DXE_MAKR Macroeconomics
MUNI, Spring 2011
Lecture Notes
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Obligatory problems: Solow - problem 2; Ramsey - problem 1; OLG - problem 2
Deadline: March 20, 2011 by midnight (per email)

Solow model

1. Consider an economy with technological progress but without population growth that is on its balances growth path. Now suppose there is a one time jump in the number of workers:

- At the time od the jump, does output per unit of effective labor rise, fall, or stay the same? Why?
- After the initial change if any in output per unit of effective labor when the new workers appear, is there any further change in output per unit of effective labor? If so, does it rise or fall? Why?
- Once the economy has again reached a balanced growth path, is output per unit of effective labor higher, lower, or the same as it was before the new workers appeared? Why?

2. Describe how, if at all, each of the following developments affects the **level and growth rate** of both **output per capital** as well as **output per effective worker**.

- The rate of depreciation increases.
- The rate of technological progress decreases.
- The production function is $f(k) = k^\alpha$ and capital's share, α, decreases.
- Workers exert more effort, so that output per unit of effective labor for a given value of capital per unit of effective labor is higher than before.

3. Suppose that the production function is Cobb-Douglas (i.e. $Y = K^\alpha (AL)^{1-\alpha}$)

- Find expressions for k^*, y^* and c^* as functions of the parameters of the model - s, n, δ, g and α.
- What is the golden -rule value of k^*?
- What saving rate is needed to yield the golden-rule capital stock?

Ramsey model

Problem 1 - Effects of changes of parameters in Ramsey model.

Describe how each of the following affect the $\dot{k} = 0$ and $\dot{c} = 0$ locus. How does c and k react immediately after change and how does the new steady state compare to the old one?

a) Permanent fall in productivity growth rate g

b) A rise in the preference for today's consumption θ

c) Proportional downward shift of production function $f(k)$

d) Increase in the depreciation rate of capital from 0 to positive δ

Problem 2 - Ramsey model with government purchases in utility function.

Consider an economy a la Ramsey with infinitely lived representative households, who provide labor services in exchange for wages, receive interest income on assets, purchase goods for consumption and save by accumulating additional assets. We will modify here the standard RCK model by assuming that government purchases affect utility from private consumption and that government purchases and private consumption are perfect substitutes. Thus the representative household maximizes its lifetime welfare

$$\int_0^\infty u(c_t, g_t)e^{-(\rho-n)t}dt = \int_0^\infty \frac{(c_t + g_t)^{1-\theta}}{1-\theta}e^{-(\rho-n)t}dt$$

subject to its flow budget constraint and the No-Ponzi-Game condition, where n is the rate of population growth, $\theta, \rho > 0$ and $\rho > n$. Assume further that the government purchases per capita $g_t = \frac{G_t}{L_t}$, which are financed by the constant tax on consumption $1 > \tau_c > 0$, are such that the government budget is balanced at any moment of time. The production sector of the economy is again according to the Ramsey model composed of representative perfectly competitive firms which produce goods, pay wages for labor input and make rental payments for capital inputs. The firms have neoclassical production function, expressed in per capita terms $y_t = Ak_t^\alpha$ where $0 < \alpha < 1$ and capital depreciates at the rate $\delta > 0$.

(a) Specify the household's dynamic optimization problem. Explain in words the meaning of the No-Ponzi-Game condition for household.

(b) Derive the first order conditions of the household's optimization problem.

(c) Write down the government's flow budget constraint. Is it in our case also necessary to specify the NPG to constraint the government behavior? Explain why or why not.

(d) Derive and explain the Euler equation.

(e) Write down and solve the problem of a profit-maximizing representative form. Using the results above specify the competitive market equilibrium.

(f) Derive the conditions for the steady-state level of capital and consumption per capita. Draw the phase diagram.

(g) Assume that the economy is initially at the steady state with $c^* > 0$. What are the effects of a temporary increase in government purchases on the paths of consumption, capital and interest rate (draw their behavior over time). How these effects will be changed if the increase in government purchases will be announced at some time T>0 before it really happens?

(h) Write down the social planner problem for this economy and derive the first order conditions. Let denote the path of optimal consumption when government spending is zero ($g_t = 0$ for all t) by $\{c_t^0\}_{t=0}^{\infty}$. Do your results support the view that any equilibrium with the time path of government spending $\{g_t\}_{t=0}^{\infty}$ (such that $g_t \leq c_t^0$ for all t) is a social optimum? Having this in mind is the competitive market equilibrium derived earlier a social optimum? How this result will be changed if the government uses a lump sum tax instead of consumption tax to finance their purchases? Which role in this sense do you think plays the assumption of perfect substitutability between private consumption and government purchases in the model?

(i) How will be the Euler equation of the original competitive market economy above changed if the elasticity of substitution between private consumption and government purchases is equal to one, i.e. the household preferences are given by $u(c,g) = \frac{(c^{\gamma} g^{1-\gamma})^{1-\theta}}{1-\theta}$ where $0 < \gamma < 1$ and $\gamma(1-\theta) < 1$.

OLG model

Problem 1 - Effect of changes of parameters in OLG model.
Consider the Diamond model with logarithmic utility function and the Cobb-Douglas production function. Describe how each of the following affects k_{t+1} as a function of k_t

a) A rise in n.

b) A downward (proportional) shift of the production function (if the production is in the form of Cobb-Douglas function $f(k) = Bk^{\alpha}$, this means the fall in B).

Problem 2 - Social security in OLG model.
Consider a Diamond model with logarithmic utility function, Coub- Douglas production function and $g = 0$

a) Pay-as-you-go social security Suppose the government taxes each young individual amount T and uses the proceeds to pay benefits to old individuals;so each old person receives (1+n).T.

   i) How, if at all, does this change affect the k_{t+1} as a function of k_t?

   ii) How, if at all, does this change affect the balanced growth path value of k (i.e. k^*)?

b) Fully funded social security Suppose the government taxes each young person amount T and uses the proceeds to purchase capital. Individuals born at t therefore receive (1+rt+1) when they are old.

   i) How, if at all, does this change affect the k_{t+1} as a function of k_t?

   ii) How, if at all, does this change affect the balanced growth path value of k (k^*)?

Problem 3 - Taxation in OLG model.
In these questions we use the overlapping generations model to examine the impact of different forms of government taxation on the steady state and dynamics of the capital stock. Assume population is constant ($n = 0$), individuals live for two periods, working and saving in the first and living off capital in the second period. Assume that capital is the only asset and it is paid its marginal product. Moreover, we assume that there is no technological progress, i.e., $g = 0$.

The individual's utility function is

$U = \log(c_{1t}) + \log(c_{2,t+1})$

The production function in per capita terms is

$y_t = k_t^{\alpha}$.

(a) Assume that there exist a government who imposes a lump sum tax of an amount T_t on each time t young individual, and that each individual maximizes her utility subject to her budget constraint.

   i. Determine the intertemporal budget constraint of each individual (life-time budget constraint).

   ii. Use the budget constraint to solve for the first period consumption c_{1t} and the first period saving, s_t.

(b) Using the saving function derived on part a) and the production function:

   i. Determine the relationship between k_{t+1} and k_t , and show it in a graph.

   ii. Write down the expression that implicitly defines the equilibrium capital stock k^* , assuming that the tax is constant: $T_t = T$.

   iii. How many equilibria does the economy exhibit? Which ones are stable?

(c) The government now decides to use a proportional tax on labor income instead of a lump-sum tax. Therefore the income of each worker (after tax) is $w_t(1 - \tau_{wt})$, where τ_{wt} is the labor income tax.

   i. Determine the new relationship between k_{t+1} and k_t , and show it in a graph. Hint : You can use the expressions you derived above, and just replace $w_t(1-\tau_{wt})$ for w_t, and remove the effect of the lump sum tax.

   ii. Write down the expression that implicitly defines the equilibrium capital stock k^* , assuming t_{wt} is constant.

   iii. Assuming that the government sets t_{wt} such that in equilibrium, the revenue is equal than with the lump-sum tax, that is $w * t_w = T$, how does the new level of equilibrium capital per capita compare with the one obtained on part (b).

(d) Consider now the case in which the government decides to tax only saving income. Therefore, the after tax interest rate on savings is $r_{t+1}(1 - \tau_r)$. Again:

   i. Determine the first period consumption $c_{1,t}$, second period consumption $c_{2,t+1}$, and the first period savings s_t. Compared to the non government case, Which of these variables is affected by the government policy?

   ii. Determine the new relationship between k_{t+1} and k_t.

   iii. Write down the expression that implicitly defines the equilibrium capital stock $k*$, assuming τ_r is constant.

iv. Assuming that the government sets t_{rt} such that in equilibrium, the revenue is equal than with the lump-sum tax, that is $s_t r_{t+1} w t_{r\,t+1} = T$, how does the new level of equilibrium capital per capita compare with the one obtained on part (b).

(e) Given a fixed level of revenue in steady state:

i. Which taxation system(s) has(have) the highest equilibrium level of output?

ii. What characteristics of the model are crucial for the results that you have obtained? Explain the economical intuition behind your findings.