

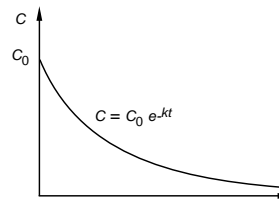
### Honors Project 0: Drug Dosage

#### Background

Two facts will be important in this project.

First, the concentration of a drug in the bloodstream from a single dose decreases with time. (Can you see why?) In general, the rate of decrease is proportional to the amount present: thus if  $C = C(t)$  is concentration (in mg/mL) as a function of time  $t$  (in hours), then

$$\frac{dC}{dt} = -kC$$



where  $k > 0$  is a constant, known as the *elimination constant* of the drug. It follows that

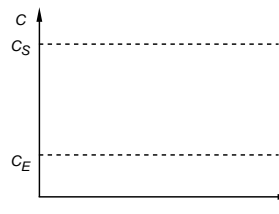
$$C = C_0 e^{-kt}$$

where  $C_0$  is the initial concentration, or

$$C = C_0 \left(\frac{1}{2}\right)^{\frac{t}{t_H}}$$

where  $t_H$  is the half-life of the drug.

Second, for most drugs there is an effective concentration level  $C_E$  above which  $C$  must stay if the drug is to be effective, and a safe concentration level  $C_S > C_E$  below which  $C$  must stay if the drug is to be safe.

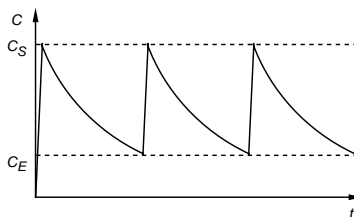


#### The Problems

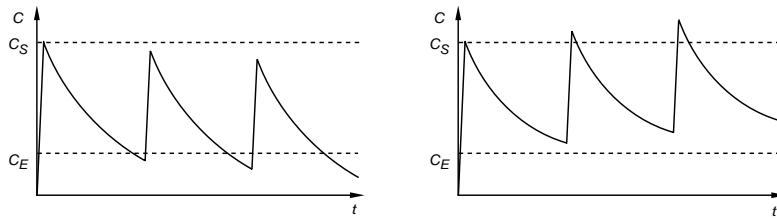
1. You are working with a drug whose elimination constant  $k$  is known. (In general, what might  $k$  depend on? How might you determine  $k$  in practice?) Design a schedule for long-term administration of the drug that will:

- a) get the concentration to an effective level as soon as possible,
- b) keep the concentration below a known safe level  $C_S$ , and
- c) keep the concentration above a known effective level  $C_E$ .

You may assume that upon administration the drug immediately becomes completely mixed in the bloodstream at its maximum concentration level. (Is this assumption reasonable?) *Hint:* Over the long-term, concentration levels should look something like this:



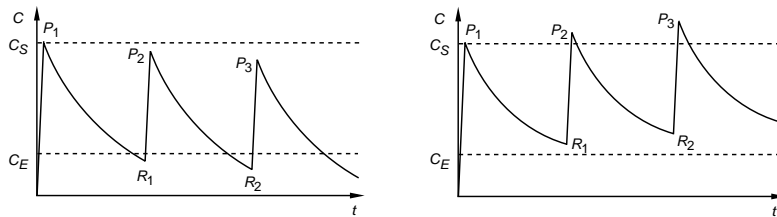
Two types of administration schedule you will want to avoid are illustrated below:



What is wrong with these schedules, and what led to the problems?

2. So far, your computations have been quite general in nature. Find data for a real drug<sup>1</sup>, and report the administration schedule you designed in concrete terms.

Two sequences of interest in understanding the long-term effects of drugs are the (sequence of) *peak concentrations*  $\{P_n\}$  and the (sequence of) *residual concentrations*  $\{R_n\}$ . The *peak concentration*  $P_n$  is the concentration of the drug immediately after the  $n$ th dose, and the *residual concentration*  $R_n$  is the concentration of the drug immediately before the  $n + 1$ st dose.



3. Assuming that a dose of a drug that raises the bloodstream concentration level by  $C_0$  is taken at regular time intervals  $t_0$ , that the elimination constant  $k$  of the drug is known, and that upon administration the drug immediately becomes completely mixed in the bloodstream at its maximum concentration level:

- find closed form expressions for  $P_n$  and  $R_n$ ,
- find  $P_\infty = \lim_{n \rightarrow \infty} P_n$ ,
- describe in pharmaceutical terms the significance of  $P_\infty$ ,
- find  $R_\infty = \lim_{n \rightarrow \infty} R_n$ , and
- describe in pharmaceutical terms the significance of  $R_\infty$ .

<sup>1</sup>Consult the *Physicians Desk Reference*, aka “the PDR”.