Project 10.2a: Direction Fields

Objective

To investigate direction fields and the graphical solution of differential equations.

Narrative

If you have not already done so, read Section 10.2 of the text. In this project we investigate direction fields and a graphical method for solving differential equations.

In this project we introduce two commands in the the package of routines `DEtools`:

`dfieldplot(<eqn>,y(x),x=a..b,y=c..d)`	plot the slope field of the differential equation $\langle eqn \rangle$ in x and y (assuming y is a function of x) for points whose x-coordinates are between a and b and whose y-coordinates are between c and d
`dsolve({<eqn>,y(a)=b},y(x))`	solves the differential equation $\langle eqn \rangle$ in x and y (assuming y is a function of x) using the initial condition $y(a) = b$ for $y = y(x)$

Note that in using the command `dfieldplot(<eqn>,x=a..b,y=c..d)` we *must* specify y by `y(x)` and y' by `diff(y(x),x)`.

Tasks

a) Type the command lines below into Maple in the order in which they are listed. These commands produce the direction field associated to the differential equation $y' = x + y$ and the three solutions corresponding to the initial conditions $y(0) = 0$, $y(0) = 1$, and $y(0) = -1$.

```
> # Project 10.2a:  Direction Fields
> restart:  with(plots):  with(DEtools):
> y0 := y(x); y1 := diff(y(x),x);
> plot0 := dfieldplot(y1 = x+y0,y0,x=-2..2,y=-2..2):
> display(plot0);
> dsolve({y1 = x+y0,y(0)=0},y(x));
> plot1 := plot(rhs(%),x=-2..2,y=-2..2,color=blue):
> display({plot0,plot1});
> dsolve({y1 = x+y0,y(0)=1},y(x));
> plot2 := plot(rhs(%),x=-2..2,y=-2..2,color=red):
> display({plot0,plot1,plot2});
> dsolve({y1 = x+y0,y(0)=-1},y(x));
> plot3 := plot(rhs(%),x=-2..2,y=-2..2,color=green):
> display({plot0,plot1,plot2,plot3});
```

At this point, make a hard-copy of your typed input and Maple's responses (both text and graphics). Then continue with the following steps.

b) On the last graphic you produced in part (a), label by hand each graph: label the graph corresponding to the initial condition $y(0) = 0$ by "$y_0 = 0$", for example.

c) On the last graphic you produced in part (a), plot and label by hand the solution to the differential equation $y' = x + y$ that passes through the point $P(0, 0.5)$.

Comments

Later we will continue our discussion of differential equations with a discussion of Euler's Method for numerically solving differential equations.