What Do Interest Rates Mean and What Is Their Role in Valuation?

Dagmar Linnertová

Measuring Interest Rates

- Different debt instruments have very different streams of cash payments to the holders
  - Cash flows
- With very different timing
  - Thus, it is important to understand
    - How we can compare the value of one kind of debt instrument with another before we see how interest rates are measured.
      - Concept of present value

Present value

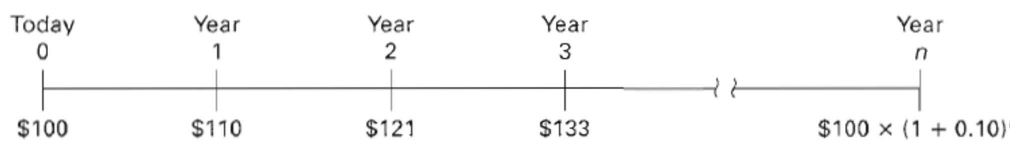
- It is base on simple idea
  - Dollar of cash flow paid to you one year from now is less valuable to you than a dollar paid to you today.
    - This is true because you can deposit a dollar in a saving account that earns interest and have more than dollar in one year.

Present value

- Simple loan
  - In this loan, the lender provides the borrower with an amount of funds (principal) that must be repaid to the lender at the maturity date, along with an additional payment for the interest.
    - At the end of on n year, your 100 $ would turn into:

$$\$100 \times (1 + i)^n$$

- The amounts you would have at the end of each year by making the 100 $ loan today can be seen in the following time line:



Present value

- This time line tells you that
  - for you is 100 in today equal as 110 in year 1 and equal as 121 in year 2, etc.
- The process of calculation today's value of dollars received in the future is called discounting of future:

$$PV = \frac{CF}{(1+i)^n}$$

Simple Present Value

What is the present value of $250 to be paid in two years if the interest rate is 15%?

Solution

The present value would be $189.04. Using Equation 1:

$$PV = \frac{CF}{(1+i)^n}$$

where

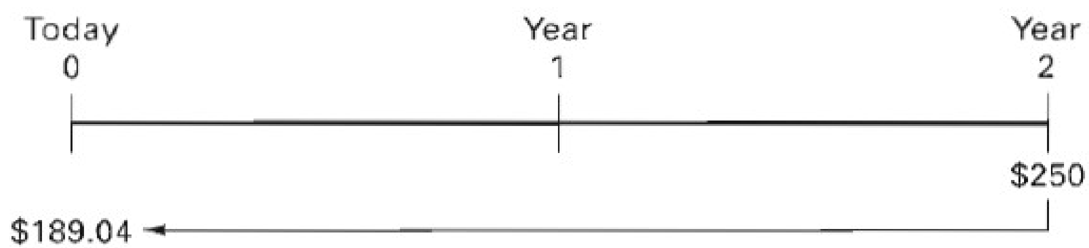
CF = cash flow in two years = \$250

i = annual interest rate = 0.15

n = number of years = 2

Thus

$$PV = \frac{\$250}{(1+0.15)^2} = \frac{\$250}{1.3225} = \$189.04$$



Present value

- The concept of present value is extremely useful because
  - You are able to figure out today's value of credit market instrument at a given simple interest rate *i* by adding up the present value of all the future cash flows received.
  - The concept of present value allows you to compare the value of two instruments with very different timing of their cash flows.

Four Types of Credit Market Instruments

- A simple loan
  - In which lender provides the borrower with an amount of funds, which must be repaid to the lender at the maturity date along with an additional payments for the interest.
  - Money markets instruments
    - Commercial loans to businesses

Four Types of Credit Market Instruments

- A fixed-payment loan (fully amortized loan)
  - The lender provides the borrower with an amount of funds, which must be repaid by making the same payment every period (such a month), consisting of part of the principal and interest for a set of year.
  - Money markets instruments
    - Installment loans (such a auto loans) and mortgages

Four Types of Credit Market Instruments

- A coupon bond
  - It pays the owner of the bond a fixed interest payment (coupon payment) every year until the maturity date when a specified final amount (face value) is repaid
  - The coupon payment is so named because the bondholder used to obtain payment by clipping a coupon off the bond and sending it to the bond issuer who then sent the payment to the holder


CONFEDERATE STATES OF AMERICA
1000
1000
$1000
1000
1000

Four Types of Credit Market Instruments

- A coupon bond is identified by three pieces of information.
    - Corporation or government agency that issues bond
    - Maturity date of the bond
    - The bond's coupon rate expressed as a percentage of the face value of the bond
  - Money markets instruments
  - Treasury bonds, corporate bonds

Four Types of Credit Market Instruments

- A discount bond (zero-coupon bond)
  - It is bought at a price below its face value (at a discount) and the face value is repaid at the maturity date.
  - A discount bond does not make any interest payments, it just pays off the face value

Four Types of Credit Market Instruments

- These four types of instruments require payments at different times:
  - Simple loans and discount bonds make payment only at their maturity dates
  - Payments loans and coupon bonds have payments periodically until maturity

Four Types of Credit Market Instruments

- How would you decide which of these instruments provides you with more income?
  - They all seem so different because they make payments at different times.
  - To solve this problem it is necessary to use the concept of present value.

Yield to Maturity

- The most important way how to calculate interest rate
  - Yield to maturity
    - The interest equates the present value of cash flows received from a debt instrument
  - This concept is considered to be one of the msot accurate measure of interest rates

Simple Loan

If Pete borrows $100 from his sister and next year she wants $110 back from him, what is the yield to maturity on this loan?

Solution

The yield to maturity on the loan is 10%.

$$PV = \frac{CF}{(1+i)^n}$$

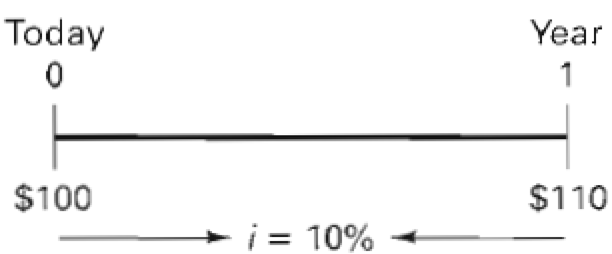
where

PV = amount borrowed = $\$100$

CF = cash flow in one year = $\$110$

n = number of years = 1

Thus



$$\$100 = \frac{\$110}{(1+i)}$$

$$(1+i)\$100 = \$110$$

$$(1+i) = \frac{\$110}{\$100}$$

$$i = 1.10 - 1 = 0.10$$

Fixed-Payment loan

- This type of loan has the same cash flow payment every year throughout the life of the loan.
- On a fixed-rate mortgage, for example, the borrower makes the same payment to the bank every month until the maturity date, when the loan is completely paid off.
- It is necessary to equate today's value of the loan with its present value.
- Because the fixed-payment loan involves more than one cash payment, the present value of the fixed-payment loan is calculated as the sum of the present values of all cash flows.

Fixed-Payment Loan

You decide to purchase a new home and need a $100,000 mortgage. You take out a loan from the bank that has an interest rate of 7%. What is the yearly payment to the bank to pay off the loan in 20 years?

Solution

The yearly payment to the bank is $9,439.29.

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \cdots + \frac{FP}{(1+i)^n}$$

where

$$\begin{aligned} LV &= \text{loan value amount} &&= \$100{,}000 \\ i &= \text{annual interest rate} &&= 0.07 \\ n &= \text{number of years} &&= 20 \end{aligned}$$

Thus

$$\$100{,}000 = \frac{FP}{1+0.07} + \frac{FP}{(1+0.07)^2} + \frac{FP}{(1+0.07)^3} + \cdots + \frac{FP}{(1+0.07)^{20}}$$

To find the monthly payment for the loan using a financial calculator:

$$\begin{aligned} n &= \text{number of years} &&= 20 \\ PV &= \text{amount of the loan } (LV) &&= -100{,}000 \\ FV &= \text{amount of the loan after 20 years} &&= 0 \\ i &= \text{annual interest rate} &&= .07 \end{aligned}$$

Then push the *PMT* button = fixed yearly payment (*FP*) = $9,439.29.

Coupon Bond

- The calculate the yield to maturity for a coupon bond, follow the same strategy used for fixed-payment loan:
  - Equate today's value of the bond with its present value.
  - Because coupon bonds also have more than one cash flow payment, the present value of the bond is calculated as the sum of the present values of all the coupon payments plus the present value of the final payment of the face value of the bond.

Coupon Bond

- For any coupon bond

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \cdots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

where
P = price of coupon bond
C = yearly coupon payment
F = face value of the bond
n = years to maturity date

Coupon Bond

Coupon Bond

Find the price of a 10% coupon bond with a face value of $1000, a 12.25% yield to maturity, and eight years to maturity.

Solution

The price of the bond is $889.20. To solve using a financial calculator:

$$
\begin{aligned}
n & = \text{years to maturity} & = 8 \\
FV & = \text{face value of the bond} & = 1000 \\
i & = \text{annual interest rate} & = 12.25\% \\
PMT & = \text{yearly coupon payments} & = 100
\end{aligned}
$$

Then push the *PV* button = price of the bond = $889.20.

Coupon Bond

TABLE 1 *Yields to Maturity on a 10% Coupon Rate Bond Maturing in Ten Years (Face Value = $1000)*

Price of Bond ($)	Yield to Maturity (%)
1200	7.13
1100	8.48
1000	10.00
900	11.75
800	13.81

- 1. When the coupon bond is priced at its face value, the yield to maturity equals to coupon rate,
- 2. The price of coupon bond and the yield to maturity are negatively related, that is, as the yield to maturity rises, the price of the bond falls. If the yield to maturity falls, the price of the bond rises.
- 3. The yield to maturity is greater than the coupon rate when the bond price is below its face value.

Perpetuity or consol

- There is one special case of a coupon bond that is worth discussing because its yield to maturity is particularly easy to calculate.
- This bond is called a perpetuity or a consol.
  - It is a perpetual bond without any maturity and repayment of principal that makes fixed coupon payments of X $ forever.

Perpetuity or consol

- The price of a perpetuity is simplifies to following:

$$P_c = \frac{C}{i_c}$$

where P_c = price of the perpetuity (consol)
C = yearly payment
i_c = yield to maturity of the perpetuity (consol)

Perpetuity or consol

Perpetuity

What is the yield to maturity on a bond that has a price of \$2000 and pays \$100 annually forever?

Solution

The yield to maturity would be 5%.

$$i_c = \frac{C}{P_c}$$

where

C = yearly payment = \$100

P_c = price of perpetuity (consol) = \$2000

Thus

$$i_c = \frac{\$100}{\$2000}$$

$$i_c = 0.05 = 5\%$$

Discount Bond

- The yield-to-maturity calculation for a discount bond is similar to that for the simple loan.
- Generally, for any one-year discount bond, the yield to maturity can be written as:

$$i = \frac{F - P}{P}$$

where F = face value of the discount bond
P = current price of the discount bond

Case of Japan

- Normally interest rates must be always positive
- Negative interest rates would imply that you are willing to pay more for a bond today than you will receive for it in the future
- Negative interest rates therefore seem like an impossible because you would do better by holding cash that the same values in the future as it does today
- In November 1998, Japan, interest rate of Japanese six-months T-bills became negative, yielding an interest rate -0,004 %, with investors paying more for the bills than their face value.
  - Weakness of Japanse economy and a negative inflation rate have driven Japanese interest rate to low levels, but they can explain the negative rates

- The answer is that large investors find it more convenient to hold these six-months bills as a store of value rather than holding cash because the bills are denominated in large amounts and can be stored electronically.
- These advantages of the Japanese T-bills result in some investors being willing to hold them, given their negative rates, even though in monetary terms the investors would be better off holding cash.
- Clearly, the convenience of T-bills only goes so far and thus their interest rates can go only a little bit bellow zero.

The Distinction Between Real and Nominal Interest Rates

- So far we have ignored the effects of inflation on the cost of borrowing.
- Real vs. nominal interest rate
- Real interest rate
  - Adjusted by expected changes in the price level
    - Reflects the true cost of borrowing
    - Ex ante interest rate
      - It is adjusted for expected changes in the price level
      - “real” interest rate

The Distinction Between Real and Nominal Interest Rates

- Fisher equation

$$i = i_r + \pi^e$$

  - States that the nominal interest rate is equal the real interest rate + the expected rate of inflation
  - When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend.
  - The distinction between real and nominal interest rates is important because the real interest rate, which reflects the real cost of borrowing, is likely to be a better indicator of the incentives to borrow and lend.
  - It is appear to be a better guide to how people will be affected by what is happening in credit markets.

The Distinction Between Real and Nominal Interest Rates

The Distinction Between Real and Nominal Interest Rates

- U.S. Treasury bill, shows that nominal and real interest rates often do not move together.
- In particular
  - Nominal rates were high in the 1970's
    - Real rates were extremely low, often negative
      - By the standard of nominal interest rates, you would thought that credit market conditions were tight in this period because it was expensive to borrow.
      - The estimation of the real rates indicate that you would have been mistaken. In real terms, the cost of borrowing was actually quite low.

The Distinction Between Interest Rate and Returns

$$R = \frac{C + P_{t+1} - P_t}{P_t}$$

where R = return from holding the bond from time t to time $t + 1$

P_t = price of the bond at time t

P_{t+1} = price of the bond at time $t + 1$

C = coupon payment

The Distinction Between Interest Rate and Returns

TABLE 2 *One-Year Returns on Different-Maturity 10% Coupon Rate Bonds When Interest Rates Rise from 10% to 20%*

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1000	503	−49.7	−39.7
20	10	1000	516	−48.4	−38.4
10	10	1000	597	−40.3	−30.3
5	10	1000	741	−25.9	−15.9
2	10	1000	917	− 8.3	+ 1.7
1	10	1000	1000	0.0	+10.0

The Distinction Between Interest Rate and Returns

- Table calculates the one-year return on several 10% coupon rate bonds when interest rates on all these bonds rise from 10% to 20%.
  - The only bond whose return equals the initial yield to maturity is one whose time to maturity is the same as the holding period
  - A rise in interest rates is associated with a fall in bond prices, resulting in capital losses on bond whose terms to maturity are longer than the holding period
  - The more distant a bond's maturity, the greater the size of the price change associated with an interest-rate change
  - The more distant a bond's maturity, the lower the rate of return that occurs as a result of the increase in the interest rate
  - Even though a bond has a substantial initial; interest rate, its return can turn out to be negative if interest rates rise

The Distinction Between Interest Rate and Returns

- A rise in the interest rate means that the price of a bond has fallen.
- A rise in interest rates therefore means that a capital loss has occurred, and if this loss is large enough, the bond can be a poor investment indeed.

Maturity and the Volatility of Bond Returns: Interest-rate Risk

- The findings that the price of longer-maturity bonds respond more dramatically to changes in interest rates helps explain an important fact about the behavior of bond markets:
  - Price and returns for long-term bonds are more volatile than those for shorter-term bonds.

Maturity and the Volatility of Bond Returns: Interest-rate Risk

- The riskiness of an asset's return that results from interest-rate changes
  - Is called interest-rate risk
- Although long-term debt instruments have substantial interest-rate risk, short-term debt instruments do not.
  - Bonds with a maturity that is as short as the holding period have no interest-rate risk.

Summary

- The return on a bond, which tell you how good an investment it has been over the holding period, is equal to the yield to maturity in only one case:
  - When the holding period and the maturity of the bond are identical
- Bonds whose term to maturity is longer than the holding period are subject to interest-rate risk:
  - Changes in interest rates lead to capital gains and losses that produce differences between the return and the yield to maturity known as the time the bond is purchased.
  - Interest-rate risk is especially important for long-term bonds, where capital gains and losses can be substantial.
    - This is why long-term bonds are not considered to be safe assets with a sure return over short holding periods.