

### Project 4.9c: Newton's Method

#### Objective

To get some experience applying Newton's Method using Maple.

#### Narrative

If you have not already done so, read Section 4.9 in the text. There you will learn Newton's Method for finding an approximation to a root  $x$  of  $f(x) = 0$  (or a *zero* of  $f$ ) given an initial approximation  $x_1$  to  $x$ . This method uses the iterative equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \dots,$$

$x_1$  being given, to obtain successive approximations to  $x$ .

#### Task

- a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands apply Newton's Method to finding the root of  $f(x) = 8x^3 - 6x + 1 = 0$  given an initial estimate of  $x_1 = 1.0$ . The effect of each command is described in the right-hand column for your reference. Note that Maple uses a period to denote a subscript.

> # Project 4.9c: Newton's Method	
> # Task 1	
> restart;	Clear Maple's memory.
> f := x -> 8*x^3-6*x+1;	Define $f$ .
> plot(f(x), x=-2..2, y=-2..4);	Plot the graph of $f$ .
> g := x -> x - f(x)/D(f)(x);	Let $g(x) = x - \frac{f(x)}{f'(x)}$ .
> x.1 := 1.0;	Let $x_1 = 1.0$ . (Make sure you type 1.0, not 1: it makes a difference to Maple!
> for n from 1 to 6 do x.(n+1) := g(x.n) od;	This computes $x_{n+1} = g(x_n)$ for $n = 1, 2, \dots$ (Since $g(x) = x - \frac{f(x)}{f'(x)}$ , it follows that $x_{n+1} = g(x_n)$ .)

At this point, make a hard-copy of your typed input and Maple's responses. Then ...

- To 8 decimal places of accuracy, what number  $x_\infty$  does it appear that  $x_n$  is converging to?
  - Plot and label the point  $P$  on the  $x$ -axis whose  $x$ -coordinate is  $x_\infty$ .
- Use Newton's Method to estimate the solutions to the equation  $\sin x = x^2 - 1$  by proceeding as follows:
    - Let  $f(x) = \sin x - (x^2 - 1)$  and plot the graph of  $f$  for  $x \in [-5, 5]$  and  $y \in [-10, 2]$ . (The graph will help you estimate the roots of  $f(x) = 0$ .)
    - Estimate each zero of  $f(x)$  by a number  $x_1$ , and iterate Newton's Method enough times to obtain an approximation to the root to 8 decimal places of accuracy. (*Note:* Your initial estimate of each root

must be close enough to the root for Newton's Method to converge: if it's not close enough, the values  $x_2, x_3, \dots$  might not converge to the value you're interested in!

Complete this task by again making a hard-copy of your typed input and Maple's responses (both text and graphics).

Your lab report will be the hard copies of your typed input and Maple's responses (both text and graphics).