CHAPTER 8 Economic Growth II

Questions for Review

1. In the Solow model, we find that only technological progress can affect the steady-state rate of growth in income per worker. Growth in the capital stock (through high saving) has no effect on the steady-state growth rate of income per worker; neither does population growth. But technological progress can lead to sustained growth.
2. In the steady state, output per person in the Solow model grows at the rate of technological progress g. Capital per person also grows at rate g. Note that this implies that output and capital per *effective* worker are constant in steady state. In the U.S. data, output and capital per worker have both grown at about 2 percent per year for the past half-century.
3. To decide whether an economy has more or less capital than the Golden Rule, we need to compare the marginal product of capital net of depreciation ($MPK - \delta$) with the growth rate of total output ($n + g$). The growth rate of GDP is readily available. Estimating the net marginal product of capital requires a little more work but, as shown in the text, can be backed out of available data on the capital stock relative to GDP, the total amount of depreciation relative to GDP, and capital's share in GDP.
4. Economic policy can influence the saving rate by either increasing public saving or providing incentives to stimulate private saving. Public saving is the difference between government revenue and government spending. If spending exceeds revenue, the government runs a budget deficit, which is negative saving. Policies that decrease the deficit (such as reductions in government purchases or increases in taxes) increase public saving, whereas policies that increase the deficit decrease saving. A variety of government policies affect private saving. The decision by a household to save may depend on the rate of return; the greater the return to saving, the more attractive saving becomes. Tax incentives such as tax-exempt retirement accounts for individuals and investment tax credits for corporations increase the rate of return and encourage private saving.
5. The rate of growth of output per person slowed worldwide after 1972. This slowdown appears to reflect a slowdown in productivity growth—the rate at which the production function is improving over time. Various explanations have been proposed, but the slowdown remains a mystery. In the second half of the 1990s, productivity grew more quickly again in the United States and, it appears, a few other countries. Many commentators attribute the productivity revival to the effects of information technology.
6. Endogenous growth theories attempt to explain the rate of technological progress by explaining the decisions that determine the creation of knowledge through research and development. By contrast, the Solow model simply took this rate as exogenous. In the Solow model, the saving rate affects growth temporarily, but diminishing returns to capital eventually force the economy to approach a steady state in which growth depends only on exogenous technological progress. By contrast, many endogenous growth models in essence assume that there are constant (rather than diminishing) returns to capital, interpreted to include knowledge. Hence, changes in the saving rate can lead to persistent growth.

Problems and Applications

1. a. To solve for the steady-state value of y as a function of s, n, g, and δ, we begin with the equation for the change in the capital stock in the steady state:

$$\Delta k = sf(k) - (\delta + n + g)k = 0.$$

The production function $y = \sqrt{k}$ can also be rewritten as $y^2 = k$. Plugging this production function into the equation for the change in the capital stock, we find that in the steady state:

$$sy - (\delta + n + g)y^2 = 0.$$

Solving this, we find the steady-state value of y:

$$y^* = s/(\delta + n + g).$$

b. The question provides us with the following information about each country:

Developed country:	$s = 0.28$	Less-developed country:	$s = 0.10$
	$n = 0.01$		$n = 0.04$
	$g = 0.02$		$g = 0.02$
	$\delta = 0.04$		$\delta = 0.04$

Using the equation for y^* that we derived in part (a), we can calculate the steady-state values of y for each country.

Developed country: $y^* = 0.28/(0.04 + 0.01 + 0.02) = 4.$

Less-developed country: $y^* = 0.10/(0.04 + 0.04 + 0.02) = 1.$

c. The equation for y^* that we derived in part (a) shows that the less-developed country could raise its level of income by reducing its population growth rate n or by increasing its saving rate s. Policies that reduce population growth include introducing methods of birth control and implementing disincentives for having children. Policies that increase the saving rate include increasing public saving by reducing the budget deficit and introducing private saving incentives such as I.R.A.'s and other tax concessions that increase the return to saving.

2. To solve this problem, it is useful to establish what we know about the U.S. economy:

A Cobb–Douglas production function has the form $y = k^{\alpha}$, where α is capital's share of income. The question tells us that $\alpha = 0.3$, so we know that the production function is $y = k^{0.3}$.

In the steady state, we know that the growth rate of output equals 3 percent, so we know that $(n + g) = 0.03$.

The depreciation rate $\delta = 0.04$.

The capital–output ratio $K/Y = 2.5$. Because $k/y = [K/(L \times E)]/[Y/(L \times E)] = K/Y$, we also know that $k/y = 2.5$. (That is, the capital–output ratio is the same in terms of effective workers as it is in levels.)

a. Begin with the steady-state condition, $sy = (\delta + n + g)k$. Rewriting this equation leads to a formula for saving in the steady state:

$$s = (\delta + n + g)(k/y).$$

Plugging in the values established above:

$$s = (0.04 + 0.03)(2.5) = 0.175.$$

The initial saving rate is 17.5 percent.

b. We know from Chapter 3 that with a Cobb–Douglas production function, capital's share of income $\alpha = MPK(K/Y)$. Rewriting, we have:

$$MPK = \alpha/(K/Y).$$

Plugging in the values established above, we find:

$$MPK = 0.3/2.5 = 0.12.$$

c. We know that at the Golden Rule steady state:

$$MPK = (n + g + \delta).$$

Plugging in the values established above:

$$MPK = (0.03 + 0.04) = 0.07.$$

At the Golden Rule steady state, the marginal product of capital is 7 percent, whereas it is 12 percent in the initial steady state. Hence, from the initial steady state we need to increase k to achieve the Golden Rule steady state.

d. We know from Chapter 3 that for a Cobb–Douglas production function, $MPK = \alpha\,(Y/K)$. Solving this for the capital–output ratio, we find:

$$K/Y = \alpha/MPK.$$

We can solve for the Golden Rule capital–output ratio using this equation. If we plug in the value 0.07 for the Golden Rule steady-state marginal product of capital, and the value 0.3 for α, we find:

$$K/Y = 0.3/0.07 = 4.29.$$

In the Golden Rule steady state, the capital–output ratio equals 4.29, compared to the current capital–output ratio of 2.5.

e. We know from part (a) that in the steady state

$$s = (\delta + n + g)(k/y),$$

where k/y is the steady-state capital–output ratio. In the introduction to this answer, we showed that $k/y = K/Y$, and in part (d) we found that the Golden Rule $K/Y = 4.29$. Plugging in this value and those established above:

$$s = (0.04 + 0.03)(4.29) = 0.30.$$

To reach the Golden Rule steady state, the saving rate must rise from 17.5 to 30 percent. This result implies that if we set the saving rate equal to the share going to capital (30%), we will achieve the Golden Rule steady state.

3. a. In the steady state, we know that $sy = (\delta + n + g)k$. This implies that

$$k/y = s/(\delta + n + g).$$

Since s, δ, n, and g are constant, this means that the ratio k/y is also constant. Since $k/y = [K/(L \times E)]/[Y/(L \times E)] = K/Y$, we can conclude that in the steady state, the capital–output ratio is constant.

b. We know that capital's share of income $= MPK \times (K/Y)$. In the steady state, we know from part (a) that the capital–output ratio K/Y is constant. We also know from the hint that the MPK is a function of k, which is constant in the steady state; therefore the MPK itself must be constant. Thus, capital's share of income is constant. Labor's share of income is 1 – [capital's share]. Hence, if capital's share is constant, we see that labor's share of income is also constant.

c. We know that in the steady state, total income grows at $n + g$—the rate of population growth plus the rate of technological change. In part (b) we showed that labor's and capital's share of income is constant. If the shares are constant, and total income grows at the rate $n + g$, then labor income and capital income must also grow at the rate $n + g$.

d. Define the real rental price of capital R as:

$$\begin{aligned} R &= \text{Total Capital Income/Capital Stock} \\ &= (MPK \times K)/K \\ &= MPK. \end{aligned}$$

We know that in the steady state, the MPK is constant because capital per effective worker k is constant. Therefore, we can conclude that the real rental price of capital is constant in the steady state.

To show that the real wage w grows at the rate of technological progress g, define:

$$TLI = \text{Total Labor Income.}$$
$$L = \text{Labor Force.}$$

Using the hint that the real wage equals total labor income divided by the labor force:

$$w = TLI/L.$$

Equivalently,

$$wL = TLI.$$

In terms of percentage changes, we can write this as

$$\Delta w/w + \Delta L/L = \Delta TLI/TLI.$$

This equation says that the growth rate of the real wage plus the growth rate of the labor force equals the growth rate of total labor income. We know that the labor force grows at rate n, and from part (c) we know that total labor income grows at rate $n + g$. We therefore conclude that the real wage grows at rate g.

4. a. The per worker production function is

$$F(K,L)/L = AK^{\alpha} L^{1-\alpha}/L = A(K/L)^{\alpha} = Ak^{\alpha}.$$

b. In the steady state, $\Delta k = sf(k) - (\delta + n + g)k = 0$. Hence, $sAk^{\alpha} = (\delta + n + g)k$, or, after rearranging:

$$k^* = \left[\frac{sA}{\delta + n + g}\right]^{\left(\frac{1}{1-\alpha}\right)}$$

Plugging into the per-worker production function from part (a) gives:

$$y^* = A^{\left(\frac{1}{1-\alpha}\right)}\left[\frac{s}{\delta + n + g}\right]^{\left(\frac{\alpha}{1-\alpha}\right)}$$

Thus, the ratio of steady-state income per worker in Richland to Poorland is:

$$\left(y^*_{Richland} / y^*_{Poorland}\right) = \left[\frac{s_{Richland}}{\delta + n_{Richland} + g} \bigg/ \frac{s_{Poorland}}{\delta + n_{Poorland} + g}\right]^{\frac{\alpha}{1-\alpha}}$$

$$= \left[\frac{0.32}{0.05 + 0.01 + 0.02} \bigg/ \frac{0.10}{0.05 + 0.03 + 0.02}\right]^{\frac{\alpha}{1-\alpha}}$$

$$= [4]^{\left(\frac{\alpha}{1-\alpha}\right)}$$

c. If α equals 1/3, then Richland should be $4^{1/2}$, or two times, richer than Poorland.

d. If $4^{\left(\frac{\alpha}{1-\alpha}\right)} = 16$, then it must be the case that $\left(\frac{\alpha}{1-\alpha}\right) = 2$, which in turn requires that α equals 2/3. Hence, If the Cobb-Douglas production function puts 2/3 of the weight on capital and only 1/3 on labor, then we can explain a 16-fold difference in levels of income per worker. One way to justify this might be to think about capital more broadly to include human capital—which must also be accumulated through investment, much in the way one accumulates physical capital.

5. How do differences in education across countries affect the Solow model? Education is one factor affecting the *efficiency of labor*, which we denoted by E. (Other factors affecting the efficiency of labor include levels of health, skill, and knowledge.) Since country 1 has a more highly educated labor force than country 2, each worker in country 1 is more efficient. That is, $E_1 > E_2$. We will assume that both countries are in steady state.
   a. In the Solow growth model, the rate of growth of total income is equal to $n + g$, which is independent of the work force's *level* of education. The two countries will, thus, have the same rate of growth of total income because they have the same rate of population growth and the same rate of technological progress.
   b. Because both countries have the same saving rate, the same population growth rate, and the same rate of technological progress, we know that the two countries will converge to the same steady-state level of capital per effective worker k^*. This is shown in Figure 8–1.

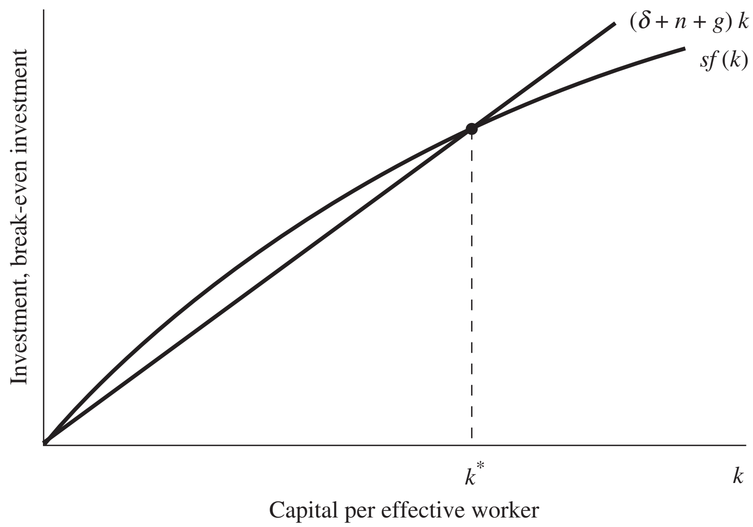

Figure 8–1

   Hence, output per effective worker in the steady state, which is $y^* = f(k^*)$, is the same in both countries. But $y^* = Y/(L \times E)$ or $Y/L = y^* E$. We know that y^* will be the same in both countries, but that $E_1 > E_2$. Therefore, $y^*E_1 > y^*E_2$. This implies that $(Y/L)_1 > (Y/L)_2$. Thus, the level of income per worker will be higher in the country with the more educated labor force.
   c. We know that the real rental price of capital R equals the marginal product of capital (*MPK*). But the *MPK* depends on the capital stock per efficiency unit of labor. In the steady state, both countries have $k_1^* = k_2^* = k^*$ because both countries have the same saving rate, the same population growth rate, and the same rate of technological progress. Therefore, it must be true that $R_1 = R_2 = MPK$. Thus, the real rental price of capital is identical in both countries.
   d. Output is divided between capital income and labor income. Therefore, the wage per effective worker can be expressed as:

$$w = f(k) - MPK \cdot k.$$

   As discussed in parts (b) and (c), both countries have the same steady-state capital stock k and the same *MPK*. Therefore, the wage per effective worker in the two countries is equal.

   Workers, however, care about the wage per unit of labor, not the wage per effective worker. Also, we can observe the wage per unit of labor but not the wage per effective worker. The wage per unit of labor is related to the wage per effective worker by the equation

$$\text{Wage per Unit of } L = wE.$$

Thus, the wage per unit of labor is higher in the country with the more educated labor force.

6. a. In the two-sector endogenous growth model in the text, the production function for manufactured goods is

$$Y = F(K,(1-u)\,EL).$$

We assumed in this model that this function has constant returns to scale. As in Section 3-1, constant returns means that for any positive number z, $zY = F(zK, z(1-u)\,EL)$. Setting $z = 1/EL$, we obtain:

$$\frac{Y}{EL} = F\left(\frac{K}{EL},(1-u)\right).$$

Using our standard definitions of y as output per effective worker and k as capital per effective worker, we can write this as

$$y = F(k,(1-u)).$$

b. To begin, note that from the production function in research universities, the growth rate of labor efficiency, $\Delta E / E$, equals $g(u)$. We can now follow the logic of Section 8-1, substituting the function $g(u)$ for the constant growth rate g. In order to keep capital per effective worker (K/EL) constant, break-even investment includes three terms: δk is needed to replace depreciating capital, nk is needed to provide capital for new workers, and $g(u)$ is needed to provide capital for the greater stock of knowledge E created by research universities. That is, break-even investment is $(\delta + n + g(u))k$.

c. Again following the logic of Section 8-1, the growth of capital per effective worker is the difference between saving per effective worker and break-even investment per effective worker. We now substitute the per-effective-worker production function from part (a), and the function $g(u)$ for the constant growth rate g, to obtain:

$$\Delta k = sF(k,(1-u)) - (\delta + n + g(u))k.$$

In the steady state, $\Delta k = 0$, so we can rewrite the equation above as:

$$sF(k,(1-u)) = (\delta + n + g(u))k$$

As in our analysis of the Solow model, for a given value of u we can plot the left- and right-hand sides of this equation:

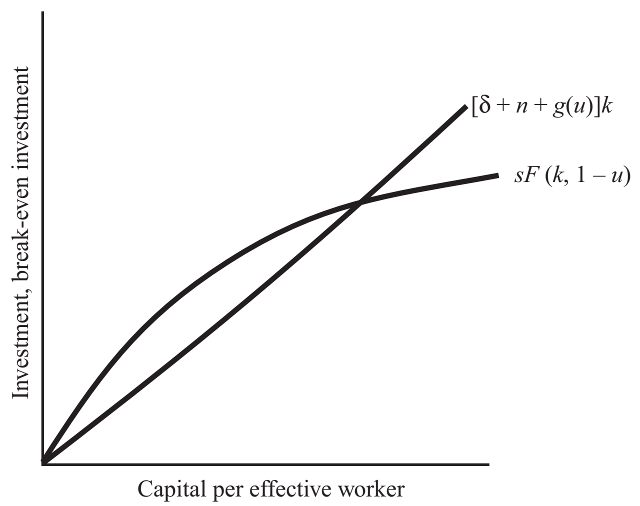

Figure 8–2

The steady state is given by the intersection of the two curves.

d. The steady state has constant capital per effective worker k as given by Figure 8–2 above. We also assume that in the steady state, there is a constant share of time spent in research universities, so u is constant. (After all, if u were not constant, it wouldn't be a "steady" state!). Hence, output per effective worker y is also constant. Output per worker equals yE, and E grows at rate $g(u)$. Therefore, output per worker grows at rate $g(u)$. The saving rate does not affect this growth rate. However, the amount of time spent in research universities does affect this rate: as more time is spent in research universities, the steady-state growth rate rises.

e. An increase in u shifts both lines in our figure. Output per effective worker falls for any given level of capital per effective worker, since less of each worker's time is spent producing manufactured goods. This is the immediate effect of the change, since at the time u rises, the capital stock K and the efficiency of each worker E are constant. Since output per effective worker falls, the curve showing saving per effective worker shifts down.

At the same time, the increase in time spent in research universities increases the growth rate of labor efficiency $g(u)$. Hence, break-even investment [which we found above in part (b)] rises at any given level of k, so the line showing break-even investment also shifts up.

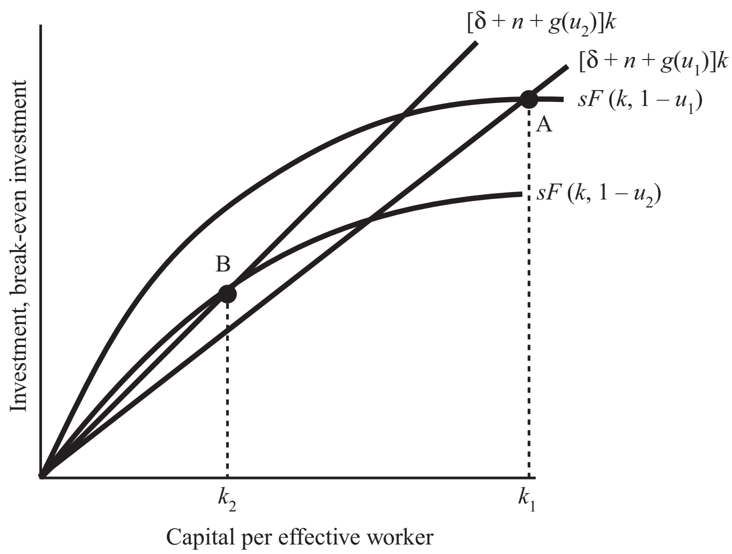

Figure 8–3

Figure 8–3 below shows these shifts:

In the new steady state, capital per effective worker falls from k_1 to k_2. Output per effective worker also falls.

f. In the short run, the increase in u unambiguously decreases consumption. After all, we argued in part (e) that the immediate effect is to decrease output, since workers spend less time producing manufacturing goods and more time in research universities expanding the stock of knowledge. For a given saving rate, the decrease in output implies a decrease in consumption.

The long-run steady-state effect is more subtle. We found in part (e) that output per effective worker falls in the steady state. But welfare depends on output (and consumption) per *worker*, not per effective worker. The increase in time spent in research universities implies that E grows faster. That is, output per worker equals yE. Although steady-state y falls, in the long run the faster growth rate of E necessarily dominates. That is, in the long run, consumption unambiguously rises.

Nevertheless, because of the initial decline in consumption, the increase in u is not unambiguously a good thing. That is, a policymaker who cares more about current generations than about future generations may decide not to pursue a policy of increasing u. (This is analogous to the question considered in Chapter 7 of whether a policymaker should try to reach the Golden Rule level of capital per effective worker if k is currently below the Golden Rule level.)

More Problems and Applications to Chapter 8

1. a. The growth in total output (Y) depends on the growth rates of labor (L), capital (K), and total factor productivity (A), as summarized by the equation:

$$\Delta Y/Y = \alpha\Delta K/K + (1 - \alpha)\Delta L/L + \Delta A/A,$$

where α is capital's share of output. We can look at the effect on output of a 5-percent increase in labor by setting $\Delta K/K = \Delta A/A = 0$. Since $\alpha = 2/3$, this gives us

$$\begin{aligned}\Delta Y/Y &= (1/3)\,(5\%)\\ &= 1.67\%.\end{aligned}$$

A 5-percent increase in labor input increases output by 1.67 percent.

Labor productivity is Y/L. We can write the growth rate in labor productivity as

$$\frac{\Delta(Y/L)}{Y/L} = \frac{\Delta Y}{Y} - \frac{\Delta L}{L}.$$

Substituting for the growth in output and the growth in labor, we find

$$\begin{aligned}\Delta(Y/L)/(Y/L) &= 1.67\% - 5.0\%\\ &= -3.34\%.\end{aligned}$$

Labor productivity falls by 3.34 percent.

To find the change in total factor productivity, we use the equation

$$\Delta A/A = \Delta Y/Y - \alpha\Delta K/K - (1 - \alpha)\Delta L/L.$$

For this problem, we find

$$\begin{aligned}\Delta A/A &= 1.67\% - 0 - (1/3)\,(5\%)\\ &= 0.\end{aligned}$$

Total factor productivity is the amount of output growth that remains after we have accounted for the determinants of growth that we can measure. In this case, there is no change in technology, so all of the output growth is attributable to measured input growth. That is, total factor productivity growth is zero, as expected.

b. Between years 1 and 2, the capital stock grows by 1/6, labor input grows by 1/3, and output grows by 1/6. We know that the growth in total factor productivity is given by

$$\Delta A/A = \Delta Y/Y - \alpha\Delta K/K - (1 - \alpha)\Delta L/L.$$

Substituting the numbers above, and setting $\alpha = 2/3$, we find

$$\begin{aligned}\Delta A/A &= (1/6) - (2/3)(1/6) - (1/3)(1/3)\\ &= 3/18 - 2/18 - 2/18\\ &= -\,1/18\\ &= -\,.056.\end{aligned}$$

Total factor productivity falls by 1/18, or approximately 5.6 percent.

2. By definition, output Y equals labor productivity Y/L multiplied by the labor force L:

$$Y = (Y/L)L.$$

Using the mathematical trick in the hint, we can rewrite this as

$$\frac{\Delta Y}{Y} = \frac{\Delta(Y/L)}{Y/L} + \frac{\Delta L}{L}.$$

We can rearrange this as

$$\frac{\Delta(Y/L)}{Y/L} = \frac{\Delta Y}{Y} - \frac{\Delta L}{L}.$$

Substituting for $\Delta Y/Y$ from the text, we find

$$\begin{aligned}\frac{\Delta(Y/L)}{Y/L} &= \frac{\Delta A}{A} + \frac{\alpha\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L} - \frac{\Delta L}{L} \\ &= \frac{\Delta A}{A} + \frac{\alpha\Delta K}{K} - \frac{\alpha\Delta L}{L} \\ &= \frac{\Delta A}{A} + \alpha\left[\frac{\Delta K}{K} - \frac{\Delta L}{L}\right].\end{aligned}$$

Using the same trick we used above, we can express the term in brackets as

$$\Delta K/K - \Delta L/L = \Delta(K/L)/(K/L).$$

Making this substitution in the equation for labor productivity growth, we conclude that

$$\frac{\Delta(Y/L)}{Y/L} = \frac{\Delta A}{A} + \frac{\alpha\Delta(K/L)}{K/L}.$$

3. We know the following:

$$\begin{aligned}\Delta Y/Y &= n + g = 3.6\% \\ \Delta K/K &= n + g = 3.6\% \\ \Delta L/L &= n = 1.8\% \\ \text{Capital's share} &= \alpha = 1/3 \\ \text{Labor's share} &= 1 - \alpha = 2/3.\end{aligned}$$

Using these facts, we can easily find the contributions of each of the factors, and then find the contribution of total factor productivity growth, using the following equations:

Output Growth	=	Capital's Contribution	+	Labor's Contribution	+	Total Factor Productivity
$\frac{\Delta Y}{Y}$		$\frac{\alpha\Delta K}{K}$		$\frac{(1-\alpha)\Delta L}{L}$		$\frac{\Delta A}{A}$
3.6%	=	(1/3)(3.6%)	+	(2/3)(1.8%)	+	$\Delta A/A$

We can easily solve this for $\Delta A/A$, to find that

3.6%	=	1.2%	+	1.2%	+	1.2%.

We conclude that the contribution of capital is 1.2% per year, the contribution of labor is 1.2% per year, and the contribution of total factor productivity growth is 1.2% per year. These numbers match the ones in Table 8–3 in the text for the United States from 1948–2002.