaction from macroeconomists working in the Keynesian tradition [Summers (1986), Mankiw (1989)]. Their criticisms focused on three main points. First, they questioned some of the parameter values used in the calibration of the model. In particular, they stressed that the model's performance required an empirically unreasonable degree of intertemporal substitution in labor supply. Second, they emphasized the model's counterfactual implications for some relative and absolute prices. The critics observed that the strongly procyclical character of the model's real wage rate was inconsistent with the findings of numerous empirical studies. They also pointed to Mehra and Prescott's (1985) earlier finding that standard preferences, such as Equation (4.2), are incompatible with the equity premium, i.e. the difference between the average rate of return to equities and the risk free rate.[37] In addition, they suggested that a productivity shock theory of the cycle should imply a strongly countercyclical price level. Third, they argued that the use of the Solow residual was highly problematic, leading to excessively volatile productivity shocks.

In retrospect, the first two criticisms of RBC analysis had a small impact on the RBC program. Rather than being fragile, the model's performance is surprisingly resilient to variations in its parameters. Much of the model's performance is anchored on three single ingredients: a highly persistent technology shock that is sufficiently volatile, a sufficiently elastic labor supply, and empirically reasonable steady-state shares of

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[37] See John Campbell (1999) for a detailed discussion.

consumption and investment in output[38]. The RBC model does not need to rely on a high degree of intertemporal substitution in labor choice. In fact some RBC models [e.g. Greenwood, Hercowitz and Huffman (1988)] assume that this elasticity is zero. However, either intertemporal or intratemporal substitution must be strong enough to produce realistic labor movements, a point to which we will return in Section 6 below. RBC researchers have produced a battery of models that lead to a relatively high elasticity of labor supply. The model's predictions for the real wage can be improved if we step away from the assumption of spot labor markets and incorporate contracts between firms and workers that allow for wage smoothing [Gomme and Greenwood (1995), Boldrin and Horvath (1995)] or other forms of labor contracts [Danthine and Donaldson (1995)]. It is hardly surprising that the assumption of spot labor markets produces unreasonable implications for the real wage. And while research on the equity premium puzzle continues, we now have models that are consistent with some aspects of the equity premium while maintaining the business cycle performance of the basic model[39]. Lastly, the studies of Kydland and Prescott (1990) and Cooley and Ohanian (1991) have concluded that the price level is indeed strongly countercyclical during most time periods.

It is the final criticism – that the Solow residual is a problematic measure of technology shocks – that has remained the Achilles heel of the RBC literature. The key issues in this area involve quantitative rather than qualitative disagreements. With the exception of the two oil shocks, it is hard to identify the macro shocks that produce the productivity variations suggested by the Solow residual. If these shocks are large and important why can't we read about them in the *Wall Street Journal*? Also, the Solow residual often declines suggesting that recessions are caused by technological regress. Finally there are several measurement problems that can make the Solow residual a bad measure of productivity at cyclical frequencies. Summers and Mankiw emphasized the importance of labor hoarding, that is, unmeasured variation in labor effort over the business cycle. Perhaps even more important than labor hoarding is the cyclical variability in capital utilization. Solow-residual based measures of technology shocks that do not account for unmeasured variations in labor and capital will tend to be more volatile and procyclical than true shocks to technology.

These difficulties arose as well in the earlier literature on growth accounting, where the Solow residual had its origins. The stated goal of that literature was to measure the long run evolution of disembodied technical progress, not the short run behavior of productivity. Its hidden agenda was to make the Solow residual negligible, that is, to measure production inputs well enough that all growth in output could be accounted for by movements in factors of production. For this reason the residual was often referred

[38] Given the high correlation between investment and output it is not surprising that the model cannot display enough investment volatility if its share is unreasonably high. As the steady-state investment–output ratio increases the volatility of investment has to converge to that of output.

[39] See Boldrin, Christiano and Fisher (1995) and Christiano and Fisher (1995). These models employ a two-sector structure and use preferences that feature habit formation.

to as a "measure of our ignorance". Growth accountants were horrified when they saw the measure of their ignorance recast as the main impulse to the business cycle. For now we will put the problems associated with the Solow residual as a measure of technology shocks on the back burner. But we return to them in Sections 7 and 8.

5. The central role of productivity shocks

In the standard RBC model, productivity shocks are central to the nature of business cycles. In this section, we will discuss three major aspects of this relationship. First, we explain that the standard RBC model requires large and persistent productivity shocks, by considering how the comparative dynamics of the model change as productivity persistence is altered. Second, we show how the assumption that agents have rational expectations matters to the nature of real business cycles. Third, we discuss why other shocks cannot easily generate real business cycles in the standard model.

5.1. Productivity shocks must be large and persistent

The simple driving process for productivity used by Prescott (1986) and Plosser (1989) provides a natural basis for discussing the volatility and persistence of productivity. These authors modeled the stochastic component of productivity as a first-order autoregressive process, $\log(A_t) = \rho \log(A_{t-1}) + \varepsilon_t$. Under this specification, the statistical behavior of the productivity process is influenced by the serial correlation parameter ρ and the standard deviation of the zero mean "innovations" ε_t. A standard result from time series textbooks [e.g., Hamilton (1994)] is that the autocovariance of $\log(A_t)$ with its own value lagged by j periods is $\rho^j \frac{\text{var}(\varepsilon_t)}{1-\rho^2}$. This autocovariance expression reveals that increases in the variability of the innovations directly raise the variability of productivity. Increases in the parameter ρ also increase the variability of the time series, since the variance of $\log(A_t)$ is $\frac{\text{var}(\varepsilon_t)}{1-\rho^2}$. However, an increase in ρ produces this additional variability by raising the persistence of the productivity series, since the jth-order correlation of $\log(A_t)$ is ρ^j. Thus, when we say that the standard RBC model requires that there must be large and persistent variations in productivity, we are making several related statements. Mathematically, these can be summarized by saying that the model requires large values of $\text{var}(\varepsilon_t)$ and of ρ.

Why does the standard model require large shocks? When we say that the model requires large shocks, we mean that there must be considerable variability in productivity. This statement is based on understanding how output, consumption, and other variables respond to shifts in ε in the basic model. In all of the models that we study in this section, for example, output responds to a one percent increase in productivity by rising by no more than two percent: there is not much *amplification* of the productivity shock by the model.

To illustrate the effect of smaller productivity shocks, we recomputed the simulation of the basic RBC model using an alternative series of productivity shocks, which have

an innovation standard deviation that is 0.0012 or about $\frac{1}{6}$ times as large as the Solow residual. The result of this is shown in panels 1 and 2 of Figure 8: real business cycles explain a very small fraction of output and labor volatility. Since the standard RBC model is approximately linear, changes in the standard deviation of the innovations, std(ε), simply work to rescale the model's fluctuations. We will return later to discuss more of the details of the computation of these alternative shocks, but at present it is sufficient to note that they were not chosen arbitrarily. Rather, they arise from correcting the Solow residual for the effects of varying capital utilization in ways that we will discuss further in Sections 6–8 below.

Why does the standard model require persistent shocks? By saying that the variations in productivity must be persistent, we mean that the series generated from the standard RBC model will display autocorrelation similar to the US data only if ρ is near one. To discuss this, we consider in detail how the standard RBC model's implications depend on the extent of serial correlation in productivity. We begin by discussing the response of the economy to a serially uncorrelated productivity shock, i.e., the solution of the model when $\rho = 0$. While the dynamic responses to this shock shown in Figure 9 are the result of a complex set of factors – the preferences of households for consumption and labor supply, the production function and the mechanism for accumulating capital, and the interaction of households and firms in general equilibrium – the key mechanisms can be easily described.

Productivity is assumed to increase by one percent ($\varepsilon = 1$) in the initial period (date 1). Given the rise in the marginal product of labor resulting from the increase in productivity, the representative household faces an unusually high opportunity cost of taking leisure in this initial period. While there are offsetting income and substitution effects, the model's preferences were chosen so that a *permanent* increase in the real wage (such as the one associated with the trend in technical progress) generates exactly offsetting income and substitution effects so that labor and leisure are left unchanged. An implication of this result is that N has to rise in response to a *temporary* productivity increase. With a temporary shock, there is a much smaller income effect and there is greater incentive to substitute intertemporally, since the current wage is high relative to expected future wages. On net, the positive labor response amplifies the productivity shock: the impact effect on output in Figure 9 is 2%. Half of this response is due to the direct effect of the productivity shock and half due to the increase in labor.

The representative agent must choose what the economy will do with all this additional output. One possibility is to consume it all in period one[40]. However, this would be inefficient given that the marginal utility of consumption is decreasing, thus inducing a preference for smooth consumption paths. It is optimal to increase consumption both today and in the future. In fact, given that there are many future

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[40] Our figures make the impact date of the shock period 1, while the earlier theoretical analysis made the initial period zero. The discussion in the text follows the dating convention in the figures.
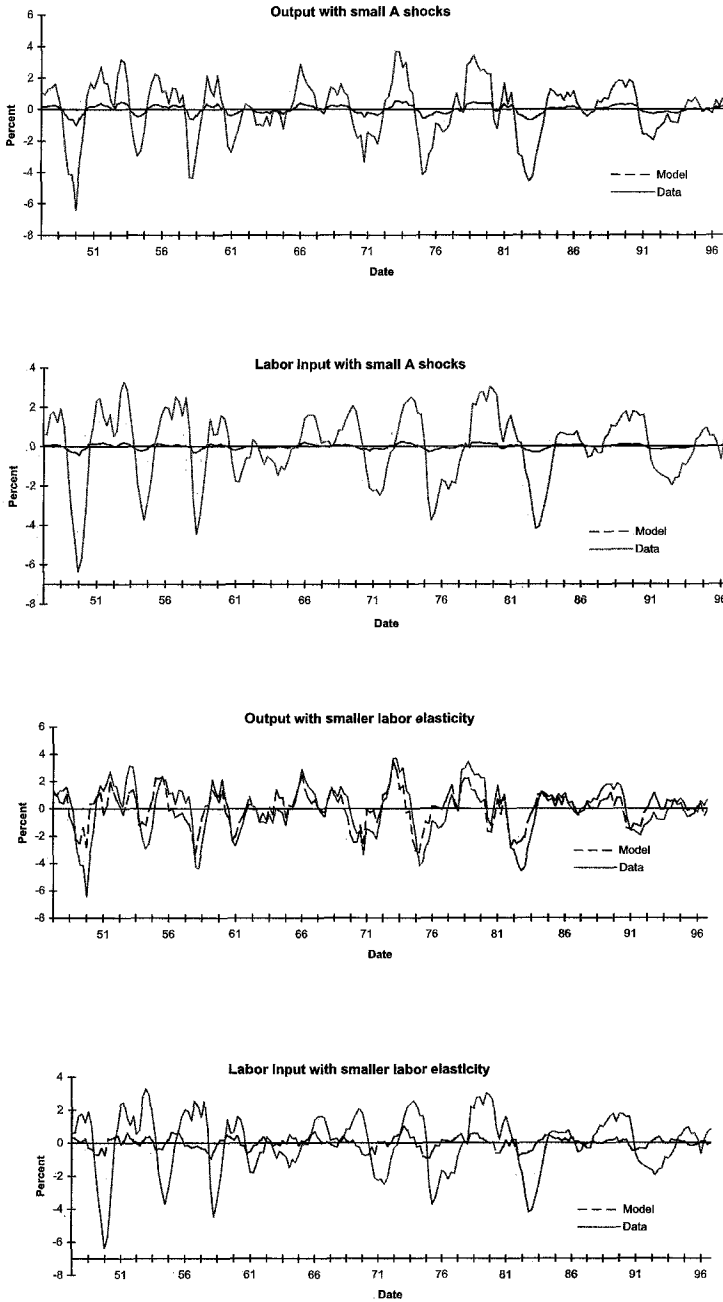
Fig. 8. Consequences of smaller shocks and smaller labor elasticity. Sample period is 1947:2–1996:4. All variables are detrended using the Hodrick–Prescott filter.
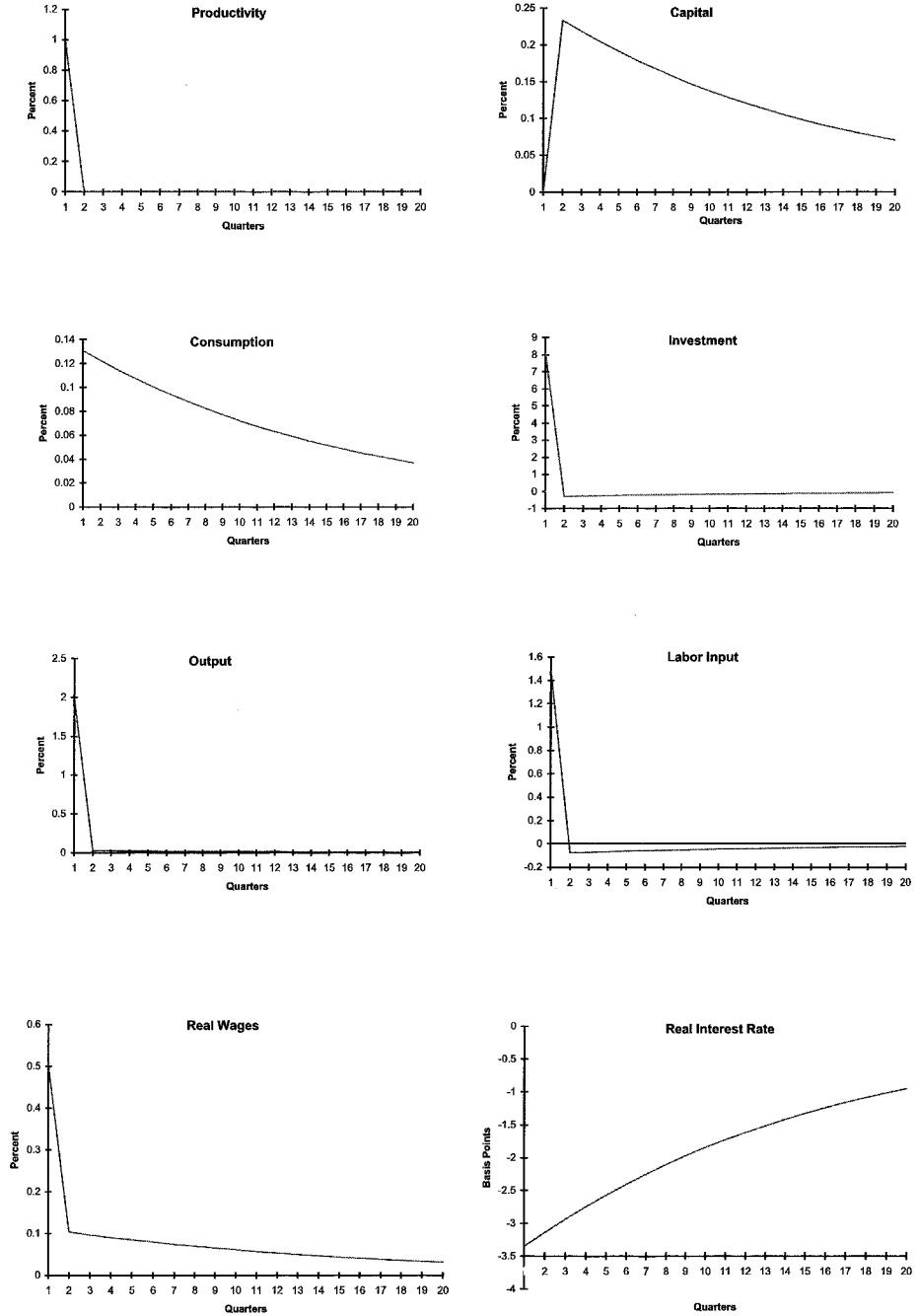
Fig. 9. Comparative dynamics to purely temporary productivity shock.

periods, only a small fraction of the output windfall will be consumed at time 1; most of it will be invested. Thus investment rises by 8% in response to a 2% increase in output. It is interesting to note that the high volatility of investment, which Keynes ascribed to "animal spirits", arises naturally in this economy as the flip side of consumption smoothing.

In the future, which begins with period 2 in Figure 9, productivity returns to its original benchmark level. The only difference relative to period zero is that the economy has accumulated some capital and only a relatively small amount since the productivity shock lasted just for one period. In line with the transitional dynamics that we discussed in Section 3 above, the optimal policy for the economy is to gradually reduce this excess capital by enjoying higher levels of consumption and leisure. The real interest rate again signals individuals to adopt these consumption and leisure paths: with a purely temporary change in productivity, the real interest rate falls in the impact period and in all future periods, making it desirable for individuals to choose consumption profiles that decline through time toward the steady-state level.

The impulse response makes it clear that there will be no tendency for a period of high output and work effort to be followed by another period which has similarly high output. That is: the basic neoclassical model does not produce substantial *internal propagation* of temporary productivity shocks, a point which has been stressed by Cogley and Nason (1995). The effects of the one-time shock are propagated over time: the large investment in period 1 leads to high values of the capital stock that keep output above its steady-state level in the following periods. But this propagation mechanism is very weak. This weakness, together with the fact that Solow residuals display substantial persistence led most RBC studies to focus on specifications in which the persistence is inherited from the shock process.

What happens with serial correlation in productivity consistent with Solow residuals? The solid line in Figure 10 depicts the effect of a serially correlated productivity shock using the estimate discussed in Section 3 above ($\rho = 0.979$). In this case, the different series exhibit realistic persistence, which is inherited from the shock. The same mechanisms are at work as in the case of a purely temporary shock, but these effects are now drawn out over time. We now have an extended interval in which productivity is above normal. During this interval, workers respond by increasing their labor supply and most of the additional output is invested. Interestingly, high productivity is now initially associated with a high real interest rate, since the marginal product of capital schedule $[A_{t+1}D_1F(k_{t+1}, N_{t+1}) - \delta]$ is shifted upwards by the productivity shock and by a higher level of future labor input, with capital responding only gradually via the accumulation of investment. However, later in the impulse responses, the rate of return is below its steady-state level because the capital stock has been built up while the stimulative effects of the productivity shock and labor input have dissipated. This leads consumption to initially grow through time and then subsequently to decline back toward the stationary level. Later in the impulse responses, as productivity converges slowly to its normal level, labor supply actually drops below the steady-state level as the economy enters a phase that resembles the
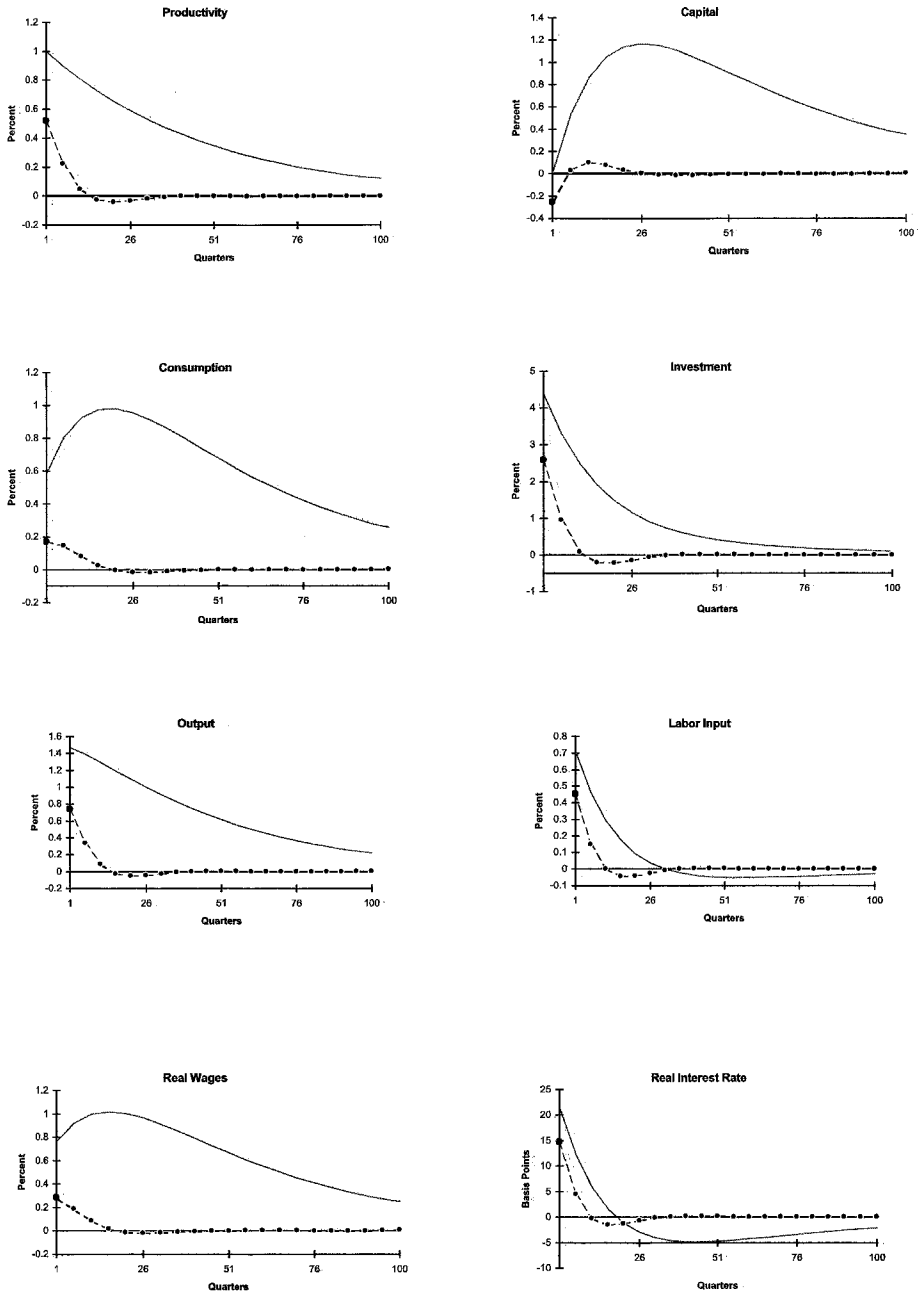
Fig. 10. Comparative dynamics to more persistent productivity shock. Circled lines are impulse responses filtered with the Hodrick–Prescott filter

transitional dynamics discussed above. Investment also eventually drops below the steady state, as the economy runs down the capital that was accumulated during the initial expansion.

As with the case of the purely temporary shock discussed above, the early part of the impulse responses is dominated by the fact that the productivity shock raises the desirability of work effort, production, investment and consumption; the latter part of the impulse response function is dominated by the transitional dynamics, i.e., reduction of capital back toward its stationary level.

These impulse responses govern the autocovariances of productivity, output and other variables. With many periods of high output, there will be positive correlation between output and its past values: expansions and recessions will persist for many periods.

Since the HP filter is so widely used in the real business cycle literature, it is worthwhile investigating its effects on the impulse response function, as an indication of the effects that it has on the moments of the different variables. In Figure 10, the HP-filtered impulse responses are given by the generally lower paths that are highlighted with the 'o' symbol. One notable feature of this filtered impulse response is that there is less tendency for series to remain above or below their normal levels, i.e., filtering reduces the persistence of the various series. This effect is particularly noticeable for output and for productivity. Filtering also flattens the response of consumption and the real wage, at the same time that it makes the capital stock largely acylical.

5.2. The influence of productivity persistence

In the basic RBC model, the persistence parameter governing the productivity process has an important influence on the effects of shocks. For example, if we compare the responses of output in the two figures that we just looked at, that there is a larger initial output response in Figure 9 (where $\rho = 0$) than in Figure 10 (where $\rho = 0.979$). In particular, the additional persistence lowers the impact effect on output from about 2 when $\rho = 0$ to about 1.5 when $\rho = 0.979$. Similarly, the impact effect on work effort is smaller and the impact effect consumption is larger in Figure 10 than it is in Figure 9.

When the productivity is very persistent, in the sense that the coefficient ρ is near unity, there are very dramatic effects of small changes in the value of ρ. In this subsection, to exposit these effects, we focus on the consequences of assuming that productivity is a random walk. Economically, this involves the plausible assumption that changes in technology are permanent and there is some empirical support for the idea that productivity contains a unit root [e.g., Nelson and Plosser (1982)]. Mathematically, this amounts to setting $\rho = 1$ and implies that all shocks to productivity are expected to have an equal effect on current and expected future productivity.

Impulse response analysis: Figure 11 shows that the impulse responses of all variables are substantially affected by changing the driving process parameter ρ from 0.979 to 1. Part of this difference involves the fact that a permanent productivity shock
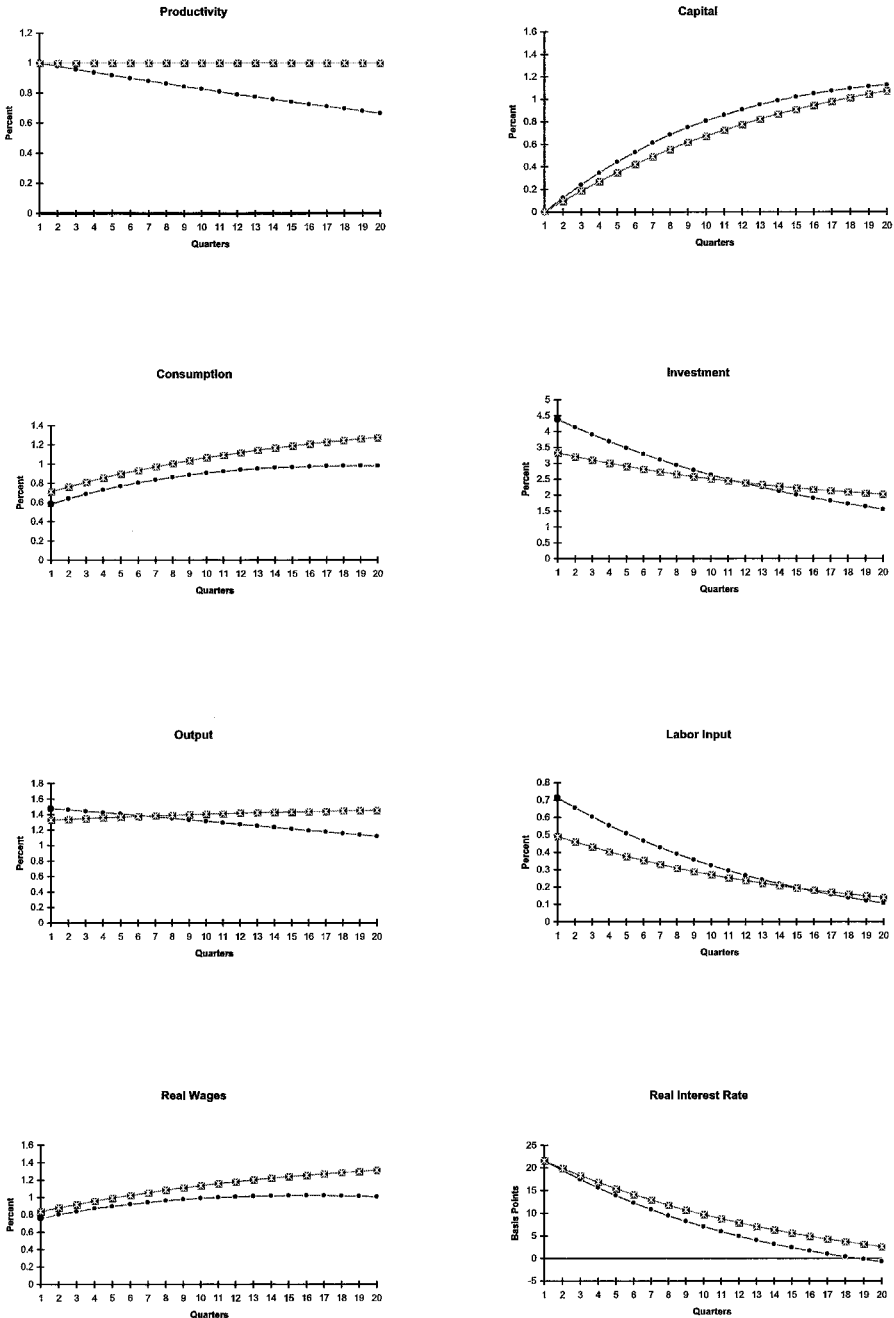
Fig. 11. Comparative dynamics to fully permanent shock. Standard model, $\rho = 0.979$ (circles); $\rho = 1$ (stars).

leads to an identical, proportional, long-run increase in consumption, investment and output, while the stationary shock has no long-run effect. There are also important differences in how the economy responds in the short-run depending on the value of ρ. For example, in the impact period of the shock ($t = 1$), there is a much smaller response of labor and output to a permanent shock (the '*' path) than the standard shock (the 'o' path). The date $t = 1$ shock is assumed to be 1% in both cases so that labor rises by 0.7% with productivity when $\rho = 0.979$ and by 0.5% when $\rho = 1.0$. Conversely, the impact effect on consumption is larger when $\rho = 1$ than it is when $\rho = 0.979$.

After the impact period of the shock, the differences between the unit root case and the base case involve a combination of the differing direct effects of the shocks as well as the differing responses to permanent and temporary shocks. For example, one year after the productivity shock, the stationary model implies that $0.919 = \rho^4$ of the initial one percent impulse to productivity is (expected to be) present when $\rho = 0.979$ and the unit root model implies that there is a one percent higher productivity level.

Explaining the influence of persistence: We previously used our intuition to explain the general shape of impulse response patterns, as in our discussion of Figures 9 and 10 above. We described how the consumption and labor supply plan of the representative household are affected by shocks that affect wealth and the time path of wages. However, in the standard RBC model, there are general equilibrium effects that are subtle to think through. Consumption and labor supply decisions also depend on the time path of interest rates as well as wages. At the same time the time path of wages is influenced by labor supply decisions.

To understand the channels of effect by which increased persistence affects the impulse responses, we adopt a version of Hick's celebrated demand decomposition that is suitable for dynamic models.[41] To exposit this decomposition, let us focus on the determination of the increase in consumption and leisure at the initial date $t = 0$. When a productivity shock occurs, the representative household understands that there is a higher amount of wealth as a result of this shock. If wages and interest rates are unchanged, then this wealth effect would be used to finance a permanently higher level of consumption and a permanently lower level of work effort. These effects are shown in Figure 12 for the persistent shock ($\rho = 0.979$) and the fully permanent shock ($\rho = 1$). There are two notable aspects of these *wealth effects*, which are computed in Hicksian fashion by finding the constant increments to the consumption and leisure paths that yield the same utility change as arises in the general equilibrium of the model. First, the wealth effects are the same for consumption and leisure in proportionate terms, which reflects the fact that the preference specification $[u(c_t, L_t) = \log(c_t) + \theta \log(L_t)]$ makes the wealth elasticities equal across goods. Second, the wealth effects are much larger when $\rho = 1$ as they are when $\rho = 0.979$.

The representative household knows the path of wages which will arise as a result of the shock to productivity. Taking into account just the change in the wage path, we can

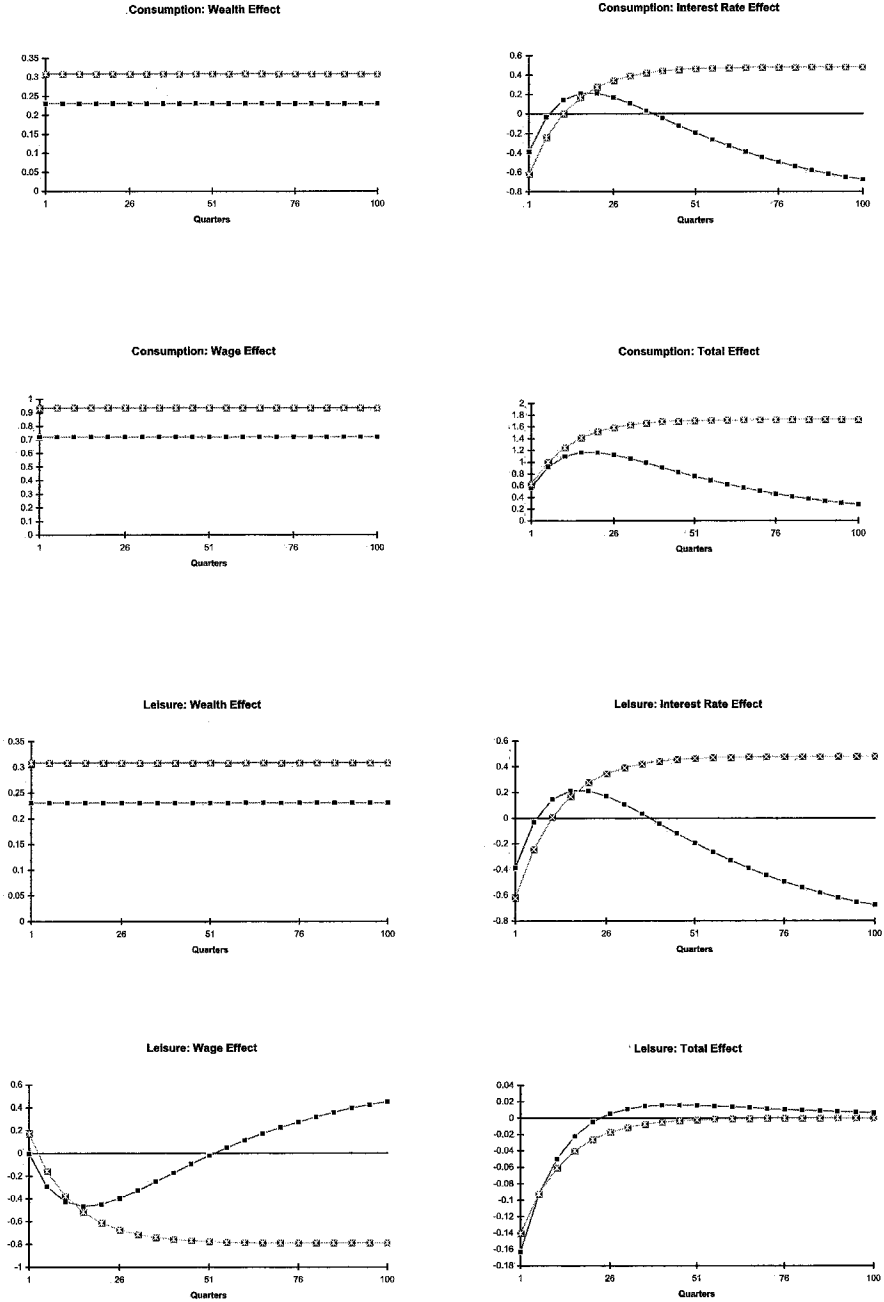---

[41] This decomposition is developed in King (1991).

Fig. 12. Substitution and wealth effects of persistent and permanent shocks: stars, permanent shocks; squares, persistent shocks.

determine the consequences for consumption and leisure at date 0, which we call the *wage effect*. This effect is analogous to the Hicksian effect of the wage on consumption and leisure, in that it holds utility fixed, tracing out a substitution response. However, in our general equilibrium model, the productivity shock implies that wages change in all periods, $\{\widehat{w}_t\}_{t=0}$. Thus, the wage effect in Figure 12 takes into account the entire change in the time path of wages, combining static and intertemporal substitutions. When $\rho = 0.979$, the representative household correctly understands that productivity will raise the path of wages at date 0 and in many future periods, but that the long-run level of the wage will be unchanged. Accordingly, the household plans to consume more at date 0. Leisure hardly changes at all because the current period is about "average"; this conclusion depends on the particular ρ value. However, this pattern is sharply altered when $\rho = 1$, for then the household recognizes that the current wage is below the long-run wage and leisure rises due to the wage effects that stem from a positive productivity shock [42].

In the general equilibrium of our RBC model, there is one additional channel: *interest rate effects* that induce intertemporal substitutions of consumption and leisure. In general, these intertemporal price effects are a powerful influence, but one that is not much discussed in informal expositions of the comparative dynamics of RBC models. In particular, permanent increases in productivity lead to high real interest rates and these induce individuals to substitute away from date 0 consumption and leisure as shown in Figure 12.

We are now in a position to describe why a permanent shift in productivity (arising when $\rho = 1$) has a smaller effect on labor than a persistent but ultimately temporary shock ($\rho = 0.979$). When the shock is temporary, there is a small wealth effect that depresses labor supply but temporarily high wages and real interest rates induce individuals to work hard. When the shock is permanent, there are much larger wealth effects and the pattern of intertemporal substitution in response to wages is reversed since future wages are high relative to current wages. However, labor still rises in this case in response to productivity shocks due to very large intertemporal substitution effects of interest rates.

5.3. Why not other shocks?

We have just seen that the basic real business cycle model driven by persistent technology shocks can produce realistic business cycle variation in real quantities. Do these same patterns emerge when the economy is buffeted by other disturbances? Shocks to fiscal and monetary policy have been long standing suspects in the search

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[42] The wage effect on consumption is constant across time in each case because the separable momentary utility function implies that efficient consumption plans do not depend on the amount of work. Equivalently, with this utility function, there is a general substitution effect on consumption at all dates that works much like a wealth effect.

for the causes of business cycles. It is thus natural to ask what are the effects of these shocks in the standard RBC model.

Shocks to government spending cannot, by themselves, produce realistic patterns of comovement among macroeconomic variables[43]. This result stems from the fact that an increase in government expenditures (financed with lump sum taxes) gives rise to a negative wealth effect that induces consumption to fall at the same time that labor and output rise. Thus, if government spending were the only shock in the model, consumption would be countercyclical[44].

Changes in labor and capital income taxes have effects that are similar to productivity shocks. However, these taxes change infrequently making them poor candidates for sources of business cycles fluctuations.

Monetary policy shocks have small effects in this class of models both in versions in which money is introduced via a cash-in-advance constraint [Cooley and Hansen (1989)] and in models that stress limited participation [Fuerst (1992), Christiano and Eichenbaum (1992b)]. Many researchers are also currently investigating the nature of business cycles in models that start with the core structure of an RBC framework but also incorporate nominal rigidities of various forms[45]. This research has not yet produced a business cycle model that performs at the same level as the RBC workhorse described in Section 4.

6. Extensions of the basic neoclassical model

Since the basic RBC model contains explicit microeconomic foundations, part of the literature has tried to improve its predictions for individual behavior. Other researchers have sought to improve the fit between model and data, focusing on moments and sample paths of macroeconomic time series. In this section, we discuss two strands of this research: work on labor supply and on capital utilization.

6.1. The supply of labor

There is a substantial body of work that focuses on the labor supply and, more generally, on the labor market in RBC models. This research is motivated by four difficulties encountered by the basic model on micro and macro dimensions. In most

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[43] There is a large literature that investigates the effects of fiscal policy in an RBC context. References include Wynne (1987), Christiano and Eichenbaum (1992a), Rotemberg and Woodford (1992), Baxter and King (1993), Braun (1994), McGrattan (1994), and Cooley and Ohanian (1997).

[44] For an early discussion of this difficulty, see Barro and King (1984). There is actually some evidence that in historical periods dominated by large shocks to government expenditures consumption was countercyclical, see Correia, Neves and Rebelo (1992) and Wynne (1987).

[45] Examples include Cho and Cooley (1995), Dotsey, King and Wolman (1996), and Chari, Kehoe and McGrattan (1996).

RBC models the implied labor supply elasticity to wage changes is very large, relative to micro studies. All of the variation in aggregate hours in the model arise due to movements in hours-per-worker, while the US experience is that most of the action comes from movements of individuals in and out of employment. Labor in the model lacks a close correspondence to labor in data (see Figure 6). Finally, labor input and its average product are very highly correlated in the model, but not in the data.

6.1.1. Estimated and assumed labor supply elasticities

Labor economists have long been interested in estimating the response of the labor supply to a change in the real wage rate. In the standard static model, an increase in the real wage produces a substitution effect which leads to an increase in N and C as well as a wealth effect which leads to a decline in N and an increase in C. While the effect of a wage increase on consumption is unambiguous, the effect on the labor supply involves conflicting substitution and wealth effects. In a dynamic model, the effect of a wage change is complicated by the fact that the size of the wealth effect depends on the anticipated duration of the wage change: temporary wage changes have a small wealth effect and permanent ones have a large wealth effect.

In a dynamic setting, the key equation that determines the supply of labor is the requirement that the marginal utility of leisure equal its cost along the intertemporal budget constraint. Many empirical studies of dynamic labor supply [e.g., MaCurdy (1981)], suppose that the utility function has the separable form (4.2), that we introduced in our discussion of the approximation of the RBC model in Section 5 above and for which we showed that

$$\hat{N} = \frac{1-N}{N\eta}(\hat{w} + \hat{\lambda}). \tag{6.1}$$

In this expression, the term $\frac{1-N}{N\eta}$ is the λ-constant elasticity of labor supply. To isolate the substitution effect, labor economists often estimate a λ-constant elasticity of labor supply and we organize our discussion of labor supply issues around this elasticity.

In the basic RBC model, with its assumption of log utility ($\eta = 1$) and a steady-state fraction of time spent working of $N = 0.2$, it follows that the implied labor supply elasticity is four: a one percent change in the wage rate calls forth a four percent change in hours worked if there is little wealth effect (λ constant), as with a temporary wage change. Yet, the microeconomic evidence on variations in hours worked is sharply at odds with the elasticity built into the RBC model. While estimates of this elasticity vary across different gender and race groups, they are typically much lower than unity [e.g. Pencavel (1986)].

6.1.2. Implications of varying the aggregate labor supply elasticity

To show the consequences of adopting a labor supply elasticity in line with microeconomic estimates, the third and fourth panels of Figure 8 show the effect of

choosing $\frac{1-N}{N\eta} = 1$ which is the upper bound suggested by Pencavel's estimates, rather than $\frac{1-N}{N\eta} = 4$ as in the model of Section 4. There is an important reduction in the volatility of output in the third panel of Figure 8. However, the model loses most of its ability to produce fluctuations in labor (see the fourth panel of Figure 8). In terms of moments, the standard deviation of output falls from 1.39 to 1.16 with the smaller labor supply elasticity and the standard deviation of labor falls from 0.67 to 0.33.

6.1.3. Modeling the extensive margin

RBC researchers have investigated ways of enhancing the aggregate labor supply response by focusing on the extensive margin. Figure 4 shows that most fluctuations in total labor input occur as households substitute between employment and nonemployment (the extensive margin) rather than between a greater or smaller number of per capita hours worked (the intensive margin). Explaing these facts seems to require that there are fixed costs of going to work or other attributes of the technology that lead to nonconvexities in the individual's opportunity set.

There are two strategies for incorporating the extensive margin into business cycle analysis. The first is to assume that households are heterogeneous with respect to their reservation values of work, probably due to differences in fixed costs of working such as travel time to the job. This is a conventional approach in labor economics [see, e.g., Rosen (1986)] that has been introduced into a business cycle model by Cho and Rogerson (1988) and Cho and Cooley (1994). In order to make such a model tractable, it is necessary to view individual agents as efficiently sharing the resulting employment risks[46]. An alternative approach, developed by Rogerson (1988) and applied to business cycles by G.D. Hansen (1985), assumes that households are identical but agree on an efficient contract which allocates some individuals to work in each period while leaving the remaining idle. A remarkable feature of both approaches is that there is a stand-in representative agent whose preferences generally involve more intertemporal substitution in work than displayed by the underlying individual agents.

For simplicity and congruence with the literature, we focus our discussion on the economies with indivisible labor and lotteries, following Rogerson (1988). Each individual in the economy has to choose between working a fixed shift of H hours and not working at all. Suppose that preferences are such that individuals would ideally like to supply a number of hours $N < H$. This arrangement is not possible because the choice set is not convex, it includes $N = 0$ (with zero labor income) and $N = H$ (with labor income wH) but no linear combinations of these two points. In this set up agents can be made better off by the introduction of lotteries which convexify their

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[46] In actual economies, variations in aggregate hours reflect changes at both the intensive and extensive margins. In a model where workers have different fixed costs of going to work, Cho and Cooley (1994) have captured both of these responses. Such a framework appears necessary to explain the differing cyclical patterns of employment and hours-per-worker in the USA and Europe that are documented by Hansen and Wright (1992).

choice set. By entering a lottery an agent can choose to work a fraction p of his days remaining unemployed a fraction $(1 - p)$ of his time. Let us use the subscript 1 to denote those agents who are assigned to work by the random lottery draw and the subscript 2 to refer to the unemployed agents. The expected utility of an individual prior to the lottery draw is

$$pu(c_1, 1 - H) + (1 - p)u(c_2, 1), \tag{6.2}$$

where p is the fraction of the population assigned to work. Feasible allocations of consumption across the employed and unemployed agents must obey

$$pc_1 + (1 - p)c_2 = c, \tag{6.3}$$

where c is per-capita consumption. Maximizing Equation (6.2) subject to condition (6.3), we find that marginal utility of consumption must be equated across types, i.e.,

$$D_1 u(c_1, 1 - H) = D_1 u(c_2, 1), \tag{6.4}$$

which is an efficient risk-sharing condition in this situation of employment lotteries as in many other contexts.

The standard indivisible labor model. The typical treatment of the indivisible labor model, as in Rogerson (1988) and Hansen (1985), involves assuming separable utility. Within the general class of utility functions (3.8), this corresponds to $\sigma = 1$ so that $u(c, L) = \log(c) + \log(v(L))$. In this case, efficient risk-sharing implies that the employed and unemployed share the same level of consumption ($c_1 = c_2$). Using this fact, expected utility can be written as

$$u(c, L) = \log(c) + (1 - L)\frac{1}{H} \log\left(\frac{v_1}{v_2}\right) + \log(v_2), \tag{6.5}$$

where $L = 1 - pH$ is the average number of hours of leisure in the economy and where $v_1 = v(1 - H)$ and $v_2 = v(1)$.

There are three notable features of this economy. First, even though each individual agent has a finite elasticity of labor supply, the macroeconomy acts as if it were populated by agents with a more elastic supply of labor. In particular, the stand-in representative agent for this economy has preferences that are linear in leisure, implying a infinite λ-constant elasticity of labor supply [see Equation (6.1) with $\eta = 0$], a feature whose consequences we explore further below. Second, contrary to conventional wisdom, this is an economy in which it is optimal to have unemployment. Finally, agents actually choose to bear uncertainty by entering the lottery arrangement instead of working a fixed number of hours in every period.

It is interesting to explore further why the individual elasticity of labor supply differs from that of the economy as a whole and the consequences of this difference for the

determination of output and labor. The individual elasticity of labor supply answers the
question "how many more hours would you work in response to a 1% raise in salary?".
But the answer to this question is irrelevant because the number of hours worked is not
flexible, it is either H or zero. In other words, the intensive margin is not operative and
hence its elasticity of response is irrelevant. Proceeding to the consequences for the
determination of labor, the preferences of the stand-in representative agent (6.5) imply
that small changes in wages and prices can lead to very large effects on quantities. To
see this, consider an isolated individual maximizing

$$\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + (1 - L_t)\frac{1}{H} \log\left(\frac{v_1}{v_2}\right) + \log(v_2) \right].$$

Along the relevant intertemporal budget constraint, suppose that the discounted cost of
a unit of leisure is $\beta^t \lambda_t w_t$. Then, for the individual to work part of the time $(0 < L_t < 1)$
in each period, it must be the case that $\lambda_t w_t = \frac{1}{H} \log(v_1/v_2)$[47]. But, if this condition is
satisfied, the individual is indifferent across all sequences of leisure which imply the
same level of $\sum_{t=0}^{\infty} \beta^t[(1 - L_t)\frac{1}{H} \log(v_1/v_2)]$: there is an infinite intertemporal elasticity
of substitution in work.

   One implication of this labor supply behavior is that it is the demand side of the
labor market which determines the quantity of employment and work effort in the
equilibrium of the indivisible labor model. From this perspective, firms choose the
quantity of labor that equates its marginal product to the real wage, with the position
of the demand schedule being shifted by the level of productivity and the capital
stock. Since the capital stock and the multiplier λ_t are endogenously determined, this
labor market equilibrium picture is incomplete, but it is a useful partial equilibrium
description.

   The indivisible labor model with more general preferences: When the indivisible
labor model is generalized, as in Rogerson and Wright (1988), there are interesting
new conclusions. To develop these, we use the utility function (3.8), with $\sigma \neq 1$.
Efficient risk-sharing condition implies that consumption allocations must satisfy

$$c_1 = c_2 \left[\frac{v_1}{v_2} \right]^{(\frac{1-\sigma}{\sigma})}. \tag{6.6}$$

According to this specification, if $\sigma > 1$ there will be more consumption allocated
to the employed (group 1) than to the unemployed (group 2)[48]. Thus, as more
individuals are allocated to the market (higher p) aggregate consumption will rise even

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[47] If $\lambda_t w_t < \frac{1}{H} \log(v_1/v_2)$, our agent spends all available time at t in leisure $(L = 1)$. If
$\lambda_t w_t > \frac{1}{H} \log(v_1/v_2)$, our agent devotes no time to leisure $(L = 0)$.
[48] This conclusion makes use of the fact that $v_2 = v(1) > v_1 = v(1 - H)$, which follows from the fact
that v is an increasing function.

if consumption of employed individuals and unemployed individuals stays relatively constant. Further, using this consumption rule along with the expected utility objective, there is a stand-in representative agent whose preferences are[49]

$$u(c, L) = \frac{1}{1 - \sigma} \{ c^{1-\sigma} v^*(L)^{1-\sigma} - 1 \} \tag{6.7}$$

where

$$v^*(L) = \left[\left(\frac{1 - L}{H} \right) v_1^{(\frac{1-\sigma}{\sigma})} + \left(1 - \frac{1 - L}{H} \right) v_2^{(\frac{1-\sigma}{\sigma})} \right]^{\frac{\sigma}{1-\sigma}}.$$

There are two points about this expression. First, the stand-in's utility function inherits the long-run invariance of hours to trend changes in productivity from the underlying utility function (3.8). Second, the stand-in's utility function inherits the original utility function's properties with respect to effects of changes in leisure on the marginal utility of consumption. In particular, when $\sigma > 1$, the marginal utility of consumption is decreasing in leisure.

Let us again think about an isolated individual maximizing lifetime utility, $\sum_{t=0}^{\infty} \beta^t u(c_t, L_t)$, but with the new momentary utility function (6.7). As with our discussion of the representative worker in Section 4 and as with our previous discussion in this section, the stand-in agent equates the marginal utility of consumption and the marginal utility of leisure to the shadow values along the economy's resource constraint $(D_1 u(c_t, L_t) = \lambda_t$ and $D_2 u(c_t, L_t) = \lambda_t w_t = \lambda_t A_t D_2 F(k_t, N_t))$. These conditions must always hold if there is an interior optimum for work effort, i.e., $0 < L_t < 1$ in each period. Taking loglinear approximations to this pair of conditions, we find

$$-\sigma \hat{c}_t + (1 - \sigma) \kappa \widehat{L}_t \qquad = \widehat{\lambda}_t, \tag{6.8}$$

$$(1 - \sigma) \hat{c}_t - \left[\frac{(1 - \sigma)^2}{\sigma} \kappa \right] \widehat{L}_t = (\widehat{\lambda}_t + \widehat{w}_t), \tag{6.9}$$

where $\kappa = \frac{LDv^*(L)}{v^*(L)}$ is pinned down by information on the steady state of the economy[50].

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[49] There are two steps to this demonstration. First, one shows that efficient risk-sharing implies that expected utility is proportional to:

$$\frac{1}{1 - \sigma} \left\{ c^{1-\sigma} \left[p v_1^{(\frac{1-\sigma}{\sigma})} + (1 - p) v_2^{(\frac{1-\sigma}{\sigma})} \right]^{\sigma} - 1 \right\} \quad \text{if} \quad \sigma \neq 1,$$

and then one substitutes in for leisure using $L = 1 - pH$.

[50] That is, $\kappa = \dfrac{LDv^*(L)}{v^*(L)} = \dfrac{LD_2 u(c, L)}{CD_1 u(c, L)} = \dfrac{Lw}{c} = \dfrac{L}{N} \dfrac{(wN/y)}{(c/y)}$.

This set of equations reveals that there is infinitely elastic labor supply even when the preference specification is not separable. That is, the pair of equations implies that

$$0 = \widehat{\lambda}_t + \sigma\widehat{w}_t$$

which is the statement that the stand-in will supply any amount of work at a particular real wage. But because preferences are nonseparable, variations in work require variations in consumption. When $\sigma > 1$, in particular, workers require more consumption than nonworkers and aggregate consumption is negatively related to leisure, i.e.,

$$\widehat{c}_t = \left[\frac{(1-\sigma)}{\sigma}\kappa\right]\widehat{L}_t - \frac{1}{\sigma}\widehat{\lambda}_t.$$

Thus, this model involves a modified form of the permanent income hypothesis, which includes the effects of changes in work effort on the marginal utility of consumption. Baxter and Jermann (1999) have argued that this type of preference nonseparability will arise in any model with household production; they have also stressed that this specification can make consumption more cyclically volatile.

6.2. Capacity utilization

In the standard version of the neoclassical model, there is a dramatic contrast between the short run and long run elasticities of capital supply. The short run elasticity of capital supply is zero: there is no way for the economy to increase the capital stock inherited from the previous period. In contrast, the long run elasticity of capital supply is infinity: there is only one real interest rate consistent with the steady state of the economy. This difference between short run and long run elasticities stems from the assumption that capital services are proportional to the stock of capital. This is an assumption we make every time we write a production function as $Y = F(K,N)$. While this assumption may be suitable for some purposes, it is clearly problematic for business cycle analysis. The third panel of Figure 3 suggests that capacity utilization displays pronounced cyclical variability. The fact that equipment and machinery are used more intensively in booms than in recessions is corroborated by the procyclical character of electricity consumption in manufacturing industries [Burnside, Eichenbaum and Rebelo (1995)] and by the fact that expansions are accompanied by the use of two and three shifts in manufacturing industries [Shapiro (1993)]. All this evidence suggests that the flow of capital services is high in expansions. In contrast, recessions are times when capital tends to lie idle, thus producing a small service flow.

Several authors have extended the basic RBC model to incorporate variable capital utilization. Kydland and Prescott (1988) showed that introducing time-varying capital utilization enhanced the amplification capability of their 1982 model. Greenwood, Hercowitz and Huffman (1988) introduced variable utilization in a model that features

shocks to the productivity of new investment goods[51]. Finn (1991) used a similar framework to study the interaction between capital utilization and energy costs. In her model, more intensive capital use accelerates the depreciation of capital and raises marginal electricity consumption. Burnside and Eichenbaum (1996) explored a model with both capital utilization and labor hoarding. They showed that these two features significantly enhance the ability of the model to propagate shocks through time[52].

Modeling variable utilization. Most studies of variable utilization assume that depreciation is an increasing function of the utilization rate[53]. The benefits from variable capital utilization can be incorporated into the production function as follows:

$$Y_t = A_t F(z_t K_t, N_t X_t) = A_t (z_t K_t)^{1-\alpha} (N_t X_t)^\alpha.$$

where z_t denotes the utilization rate[54]. The costs of variable capital utilization are imbedded in the following law of motion for the capital stock:

$$K_{t+1} = I_t + [1 - \delta(z_t)] K_t,$$

where $\delta(\cdot)$ is a convex, increasing function of the utilization rate[55].

To determine its optimal rate of utilization, a firm maximizes its profits holding fixed its future capital stock. The marginal benefit of a higher utilization rate is additional output $(A_t D_1 F(z_t K_t, N_t X_t) K_t)$ and the marginal cost is higher (replacement) investment $(dI_t = D\delta(z_t) K_t)$. Equating these and using the Cobb–Douglas production function, we find that efficient utilization implies

$$(1 - \alpha) A_t (z_t)^{-\alpha} (K_t)^{1-\alpha} (N_t X_t)^\alpha = D\delta(z_t) K_t, \tag{6.10}$$

which is the requirement that the marginal benefit in terms of additional output produced be equated to the marginal cost in terms of additional units of capital being worn out.

The consequences of variable utilization. To explore how efficient variation in the utilization rate affects the linkages in the economy, we linearize Equation (6.10) to

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[51] These shocks tend to make consumption and investment move in opposite directions. Introducing capital utilization eliminates this counterfactual correlation between consumption and investment.

[52] Their model is also capable of producing a humped shape response of investment to technology shocks – a feature that is common in empirical impulse response functions estimated using VAR techniques.

[53] An exception is Kydland and Prescott (1988).

[54] For simplicity, we use the Cobb–Douglas form throughout our discussion of capital utilization.

[55] Thus, it has a positive first derivative $D\delta(\cdot)$ and a positive second derivative $D^2\delta(\cdot)$.

obtain an expression for $\hat{z}_t$ and substitute this result into the linearized production function:

$$
\begin{aligned}
\hat{y}_t &= \hat{A}_t + \alpha \hat{N}_t + (1 - \alpha)(\hat{k}_t + \hat{z}_t) \\
&= \hat{A}_t + (1 - \alpha)\hat{k}_t + \alpha \hat{N}_t + \frac{1 - \alpha}{\alpha + \xi}\left(\hat{A}_t - \alpha\hat{k}_t + \alpha\hat{N}_t\right).
\end{aligned}
\tag{6.11}
$$

In this expression ξ represents the elasticity of $D\delta(z_t)$, which is positive if there is increasing marginal depreciation cost of higher utilization[56]. The model without utilization occurs as a special case in which ξ is driven to infinity, since in that case the quantity of capital services does not respond to changes in the marginal product of these services. At the other extreme, as ξ is driven toward zero, the response of output becomes $\hat{y}_t = \frac{1}{\alpha}\hat{A}_t + \hat{N}_t$. For this reason, time-varying capital utilization is sometimes described as leading to a short-run production function that is nearly linear in labor.

Variable utilization makes the marginal product of labor – the real wage rate – less responsive to changes in labor input. The comparable log-linear expression for the real wage rate is

$$
\widehat{w}_t = (\hat{y}_t - \hat{N}_t) = \hat{A}_t + (1 - \alpha)\hat{k}_t + (\alpha - 1)\hat{N}_t + \frac{1 - \alpha}{\alpha + \xi}\left(\hat{A}_t - \alpha\hat{k}_t + \alpha\hat{N}_t\right).
\tag{6.12}
$$

and, as ξ is driven toward zero, the response of the real wage approaches $\widehat{w}_t = \frac{1}{\alpha}\hat{A}_t$. In other words, the labor demand schedule drawn in (w, N) space "flattens" as depreciation becomes less costly on the margin (ξ falls). When ξ is driven to zero, the labor demand curve becomes completely flat.

7. Remeasuring productivity shocks

We have seen that productivity shocks are an essential ingredient of real business cycle models. In the absence of measurement error in labor and capital services, these shocks coincide with the Solow residual. Prescott (1986) used the Solow residual as a measure of technology shocks to conclude that these shocks "account for more than half the fluctuations in the postwar period with a best point estimate near 75%".

There are three reasons to distrust the standard Solow residual as a measure of technology shocks. First, Hall (1988) has shown that the Solow residual can be forecasted using variables such as military spending, which are unlikely to cause changes in total factor productivity. Similarly, Evans (1992) showed that lagged values of various monetary aggregates also help forecast the Solow residual. Second, the conventional Solow residual implies probabilities of technological regress that are implausibly large. Burnside, Eichenbaum and Rebelo (1996) estimate that the

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[56] It can be shown that $\xi = z(D^2\delta)/D\delta > 0$.

probability of technological regress associated with the conventional Solow residual is 37% in US manufacturing. Finally, cyclical variations in labor effort ("labor hoarding") and capital utilization can significantly contaminate the Solow residual.

There are two strategies for dealing with these extra, hard-to-measure sources of factor variation. The first strategy is to use an observable indicator to proxy for the unobserved margin. For example, since individuals working harder may have more accidents in an industrial setting, the frequency of worker accidents could be used as an indicator of unobserved effort[57]. More commonly, electricity consumption in manufacturing industries is taken as an indicator of capacity utilization. The second strategy is to use implications of the model to solve out for the unobserved factor variation and then to examine other implications of the model economy. We discuss application of each of these strategies to measuring capacity utilization in the remainder of this section.

Capital utilization proxies: Burnside, Eichenbaum and Rebelo (1996) employ electricity use as a proxy for capacity utilization. In particular, assuming that the utilization rate is proportional to electricity utilization, they can use the Solow decomposition in modified form,

$$\log SR_t^* = \log Y_t - \alpha \log N_t - (1 - \alpha)[\log K_t + \log(z_t^*)], \tag{7.1}$$

where $\log(z_t^*)$ is the log of electricity use. They find that when electricity use is employed as a proxy for capital services the character of the Solow residual associated with the manufacturing sector changes dramatically: (i) there is a 70% drop in the volatility of the growth rate of productivity shocks relative to output, implying that a successful model must display much stronger amplification than the basic RBC model; (ii) the hypothesis that the growth rate of productivity is uncorrelated with the growth rate of output cannot be rejected; and (iii) the probability of technological regress assumes much more plausible values, dropping to 10% in quarterly data and to 0% in annual data. These corrections to the Solow residual significantly reduce the fraction of output variability that can be explained as emanating from shocks to technology[58].

Using the model to measure capacity utilization: An alternative strategy is to use the model's implications for efficient utilization to solve for the unobserved

---

[57] Several variants of this proxy strategy have been used to shed indirect light on the presence of labor hoarding. Bils and Cho (1994) use time and motion studies to document the presence of variability in effort. Shea (1992) uses data on on-the-job accidents to construct an indirect measure of labor hoarding. Burnside, Eichenbaum and Rebelo (1993), Sbordone (1997), and Basu and Kimball (1997) postulated a model of labor hoarding that they proceeded to use to purge the Solow residual of variations in the level of effort.

[58] Aiyagari (1994) proposed a method to compute a lower bound on the contribution of technology shocks to output volatility. His procedure relies on knowledge of two moments in the data: the variability of hours relative to the variability of output and the correlation between hours worked and labor productivity (which is essentially zero in the data). Unfortunately, his method is not robust to the presence of labor hoarding or capacity utilization.

utilization decision, i.e., z_t. In essence, this empirical strategy corresponds to our theoretical method in the previous section, when we solved out for z_t in order to derive Equation (6.11), which describe how output responds to changes in labor, capital and productivity when utilization is efficiently varied. One possibility would be to exactly follow this strategy, substituting observed variations in labor and capital into Equation (6.11) to compute the productivity residual, but we use a more "reduced form" approach that we describe more fully in the next section.

8. Business cycles in a high substitution economy

Motivated by the vanishing productivity shock, we now construct an economy in which small variations in productivity can have large effects on macroeconomic activity, i.e., an RBC model in which there is substantial amplification of shocks. There are two central ingredients to this model. First, as in Section 6.1, we assume that there is indivisible labor. This makes the supply of aggregate hours strongly responsive to changes in wages and intertemporal prices. Second, as in Section 6.2, we assume that there is variable capacity utilization. This makes the supply of capital services strongly responsive to changes in the level of aggregate hours. Taken together, these ingredients mean that the economy has high substitution in all factors of production. Further, the Solow residual is a very poor measure of technology shocks in our model economy. However, the very same structural feature that makes the Solow residual a bad measure of technology shocks (unmeasured variation in capital services) also provides a powerful amplification mechanism that allows our model to account for the observed output variation with much smaller shocks. Finally, our model provides a means of implicitly measuring the smaller shocks that occur, which can be viewed as a variant of Solow's approach[59].

8.1. Specification and calibration

The specification and calibration of the model follows the same general approach that we used in Section 4, but with some relatively minor modifications.

Restrictions on the steady state: First, we know that the production side of the basic model determines most aspects of the steady state and that continues to be true with variable capital utilization. The efficiency condition for utilization in the steady state determines a steady-state utilization rate such that $r + \delta(z) = D\delta(z)$, with the remainder of the steady-state relative prices and great ratios then adjusted to reflect the fact that the flow of capital services is zK rather than K.

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[59] The approach was suggested by Mario Crucini in unpublished research many years ago, so perhaps we should call these "Crucini residuals". Another application is contained in Burnside, Eichenbaum and Rebelo's (1993) study of unobserved effort (labor hoarding). Ingram, Kocherlakota and Savin (1997) use a similar procedure to infer information on observed shocks to the home production sector.

Table 4
Calibration of high substitution economy

σ	b	γ	α	δ	ξ	ρ	σ_ε
3	0.984	1.004	0.667	0.025	0.1	0.9892	0.0012

Second, since we are assuming an indivisible labor model, there is a different calibration of the preference side of the model. Evidence from asset pricing studies suggests that σ is larger than the unit value used in the basic model; this means that our model will have the realistic implication that more consumption is allocated to working individuals than to nonworking individuals. Drawing on Kocherlakota's (1996) review, we use $\sigma = 3$. We assume that 60% of the population is employed in the steady state and that employed individuals work 40 hours. This implies that an *average* individual's hours are $N = 0.214$, i.e., 24 hours out of a weekly 112 hours of nonsleeping time. Then, this information (including the assumed value of σ) determines the ratio $v(1)/v(1-H)$ which dictates the ratio of consumptions of the two types of individuals. It turns out that the ratio c_1/c_2 is 3.31 so that workers have substantially higher consumption than nonworkers. Table 4 summarizes our parameter assumptions. Unless otherwise discussed, the parameters are the same as in Table 2.

Measuring technology shocks: We use the implications of our model as discussed in the last section to produce a series on technology shocks which is consistent with unobserved variation in capacity utilization[60]. In particular, we start by assuming a value for the persistence and volatility of technology shocks and solve the model. The decision rule for output can be written as

$$\hat{y}_t = \pi_{yk}\hat{k}_t + \pi_{yA}\hat{A}_t.$$

Using this decision rule together with data for output and capital (which we logged and linearly detrended), we can compute an initial guess about the time series for technology shocks[61]:

$$\hat{A}_t = \frac{1}{\pi_{yA}}\hat{y}_t - \frac{\pi_{yk}}{\pi_{yA}}\hat{k}_t.$$

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[60] There is no unique way of computing this shock process, but rather any of the model's decision rules could be used in this way or these rules could be combined with other relationships in the model. For example, one could exploit the decision rule for utilization as in Burnside, Eichenbaum and Rebelo's (1993) analysis of labor hoarding, $\hat{z}_t = \pi_{yk}\hat{k}_t + \pi_{yA}\hat{A}_t$, and combine this with the modified Solow decomposition (7.1). This alternative method would produce a different shock process, which lead to broadly similar, but somewhat less dramatic results. The difference between these two productivity measures lies in whether labor in Equation (7.1) is taken from the data or from the model.

[61] We should not use the empirical capital stock series since these are flawed in the eyes of the model: they are computed assuming constant rates of depreciation. This can be circumvented by using a second decision rule to compute the "true" capital stock series. In practice this has little impact on the results.

This guess is not exactly right because the serial correlation coefficient (ρ) for this $\hat{A}_t$ series need not match that used to solve the model and to construct the π coefficients. Therefore, once we obtain a time series for $\hat{A}_t$, we compute its persistence (ρ) and use this new value to solve the model again. Using the new decision rule, we recompute $\hat{A}_t$ and once again its calculate its persistence. We continue this process until the new and old estimates for the serial correlation of $\hat{A}_t$ are the same. This iterative procedure yielded an estimate of 0.9892 for the first-order serial correlation and 0.0012 for the standard deviation of the ε_t.

8.2. Simulating the high substitution economy

With a series of productivity shocks in hand, we simulated our model economy's response to these shocks just as we previously did for the standard RBC model. Figure 13 displays the results, which we think are dramatic. Panel 1 shows the model and actual paths for output, which are virtually identical. In part, this is an artifact of our procedure for constructing the technology shock, which is a weighted average of output and capital as we just discussed. For this reason, we think that the performance of the model should not be evaluated along this dimension. Instead, the model has to be judged by its predictions for other variables of interest. The remaining panels of Figure 13 display the model's implications for total hours worked, consumption and investment, with all of these series detrended with the HP filter. The correlation between the empirical and the simulated series is 0.89 for labor, 0.74 for consumption and 0.79 for investment! This remarkable correspondence leads to three sets of questions, similar to those which arose in the analysis of the standard RBC model. First, how do small variations in productivity have such dramatic effects? Second, what are the properties of the technology shocks? Third, how sensitive are the results?

8.3. How does the high substitution economy work?

The high substitution economy contains four mechanisms that substantially amplify productivity shocks and lead to strong comovements of output, labor, consumption and investment. To begin, variable capacity utilization makes output respond more elastically to productivity shocks in Equation (6.11), which we repeat here for the reader's convenience:

$$\hat{y}_t = \hat{A}_t + (1 - \alpha)\,\hat{k}_t + \alpha\hat{N}_t + \frac{1 - \alpha}{\alpha + \xi}\left(\hat{A}_t - \alpha\hat{k}_t + \alpha\hat{N}_t\right).$$

Since utilization of capital increases when there is a positive productivity shock, there is a direct effect which is part of the amplification mechanism. In the limiting case of $\xi = 0$ for example, a labor's share of $\alpha = \frac{2}{3}$ implies that the productivity shock raises output by $\frac{1}{\alpha}$ or $\frac{3}{2}$ times its direct effect. We use a value of $\xi = 0.1$ in constructing our simulations, so that the effect with $\alpha = \frac{2}{3}$ is $1 + \frac{1-\alpha}{\alpha+\xi} = 1 + \frac{0.33}{0.77} = 1.43$. Thus, variable utilization helps create amplification, but only in a modest manner.
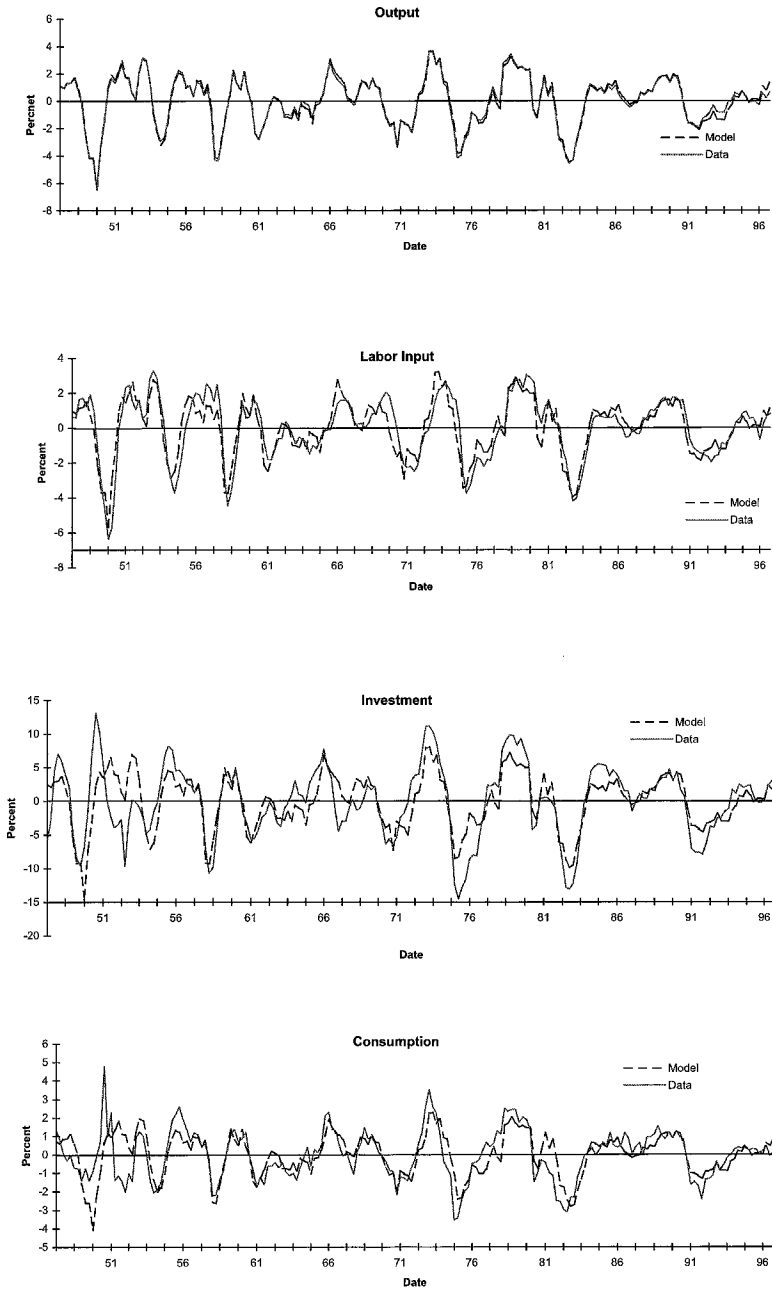
Fig. 13. Capacity utilization model: simulated business cycles. Sample period is 1947:2–1996:4. All variables are detrended using the Hodrick–Prescott filter.

Relative to the standard RBC model that we discussed in Section 4, most of the increased amplification in the model of this section comes from greater elasticity of the labor demand and labor supply schedules. Highly elastic labor supply is due to indivisible labor: work effort is highly responsive to small changes in its rewards. In fact, we have previously argued that it is the demand side which approximately determines this quantity in indivisible labor economies. Variable capacity utilization makes the labor demand more elastic. As discussed above, labor demand is implicit in the equation:

$$\widehat{w}_t = (\hat{y}_t - \widehat{N}_t) = \hat{A}_t + (1 - \alpha)\,\hat{k}_t + (\alpha - 1)\,\hat{N}_t + \frac{1 - \alpha}{\alpha + \xi}\left(\hat{A}_t - \alpha\hat{k}_t + \alpha\hat{N}_t\right).$$

In the model without variable utilization (or with $\xi = \infty$), a one percent increase in labor quantity causes the real wage to fall by 0.333 percent when $\alpha = \frac{2}{3}$, since the coefficient on $\hat{N}_t$ is $(\alpha - 1)$. At the other extreme, as ξ is driven toward zero, the response of the real wage to a productivity shock approaches $\widehat{w}_t = \frac{1}{\alpha}\hat{A}_t$, i.e., the labor demand schedule becomes more elastic until it is completely elastic in the limit. With variable utilization, the combined coefficient on labor is $(\alpha - 1) + \frac{1-\alpha}{\alpha+\xi}\alpha$. Using $\alpha = \frac{2}{3}$ and $\xi = 0.10$, as in our simulations, we find that the combined coefficient is $(0.67 - 1) + \frac{0.33}{0.77}0.67 = -0.043$: a one percent change in labor requires a decline in the wage that is an order of magnitude smaller than in the standard model. With indivisible labor *and* variable utilization, a small productivity shock shifts up labor demand and calls forth a large increase in labor supply. In order to determine the exact size of this change, however, it is essential that we simultaneously determine the path of capital (k_t) and the multiplier (λ_t).

The final structural feature that is important for the simulated time series is the nonseparable form of the utility function. In the standard Hansen-Rogerson case of log utility, most of the model's change in output goes into investment rather than consumption. However, since the efficient plan calls for the allocation of more consumption to employed individuals when $\sigma > 1$, the high substitution economy displayed in Figure 13 involves more volatile consumption that corresponds closer to the data. We return to a discussion of this feature in the context of impulse responses later in this section.

8.4. What are the properties of the shocks?

Is this remarkable coherence between data and model achieved by using an empirically unpalatable productivity shock as a driving force? Figure 14 answers this question. The first panel depicts the level of the productivity, which involves a combination of the deterministic trend and stochastic component (i.e., $A_t X_t^{\alpha}$). It increases through time smoothly in the manner that many economists believe is appropriate for the level of technology. The second panel of Figure 14 shows the growth rate of productivity in our economy. This graph shows that the average rate of technical progress is large

A. Model Productivity Level

B. Model Productivity Growth

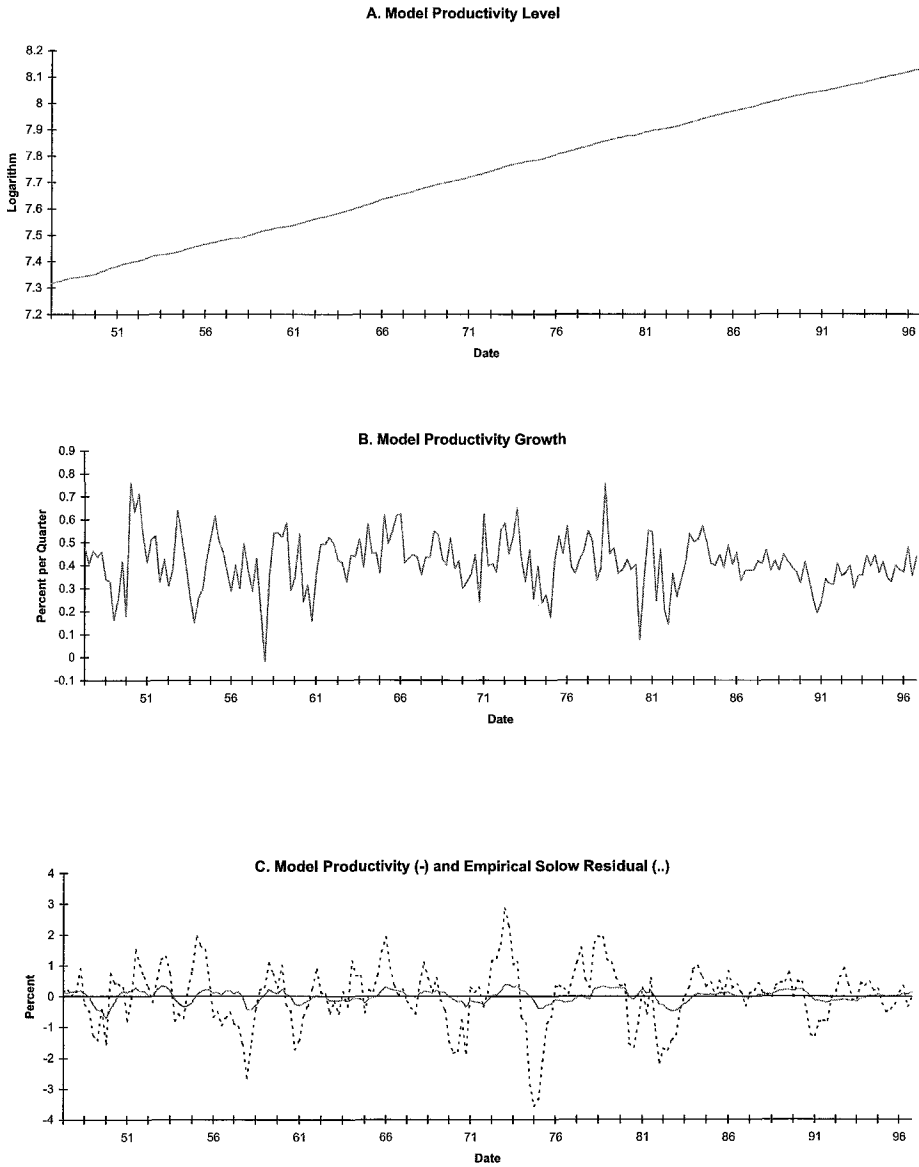**C. Model Productivity (-) and Empirical Solow Residual (..)**

Fig. 14. Capacity utilization model: productivity and the Solow residual.

enough that a measured technological regress occurs only very rarely, given the low variability of technology shocks. The third panel of Figure 14 graphs business cycle variation in the conventional Solow residual and the productivity shock used in our model, using the HP filter to create these components. Figure 14 illustrates that it

Table 5
Sensitivity analysis to different ξ values

ξ value	Standard deviations						Persistence parameter (ρ)	Likelihood of technical regress
	Y	c	I	N	A	ε		
∞	1.36	1.01	2.62	0.90	0.79	0.0061	0.9783	0.1859
1	1.39	0.94	2.86	1.07	0.45	0.0034	0.9798	0.1106
$\frac{1}{2}$	1.40	0.92	2.94	1.13	0.34	0.0026	0.9822	0.0653
$\frac{1}{3}$	1.40	0.91	2.99	1.16	0.28	0.0021	0.9841	0.0352
$\frac{1}{5}$	1.41	0.91	3.05	1.20	0.22	0.0017	0.9866	0.0101
$\frac{1}{7}$	1.41	0.90	3.09	1.23	0.19	0.0014	0.9880	0.0050
$\frac{1}{10}$	1.42	0.89	3.15	1.26	0.15	0.0012	0.9892	0.0050

is possible to explain the variability of US macroeconomic activity with productivity shocks that are much smaller than those conventionally used in the literature.

8.5. How sensitive are the results?

We next discuss the sensitivity of our results to the choice of parameters and to the measurement of output.

Sensitivity to parameterization. The value chosen for the parameter ξ is a key ingredient in the results. This is not surprising since we know that when ξ equals infinity the model with capital utilization reduces essentially to the standard model. Table 5 shows how some key model statistics change with different values for ξ. For every value of ξ we used the iterative process described above to ensure that the stochastic process assumed for A_t is in fact consistent with the properties of the technology shock implied by the model. In every case we report the persistence of the shock (ρ) and the standard deviation of the innovation (ε) as well as the implied probability of technological regress. Low probabilities of technological regress can be obtained for values of ξ that are lower than $\frac{1}{3}$.

As an alternative check on the sensitivity of the model to ξ, Figure 15 depicts the impulse response for this model for three values of ξ: ∞, $\frac{1}{5}$ and $\frac{1}{10}$. To simplify the comparison between these impulse responses we did not adjust the stochastic process for the technology shock. All three responses were computed with the same standard deviation of innovation ($\sigma_\varepsilon = 0.0072$) and same persistence ($\rho = 0.979$).

The three impulse response functions depicted in this figure have similar dynamic properties, but vary mostly in the degree of amplification. The solid line is a fixed capital utilization model ($\xi = \infty$) like the basic RBC model of Section 4, but with indivisible labor. In this model, a productivity shock has a larger effect on output than in the standard RBC model: when there is a one percent productivity shock, output rises by just less than two percent on impact with fixed utilization ($\xi = \infty$). However, the
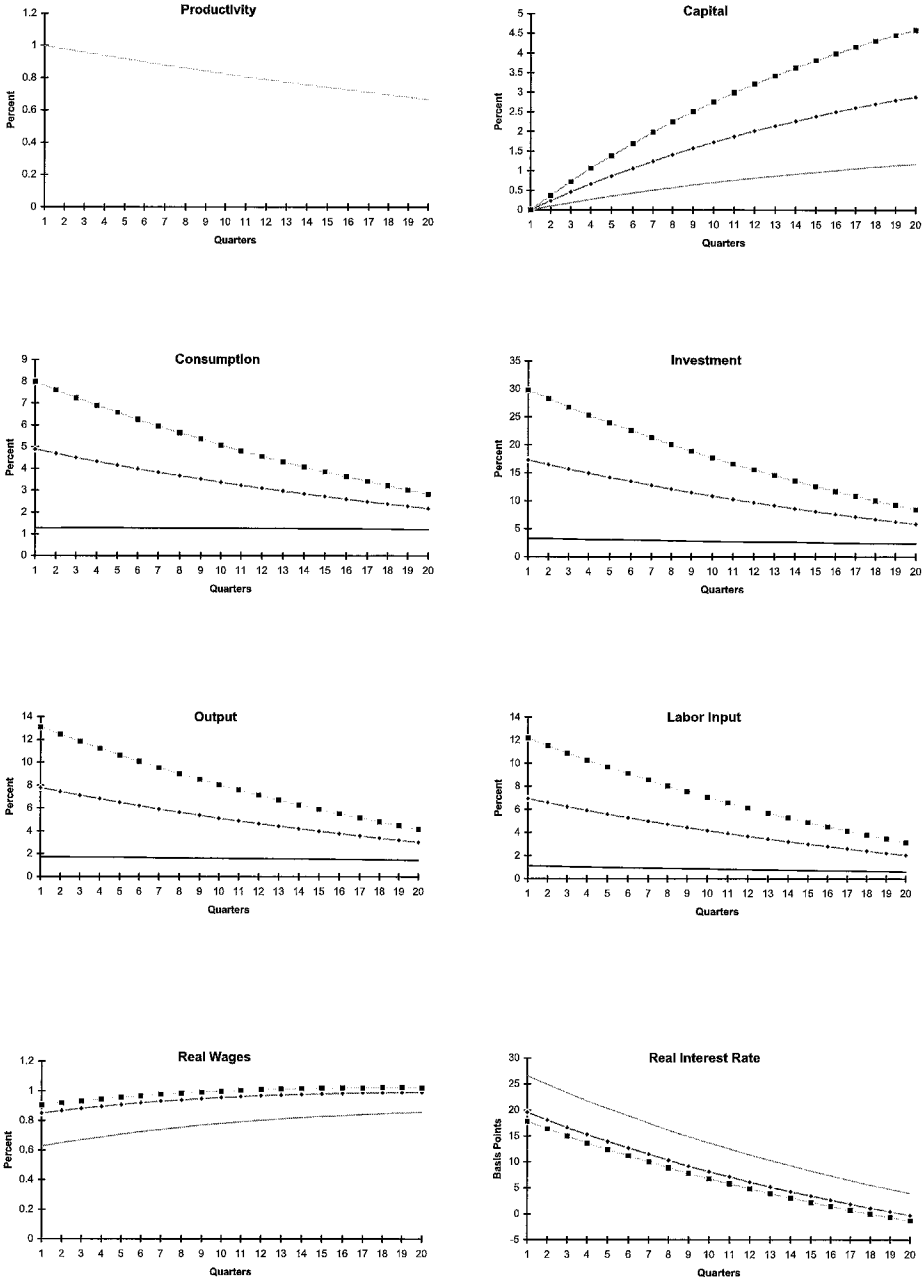
Fig. 15. Sensitivity analysis to alternative utilization costs (ξ): solid line, $\xi = \infty$; diamonds, $\xi = \frac{1}{5}$; squares, $\xi = \frac{1}{10}$.

increase in amplification is small relative to what happens when indivisible labor and capacity utilization are introduced simultaneously. A one percent productivity shock has an impact effect on output of 8 percent when $\xi = 0.2$ and of 13 percent when $\xi = 0.1$. From various experiments with this model economy, it is clear that values of ξ less than one are important to obtain substantial amplification. For example, if there is a value of $\xi = 1$ then there continues to be on average a regress in the level of productivity every ten quarters (see Table 5 above). It is important that econometric evidence be produced on the cost of varying capital utilization, so as to determine the extent to which this high substitution economy is realistic[62].

One specific feature of the impulse response in Figure 15 is worth some additional discussion. In all of the cases, the real interest rate increases in response to a positive productivity shock, at least for the first twenty quarters shown in the graph. In all cases, the level of consumption broadly resembles the level of output and the growth rate of consumption is negative, even though the real interest rate is high by comparison to its steady-state level. This behavior of consumption reflects the fact that aggregate consumption is the sum of consumptions by individuals that are working and those who are not. Since working agents have more consumption, an increase in the fraction of individuals working makes aggregate consumption rise and fall with aggregate employment[63].

Sensitivity to the measurement of output. We have seen that variable utilization and indivisible labor produce an economy in which (i) small productivity shocks have large effects on output; (ii) the standard Solow residual is substantially mismeasured; and (iii) labor and output move together on an approximately one-for-one basis. In this economy, however, there is an important sense in which output is mismeasured. There is a standard line of intuition which suggests that "intermediate" activities such as utilization should not be too important for economic activity and, in this case, suggests that the large effects of productivity on output and the strong comovement of output and labor are simply artifacts of output mismeasurement. To explore these ideas, output net of depreciation can be defined as

$$\phi_t = y_t - \delta(z_t)k_t = A_t F(z_t k_t, N_t) - \delta(z_t)k_t \tag{8.1}$$

and this expression can be used to make four important points. First, output is also mismeasured in the standard neoclassical model, i.e., even in the absence of a variable depreciation rate. Second, with efficient utilization, changes in net output are

$$
\begin{aligned}
\mathrm{d}\phi_t &= F(z_t k_t, N_t)\,\mathrm{d}A_t + A_t D_2 F(z_t k_t, N_t)\,\mathrm{d}N_t \\
&\quad + A_t D_1 F(z_t k_t, N_t)(k_t\,\mathrm{d}z_t + z_t\,\mathrm{d}k_t) - D\delta(z_t)\,k_t\mathrm{d}z_t - \delta(z_t)\,\mathrm{d}k_t \\
&= F(z_t k_t, N_t)\,\mathrm{d}A_t + A_t D_2 F(z_t k_t, N_t)\,\mathrm{d}N_t + A_t D_1 F(z_t k_t, N_t)\,z_t\,\mathrm{d}k_t - \delta(z_t)\,\mathrm{d}k_t,
\end{aligned}
$$

[62] Basu and Kimball (1997) provide an estimate of a parameter that is essentially our ξ. Their point estimate is about unity, but the parameter is very imprecisely estimated.

[63] Baxter and Jermann (1999) stress that equilibrium models with nonseparable preferences can generate apparent excess sensitivity of consumption to income, working in a model where labor supply variation is on the intensive rather than extensive margin.

where the latter equality follows from $A_t D_2 F(z_t k_t, N_t) k_t \, dz_t - D\delta(z_t) k_t dz_t = 0$ when utilization is efficient. Thus, there is a sense in which the standard intuition is correct because net output does not respond to utilization. Third, near the steady state, the Solow decomposition for net output is

$$\hat{\phi}_t = m\hat{A}_t + ms_N \widehat{N}_t + m \left(s_k - \frac{\delta k}{y} \right) \hat{k}_t, \tag{8.2}$$

where $m = \frac{y}{\phi} = [1 - \frac{\delta k}{y}]^{-1}$. This modification takes into account the fact that the net production function is more labor intensive and the fact that productivity shocks affect gross output but not depreciation. Thus, for example, if depreciation investment is 10% of output, then $m = 1.11$. Thus, if output is measured as net of depreciation – independent of whether capacity utilization affects depreciation – then this will tend to strengthen the magnitude of labor's effect on output. Fourth, most importantly for our purposes, the net production function $\phi_t = A_t F(N_t, z_t k_t) - \delta(z_t) k_t$ has the same marginal product schedule for labor, $A_t D_1 F(N_t, z_t k_t)$ as does the standard production function. Thus, our analysis of the "labor demand" consequences of efficient utilization are unaffected by whether depreciation costs are deducted from output or whether they are not. Returning to Equations (6.11) and (6.12), we can thus see that a "net output" measurement requires that we replace Equation (6.12) with the modified growth accounting expression (8.2), but that we need not change the labor demand schedule (6.11) at all. Further, it is a highly elastic labor demand that is the key force behind the great amplification present in our high substitution economy.

9. Conclusions

This chapter provides a perspective on developments in the literature on real business cycles over the last decade. We discussed the structure of these models, their successes and their deficiencies. We also argued that three main criticisms levied against first-generation real business cycle models have been largely overcome. First, the performance of the basic RBC model has proved to be remarkably resilient to alternative parameterizations, including versions in which the elasticity of labor supply is small at an individual level but large in the aggregate economy. Second, the model has been usefully extended to accommodate more realistic price behavior. Finally, we showed by example that there are RBC models which can provide enough amplification so that the underlying technology shocks can be small and involve a low probability of technological regress[64]. Our example made clear that major

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[64] Just as this first round of problems is set to rest, new challenges arise for the RBC model regarding the comovement between productivity and economic activity. In a recent paper, Gali (1996) argues, using VAR techniques, that technology shocks actually reduce input usage in the aggregate economy.

amplification of productivity shocks requires highly elastic labor supply and readily variable capital utilization[65].

Although we have concentrated on the one sector neoclassical model, which has been the central laboratory for most work on real business cycles, the next stages of RBC research will likely use richer frameworks, as we discuss next. However, we believe that the exploration of these richer frameworks will require consideration of the structural features that we have stressed in this chapter.

One exciting research direction is the exploration of models with multiple sectors, i.e., a long overdue continuation of the trail scouted by Long and Plosser (1983). Interesting recent work on these models retains most of the assumptions on preferences and production opportunities commonly incorporated in the one-sector RBC model [Horvath (1997), Dupor (1998)]. The one-to-one movement between hours and output observed in aggregate data also holds at a sectoral level. To us, this suggests the importance of introducing variable capacity utilization into sectoral production structures. Another promising direction is work on models with heterogeneous agents [Krusell and Smith (1998), den Haan (1993)]. This work seems particularly important for enriching labor market dynamics and modeling unemployment[66]. Fleshing out labor market dynamics is important on its own terms. But this work may also provide us with an alternative way of obtaining a highly elastic aggregate labor supply which appears necessary for RBC modeling. A third interesting research direction seeks to understand the industry dynamics that seem intimately related to the business cycle [Hopenhayn and Rogerson (1993), J.R. Campbell (1998)]. Finally, there are many aspects of microeconomic activity in addition to employment in which discrete choice seems very important. It has frequently been suggested, for example, that the volatility of investment is related to the fact that much of firm investment is lumpy in character [Abel and Eberly (1995), Caballero and Engel (1994)]. The incorporation of lumpy investment decisions into the RBC model and its implications for aggregate dynamics is an exciting new direction of research on which some initial progress has been made [Veracierto (1996), Thomas (1997)]. The interaction of lumpy investment with costly capacity utilization seems a particular important topic of investigation. All four of

This finding receives indirect support from two sources that do not rely on the VAR methodology: Burnside, Eichenbaum and Rebelo (1996) document that a sectoral capital-utilization adjusted measure of technology shocks is essentially uncorrelated with production in 2-digit SIC manufacturing industries. Basu, Fernald and Kimball (1997) show that, for this same set of industries, input usage is negatively correlated with technology shocks. One interpretation of these facts is that they reflect the presence of nominal rigidities that keep nominal aggregate demand fixed and lead inputs to contract in response to a productivity increase [Gali (1996)]. An alternative flexible-price explanation for these same facts involves a multisector model in which goods are complements so that a technology shock to an individual sector does not necessarily warrant an expansion of input usage in that sector.

[65] An interesting and open question is whether these same mechanisms can amplify other shocks besides productivity shocks sufficiently that these can produce realistic business cycles.

[66] Some recent examples include Andolfatto (1996), Merz (1995), Gomes, Greenwood and Rebelo (1997) and den Haan, Ramey and Watson (1997).

these lines of inquiry involve enriching the RBC model in ways that seemed virtually impossible a decade ago.

While we think that economists may have prematurely dismissed the idea that the business cycle may originate from real causes, we also think that many of the lessons drawn from current and future RBC research are likely to be independent of the main source of business fluctuations. This is one important reason why the RBC literature has been a positive technology shock to macroeconomics.

Appendix A. Dynamic theory

This Appendix discusses some theoretical aspects that underlie the construction of Real Business Cycle Models.

A.1. Assumptions on preferences and technology

The specific forms of the momentary utility and production functions used in RBC models may seen arbitrary, but they are typically chosen on the basis of economic theory and empirical observation.

Preferences: In order for preferences to be consistent with steady-state growth in a deterministic version of the basic RBC model they must have two properties: (i) households must be willing to expand their consumption at a constant rate when the real interest rate is constant; and (ii) it must be optimal for households to supply a constant number of hours when the real interest rate is constant and the real wage rate grows at a constant rate.

King, Plosser and Rebelo [1988a] study the admissible utility specifications when utility depends on "pure leisure" (L) [67]. These two requirements imply that momentary utility must have the form

$$u(C, L) = \begin{cases} \frac{1}{1-\sigma}[C\nu(L)]^{1-\sigma} - \frac{1}{1-\sigma} & \text{if} \quad \sigma > 0, \quad \sigma \neq 1, \\ \log(C) + \log \nu(L) & \text{if} \quad \sigma = 1. \end{cases} \tag{A.1}$$

It is easy to verify two properties of these specifications. First, if agents have a budget constraint for goods and leisure of the form $c + wL \leqslant w$, where w is the real wage rate and 1 is the time endowment, then there is invariance of L to the level of w [68].

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[67] Another possibility is that utility depends on leisure in efficiency units, i.e., on leisure augmented by technological progress $(L_t X_t)$. In this case it is sufficient to assume that $u(C, LX)$ is homogeneous, of class C^2, and concave. The dependency of utility on leisure measured in efficiency units can be justified by introducing home production into the model. See Greenwood, Rogerson and Wright (1995, pp. 161–162) for a discussion.

[68] This invariance extends to a setting where the budget constraint includes nonwage income which grows at the same rate as the real wage.

Second, using L'Hopital's rule, the second case is the limiting expression of the first as $\sigma \rightarrow 1$. We require that utility be sufficiently differentiable as well as concave and increasing in consumption and leisure; this implies restrictions that must be placed on v which depend on the value of σ [69]. Differentiability allows us to characterize efficient allocations using variational methods. When combined with convexity of the constraint set, concavity of preferences insures that the solution to the planner's problem is unique, whenever lifetime utility (U) is finite [70]. Since, as we will see shortly, the competitive equilibrium under rational expectations coincides with the solution to the planner's problem, this guarantees that the competitive equilibrium is also unique.

Technology: The production function $F(\cdot)$ is also twice continuously differentiable, concave and homogeneous of degree one. Constant returns to scale implies that the number of firms in the competitive equilibrium is undetermined. With increasing returns to scale a competitive equilibrium does not exist because it would entail negative profits for all firms [71]. In contrast, with decreasing returns to scale we would see an infinite number of infinitesimal firms whose total output would be infinite [72]. Alternatively, firms would earn economic profits if, for some reason, entry were limited.

We assume that $F(\cdot)$ satisfies the following limiting conditions, often referred to as Inada conditions [73]:

$$\lim_{K \rightarrow \infty} D_1 F(K, N) = 0, \qquad \lim_{K \rightarrow 0} D_1 F(K, N) = \infty.$$

These conditions ensure the existence of a steady state in which the level of capital is strictly positive. One can also show that they imply that labor is essential in production: $F(K, 0) = 0$.

---

[69] More specifically, we assume that the functions v_i are twice continuously differentiable. If $\sigma = 1$, then concavity requires that the function $\log(v)$ must be increasing and concave. If σ is not equal to 1, then $v^{1-\sigma}$ must be increasing and concave if $\sigma < 1$ and decreasing and convex if $\sigma > 1$. In addition we need $-\sigma v(L) v''(L) > (1 - 2\sigma)[v'(L)]^2$ to assure the overall concavity of u.

[70] Whenever there is one path that yields infinite utility it is always possible to construct other paths (in fact a continuum of paths) that also yield infinite utility. Thus, to ensure that there is only one solution to the planner's problem we need to constrain the discount factor so that life-time utility, U, is finite. The requirement $(b\gamma^{1-\sigma} < 1)$ involves the interaction of preferences and technology. See Alvarez and Stokey (1999) for a discussion of this type of conditions.

[71] See Hornstein (1993), Rotemberg and Woodford (1995), and Chatterjee and Cooper (1993) for a discussion of models that move away from perfect competition and incorporate increasing returns to scale.

[72] Suppose, for example, that the production function is Cobb–Douglas and that there is a stock of capital K and a number of labor hours N which will be divided equally among n firms. Total production will be given by $Y = nA(K/n)^{\alpha_1} (N/n)^{\alpha_2} = AK^{\alpha_1} N^{\alpha_2} n^{1-\alpha_1-\alpha_2}$. With decreasing returns to scale $\alpha_1 + \alpha_2 < 1$ and $\lim_{n \rightarrow \infty} Y = \infty$.

[73] As in the main text we use the notation $D_i F(\cdot)$ to refer to the partial derivative of $F(\cdot)$ with respect to its ith argument. We use $DF(\cdot)$ to refer to the total derivative of a function of a single variable.

A.2. The dynamic social planning problem

Let us consider first the case in which allocation decisions are made by a benevolent planner who maximizes the welfare of the representative agent. The solution to this problem will be a symmetric Pareto-optimum in which all agents receive the same consumption and leisure allocations.

The stationary economy: In the steady state of a deterministic version of this economy Y, C, I, and K all grow at rate γ, i.e., the model captures the Kaldor growth facts. This suggests that it is useful to write the planner's problem for this economy in terms of variables that are constant in the steady state: $y = Y/X$, $c = C/X$, $i = I/X$, $k = K/X$. Using these stationary variables the planner's problem is given by

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - N_t) \tag{A.2}$$

subject to:

$$y_t = A_t F(k_t, N_t), \tag{A.3}$$

$$y_t = c_t + i_t, \tag{A.4}$$

$$\gamma k_{t+1} = i_t + (1 - \delta)k_t, \tag{A.5}$$

$$k_0 > 0. \tag{A.6}$$

where $\beta \equiv b\gamma^{1-\sigma}$.

In a deterministic environment the solution to the problem of maximizing Equation (A.2) subject to conditions (A.3)–(A.5) would be a sequence of consumption, labor supply and capital accumulation decisions: $\{c_t\}_{t=0}^{\infty}$, $\{N_t\}_{t=0}^{\infty}$, and $\{k_t\}_{t=1}^{\infty}$. These decisions could be made at time zero, since no relevant information is revealed later on. In contrast, in a stochastic economy agents learn over time the realizations of the random shocks that affect their environment. It would be inefficient to ignore this information that will be available later on and cast in stone the consumption and leisure decisions at time zero. For this reason, the solution to the utility maximization problem is a set of contingency rules, which specify how much to consume and work at each point in time as a function of the state of the economy in that period. Since the state of the economy can be, at any point in time summarized by two variables, the value of A_t, which influences current output and helps predict future productivity, and the value of the stock of capital. Thus contingency rules take the form $c = c(k, A)$ and $N = N(k, A)$.

Dynamic programming: To use this approach, we write the planner's problem in recursive form as

$$V(k, A) = \max_{c, N, k'} \{u(c, 1 - N) + \beta E V(k', A')\}, \tag{A.7}$$

subject to: $\quad c + \gamma k' - (1 - \delta)k = A F(k, N). \tag{A.8}$$

where we use primes $(')$ to denote the value of a variable in the next period. The value function $V(k, A)$ represents the expected life-time utility of the representative

agent of an economy with a capital stock equal to k and a level of productivity
equal to A. Equation (A.7) decomposes this life-time utility into two parts: the utility
flow that accrues in the current period, $u(c, L)$, and the expected utility that results from
starting tomorrow with a stock of capital k' and a shock A' and proceeding optimally
from then on. The planner will decide today on the value of k', so this variable is
known with certainty at time t. However, the value of A' will only be known in the
next period, so we have to compute the expectation of $\beta V(k', A')$ with respect to A':
$\beta EV(k', A') = \beta \int V(k', A')H(dA', A)$. Bellman's Principle of Optimality guarantees
that the solution to the problem (A.2)–(A.5) coincides with the solution to the recursive
problem (A.7)–(A.8) [see Stokey, Lucas and Prescott (1989), Section 9.1].

The efficiency conditions for the planning problem can be computed forming a
Lagrangian in which Equation (A.7) is the objective and Equation (A.8) the constraint.
The optimal value of c is dictated by

$$D_1 u(c, 1 - N) = \lambda, \tag{A.9}$$

where λ is the multiplier associated with the constraint (A.8).

The optimal value of N, which we assume has an interior solution $(0 < N < 1)$, is
given by

$$D_2 u(c, 1 - N) = \lambda A D_2 F(k, N). \tag{A.10}$$

The optimal k' is given by

$$\lambda \gamma = \beta E D_1 V(k', A')$$

This condition involves the expectation of the term $D_1 V(k', A')$, which is unknown,
since we do not know the form of the value function. Information about $D_1 V(k, A)$
can, however, be obtained differentiating the Lagrangian with respect to k:

$$\begin{aligned}
D_1 V(k, A) = {}& \lambda \left[A D_1 F(k, N) + (1 - \delta) \right] \\
& + \left[\lambda A D_2 F(k, N) - D_2 u(c, 1 - N) \right] \frac{dN}{dk} \\
& + \left[\beta E D_1 V(k', A') - \lambda \gamma \right] \frac{dk'}{dk}.
\end{aligned}$$

Using the same logic as in the derivation of the "envelope theorem" in demand theory,
this equation can be greatly simplified by using the first-order conditions (A.9) and
(A.10) to set the two bracketed terms equal to zero. Intuitively, given that the values
of N and k' were optimally chosen, there are zero net benefits from the adjustments
in these quantities that will arise from a change k. Thus $D_1 V(k, A)$ can be simplified
to

$$D_1 V(k', A') = \lambda \left[A' D_1 F(k', N') + (1 - \delta) \right]. \tag{A.11}$$

Finding the decision rules: Conditions (A.9) and (A.10) can be used to solve for c
and N as a function of λ, k and A. These functions are not quite the decision rules for

consumption and labor, since they depend on λ which we have not yet determined. We specify the resulting functions as $N = \tilde{N}(k, \lambda, A)$ and $c = \tilde{c}(k, \lambda, A)$. To find the decision rule for capital, we proceed as follows. Using $N = \tilde{N}(k, \lambda, A)$ and $c = \tilde{c}(k, \lambda, A)$, we can express the optimization conditions as a first-order system of nonlinear stochastic difference equations in λ and k:

$$\lambda \gamma = \beta E \lambda' \left[A' D_1 F(k', \tilde{N}(k', \lambda', A')) + (1 - \delta) \right], \tag{A.12}$$

$$AF[k, \tilde{N}(k, \lambda, A)] = \tilde{c}(k, \lambda, A) + \gamma k' - (1 - \delta)k.$$

The solution to this system is a pair of decision rules $k' = h(k, A)$ and $\lambda = \lambda(k, A)$. In turn, these imply decision rules for consumption and labor $N = N(k, A)$ and $c = c(k, A)$. Taking these decision rules for c, N, λ, k', we have a complete description of how quantities in the real economy will efficiently evolve through time.

The steady state of the optimal economy: The stationary distribution of A is given by the function $G(A)$ such that: $G(A') = \int H(A', A)G(\mathrm{d}A)$. Given this stationary distribution the mean value of A can be computed as $A^* = \int AG(\mathrm{d}A)$. If we ignore, for the moment, the stochastic nature of A and set it equal to its mean, $A = A^*$, the model reduces to a variant of the Cass–Koopmans neoclassical model. It is well known that this deterministic model has a unique non-trivial steady state which is globally stable [Stokey, Lucas and Prescott (1989), Section 6.1].

Replacing A and A' by A^* in Equations (A.8)–(A.11) we obtain the system of equations that characterize the steady state:

$$A^* D_1 F(k^*, N^*) + (1 - \delta) = \gamma/\beta, \tag{A.13}$$

$$A^* D_2 F(k^*, N^*) D_1 u(c^*, 1 - N^*) = D_2 u(c^*, 1 - N^*), \tag{A.14}$$

$$\gamma k^* = A^* F(k^*, N^*) - c^* + (1 - \delta)k^*. \tag{A.15}$$

We use an asterisk to denote the steady-state values of the different variables. This system of equations is recursive. Equation (A.13) determines the value of k^*/N^*; recall that F is homogeneous of degree one and thus $D_1 F$ is homogeneous of degree zero implying that $D_1 F(k^*, N^*) = D_1 F(k^*/N^*, 1)$. Equations (A.14) and (A.15) jointly determine c^* and N^*. We will return below to discussing the nature of the steady state in the competitive economy.

A.3. A dynamic competitive equilibrium interpretation

Our theoretical discussion so far has focused on a planning problem. However, the stylized facts described in Section 2 pertain to market economies where economic decisions are made in a decentralized manner. For this reason we now turn our attention to this economy's competitive equilibrium under rational expectations.

There are several ways of decentralizing the basic RBC model economy. Here we will focus on a sequential competitive equilibrium in which households own the firms

and the stock of capital and make three inter-related decisions: how much labor to supply (N_s), how much capital to accumulate (k_s'), and how much to consume (c). In this decentralization scheme, households have to take into account the law of motion for the wage rate (w) and for the rental price of capital (R). Both of these prices are a function of the state of the economy, as summarized by the productivity level A and the aggregate capital stock k:

$$w = w(k, A), \tag{A.16}$$
$$R = R(k, A). \tag{A.17}$$

To forecast these prices, agents have to know the functions w and R and the law of motion for A and k. The variable A evolves according to $H(A', A)$, while the law of motion for the aggregate capital stock will be described as

$$k' = g(k, A).$$

The Household Problem. With these preliminaries in place we can now write the household problem as:

$$v(k_s; A, k) = \max_{c, N_s, k_s'} \{u(c, 1 - N_s) + \beta E v(k_s'; A', k')\},$$

subject to:
$$c + \gamma k_s' = w(k, A) N_s + (1 + R(k, A) - \delta) k_s + \pi. \tag{A.18}$$

where v is the value function of the household and π denotes the firms' profits, which, as we will see in a moment, are always equal to zero.

It is useful to define the real interest rate as the rental price of capital net of depreciation:

$$r(k, A) = R(k, A) - \delta.$$

The solution to the household problem is described by the following contingent rules:

$$k_s' = k_s(k_s, k, A),$$
$$c = c(k_s, k, A),$$
$$N_s = N(k_s, k, A).$$

These rules must satisfy the following set of efficiency conditions:

$$D_2 u(c, 1 - N_s) = D_1 u(c, 1 - N_s) w(k, A), \tag{A.19}$$
$$\gamma D_1 u(c, 1 - N_s) = \beta E D_1 u(c, 1 - N_s)[R(k', A') + (1 - \delta)]. \tag{A.20}$$

The Firm's Problem. The firms in this economy solve a static problem. They have to decide how much capital and labor to hire in the spot competitive markets for both of these factors:

$$\max_{k_d, N_d} \pi = AF(k_d, N_d) - wN_d - Rk_d.$$

The familiar optimization conditions for this problem are:

$$AD_1 F(k_d, N_d) = R(\mathbf{k}, A), \tag{A.21}$$

$$AD_2 F(k_d, N_d) = w(\mathbf{k}, A). \tag{A.22}$$

Given that the production function exhibits constant returns to scale, profits will always be equal to zero:

$$\pi = AF(k_d, N_d) - AD_2 F(k_d, N_d)N_d - AD_1 F(k_d, N_d)k_d = 0.$$

Market Clearing. There are three markets in this economy: spot markets for capital, labor and output. By Walras's law if two of these markets are in equilibrium the third market will also have to be in equilibrium. Thus we can state the equilibrium conditions limiting ourselves to the factor markets:

$$k_d = k_s = \mathbf{k}, \qquad N_d = N_s.$$

To ensure that this is a competitive equilibrium under rational expectations, the law of motion conjectured by households for the competitive equilibrium has to coincide with the actual aggregate law of motion for this variable:

$$k_s(\mathbf{k}, \mathbf{k}, A) = g(\mathbf{k}, A).$$

The steady state in the market economy: If we treat N as fixed for the moment, we can interpret Equation (A.13) as equating the long run demand and supply for capital. The real rate of return to capital in a decentralized version of this economy is given by $r = AD_1 F(k, N) - \delta$. This can be seen as a demand schedule; given the value of r (and the value of N) it tells us the value of k that the economy would choose. The long run supply of capital is given by $r = \gamma/\beta - 1$ and is thus perfectly elastic: the capital stock of the economy always adjusts so that the steady-state real interest rate is $r = \gamma/\beta - 1$.

A.4. The welfare theorems

To show heuristically the connection between the competitive equilibrium and the Pareto Optimum we can now compare the first-order conditions of the competitive equilibrium with those of the planner's problem to show that they coincide. Replacing

Equations (A.21) and (A.22) in (A.18) and exploring the fact that $F(\cdot)$ is homogeneous of degree one, we obtain a resource constraint that is equivalent to the one implied by Equations (A.3)–(A.5). Making use of Equations (A.21) and (A.22) it can also be readily shown that (A.19)–(A.20) are equivalent to (A.9)–(A.11). Notice that the assumption of rational expectations is crucial for this comparison. The equivalence between the conditions that characterize the two problems underlies the two welfare theorems that apply to this economy: the competitive equilibrium is Pareto Optimal and a Pareto Optimal allocation can be decentralized as a competitive equilibrium.

The fact that the competitive equilibrium can be solved as a solution to a planning problem has important technical implications. Since the planners problem involves maximizing a continuous function defined over a compact set we know that a solution to the problem exists. Furthermore, since the planner's problem is strictly concave, its solution is unique. Thus, the existence and uniqueness of the competitive equilibrium can then be established by exploring its equivalence to the planner's problem.

There are many instances in which we may want to explore economies where the first welfare theorem does not hold. Examples include economies with distortionary taxes, externalities, or monopolistic competition. Rarely can the competitive equilibrium for these economies be mapped into a concave planning problem[74]. We can still linearize the system of equations that characterizes the competitive equilibrium to explore some of its properties. However, we no longer have the guarantee that the equilibrium exists or that it is unique. This is the reason why the multiple equilibrium literature discussed in Farmer (1993) focuses on economies in which the competitive equilibrium is suboptimal.

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[74] See Danthine and Donaldson (1985) for an exception.

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