DXE_MAKR Macroeconomics
MUNI, Spring 2011
Assignment 2
Eva Hromádková

Obligatory problems: Endog - problem #1, RBC - problem #1
Deadline: April 14, 2011 by midnight (per email)

Endogenous growth model

Problem 1 - Utility Enhancing Government Expenditure.
Consider the growth model where firms have the linear production function $y = Ak$ where $A > 0$ is capital per capita and y is output per capita (for simplicity there is no population growth and the size of the population is equal to one). Assume further that the government expenditure g, which is a fixed fraction of output τ, i.e. $g = \tau y = \tau Ak$, contribute to the welfare of hte private agent. The government expenditure is financed by lump-sum tax T. Thus infinitely lived household maximizes the following utility

$$\int_0^\infty \frac{(c_t g_t^\alpha)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

with $\alpha > 0$.

1. Assuming perfect competition framework in production factor markets derive the expression for the interest rate and wage rate. Assume for simplicity that the rate of capital depreciation is equal to zero.
2. Set up the representative household's optimisation problem. What are the state and control variables? What is the **current-value** Hamiltonian?
3. Derive the F.O.C's.
4. Derive the Euler equation. What is the growth rate of consumption at the steady state? What is the steady state growth rate of capital?
5. Set up a social planner problem. What are the control variables? Discuss possible sources of inefficiency of the decentralized equilibrium in this model.

Problem 2 - Effects of changes of parameters in Ramsey model.
Consider economy with knowledge spillovers where the representative household maximizes its lifetime welfare subject to its flow budget constraint and No-Ponzi-Game condition. Assume that the utility function is of CRRA type and that the size of the household grows at the rate $n > 0$. There is a continuum of identical perfectly competitive firms of mass one. The firm i has the production function $Y_i = AK_i^\alpha L_i^{1-\alpha} K^\alpha$ where

$0 < \alpha < 1$ and $K = \int_0^1 K_i di$ is the aggregate capital. Assume further that there is a government which takes household consumption and labor income at constant rates τ_c and τ_w, respectively, and that the firms pay taxes from renting capital at constant rate τ_r. Let the government consume the amount of the tax revenues from taxing consumption and the rest is returned back to the households in the form of lump-sum transfers.

- Write down the household's problem, firm;s problem and government budget constraints
- Derive the F.O.C's.
- Solve for the growth rates of consumption, capital and output along the balanced growth path.
- Which taxes can affect the long run growth rate and which cannot? Explain.

Problem 3 - Intermediate Inputs as Durables.

Assume the same setup as in the R&D model presented on the lecture. Suppose now that the intermediate inputs, X_{ij}, are infinite-lived durable goods. New units of these durables can be formed from one unit of final output. The inventor of the jth type of intermediate good charges the rental price R_j and the competitive producers of final goods treat R_j as given.

1. How is R_j determined?
2. In the steady-state, what is the quantity X_j of each type of intermediate good?
3. What is the steady-state growth rate of the economy? How does this answer differ from the case we had in class in which the intermediate inputs were perishable goods?
4. If the intermediate goods are durables, then what kinds of dynamic effects arise in the transition to the steady-state?

Problem 4 - Production function of new ideas discovery.

Consider the Romer (1990) version of the model with a variety of producer products, as we had in class with the production function for firm i given by

$$Y_i = L_{Y,i}^{\beta} \sum_{j=1}^{A} X_{ij}^{1-\beta}$$

where $0 < \beta < 1$, Y_i is output, $L_{Y,i}$ is labor output and X_{ij} is the employment of the jth type of specialized intermediate (nondurable) output.

- Derive the final good's sector demand for labor and intermediate goods as a function of the prices w and P_j.

- We assume that the inventor of good j retains a perpetual monopoly right over the production and sale of the good X_j. As in class, one unit of final good combined with a design can be transformed into one unit of intermediate good. Solve the profit maximization problem for an intermediate-goods firm that owns a patent and show that all intermediate goods sell for the same price. What is the instantaneous profit π earned by an I-firm as a function of labor used in the final-good sector?

- Assume that the production function in R&D sector for discovering new ideas is more general than we had in class: the quantity of labor necessary to invent a new product is $\hat{\eta} = \frac{\eta}{L_R^{\lambda-1} A^{\epsilon}}$ capturing the externality effects of the stock of ideas A and the total labor searching for new ideas L_R. Assume that $\epsilon < 1$ and $0 < \lambda < 1$. Assume further that population L grows at rate $n = \frac{\dot{L}}{L}$.

  1. Explain the meaning of $\dot{A} = \frac{L_R}{\eta}$ and interpret the meaning of $\hat{\eta}$. Using the above general production function specify the rate of technological change $\frac{\dot{A}}{A}$ as a function of the labor used in R&D sector and of the level of technology? Derive the expression for the rate of technological change along a balanced growth path (BGP). Is this model a true endogenous growth model? Why or why not? [Hint: Use the fact that along a BGP $\left(\frac{\dot{L}_R}{L_R}\right)^* = n$.]

  2. Write down the formula for the present discounted value of a blueprint for an I-good $V(t)$. Show that it implies the arbitrage condition $r = \frac{\pi(t)}{V(t)} + \frac{\dot{V}(t)}{V(t)}$. Explain the meaning of the arbitrage condition. Prove that the profit grows at rate $\frac{\dot{\pi}(t)}{\pi(t)} = n$. Using this and the arbitrage condition along a BGP derive the expression for the present value $V^*(t)$ along the BGP.

  3. Explain the meaning of free entry into the R&D sector. Write down the expression which captures free entry into the R&D sector for this economy.

  4. Assume that the household divides his unit time endowment between the working time in the R&D - l_R and the working time in the final goods sector - l_Y. So $l_R + l_Y = 1$ and the aggregate labor in the R&D and the final good sector are $L_R = l_R L$ and $L_Y = l_Y L$ respectively. Write down the household problem for this economy. What are the state and control variables for this problem?

  5. Explain in words the nature of all market failures (inefficiencies) that emerge from the decentralized model.

RBC

Problem 1 - RBC model, optimisation under uncertainty

Consider the following simple RBC model. Preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [bc_t^{1-\eta} + (1-b)l_t^{1-\eta}]^{\frac{1}{1-\eta}} \qquad 0 < \beta < 1$$

where $l = 1 - n$ is leisure and production technology is given by

$$y_t = e^{z_t} k_t^{\alpha} n_t^{1-\alpha}$$

and the resource constraint

$$c_t + k_{t+1} - (1-\delta)k_t = y_t$$

1. Set up the social planner's problem for the economy and derive the first order conditions.
2. Instead of a social planner, assume there are households that maximize utility and firms that maximize profit. Households and firms interact in competitive markets. Write down the decision problem of the representative households and firms. Derive the first order conditions and show that they imply the same equilibrium as derived from the social planner approach.
3. For a given value of consumption and wage, how does the increase in b affect the labor supply curves?

Problem 2 - RBC model with endogenous labor and capital utilisation

YOU CAN ALSO TRY TO SOLVE IT USING HAMILTONIAN, I JUST PUT IT HERE IF YOU WOULD LIKE TO TRY SOME RECURSIVE.

There is a centralized economy with one good that can be consumed and invested. The social planner maximizes expected value of the discounted lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\chi}}{1+\chi} \Big), \quad \sigma > 0, \quad \chi > 0$$

where C and N are consumption and working hours. The budget constraint and technology are given by:

$$\begin{aligned} K_{t+1} &= Y_t + (1-\delta(u_t))K_t - C_t \\ Y_t &= A_t (K_t u_t)^{\alpha} N_t^{1-\alpha} \\ A_{t+1} &= A_t^{\rho} v_t, \qquad\qquad K(0), A(0) \quad \text{given.} \end{aligned}$$

The depreciation rate is endogenous and given by $\delta(u_t) = \mu u_t^\theta$, where $\mu > 0$ and $\theta > 1$ are parameters and u_t is capital utilization rate chosen by social planner. A_t is autocorrelated productivity shock, and v_t is the i.i.d random variable such that $E[\ln v_t] = 0$.

1. (3 points) Write down Bellman equation for the transformed problem. Clearly denote state and control variable(s).
2. (3 points) Derive First Order Condition(s) for the problem.
3. (1 point) Write down Envelope Theorem condition(s)
4. (2 points) Plug in ET into FOC and derive the Euler equation for this problem.
5. (5 points) Log-linearize equations of the model: capital law of motion, FOC's, Euler equation, productivity shock law of motion
6. (4 points) Assume $A_t = \bar{A}$ for all times t. Derive marginal product of capital and the real interest rate in the steady state. Interpret your answer.