CHAPTER FIVE 5

LEARNING ABOUT RETURN AND RISK FROM THE HISTORICAL RECORD

PART II

CASUAL OBSERVATION AND formal research both suggest that investment risk is as important to investors as expected return. While we have theories about the relationship between risk and expected return that would prevail in rational capital markets, there is no theory about the levels of risk we should find in the marketplace. We can at best estimate the level of risk likely to confront investors by analyzing historical experience.

This situation is to be expected because prices of investment assets fluctuate in response to news about the fortunes of corporations, as well as to macroeconomic developments that affect interest rates. There is no theory about the frequency and importance of such events; hence we cannot determine a "natural" level of risk.

Compounding this difficulty is the fact that neither expected returns nor risk are directly observable. We observe only *realized* rates of return after the fact. Hence, to make forecasts about future expected returns and risk, we first must learn how to "forecast" their *past* values, that is, the expected returns and risk that investors actually anticipated, from historical data. (There is an old saying that forecasting the future is even more difficult than forecasting the past.) In this chapter, we present the essential tools for estimating expected returns and risk from the historical record and consider the implications of this record for future investments.

We begin by discussing interest rates and investments in safe assets and examine the history of risk-free investments in the U.S over the last 80 years. Moving to risky assets, we begin with scenario analysis of risky investments and the data inputs necessary to conduct it. With this in mind, we develop statistical tools needed to make inferences from historical time series of portfolio returns. We present a global view of the history of returns over 100 years from stocks and bonds in various countries and analyze the historical record of five broad asset-class portfolios. We end the chapter with discussions of implications of the historical record for future investments and a variety of risk measures commonly used in the industry.

5.1 DETERMINANTS OF THE LEVEL OF INTEREST RATES

Interest rates and forecasts of their future values are among the most important inputs into an investment decision. For example, suppose you have $10,000 in a savings account. The bank pays you a variable interest rate tied to some short-term reference rate such as the 30-day Treasury bill rate. You have the option of moving some or all of your money into a longer-term certificate of deposit that offers a fixed rate over the term of the deposit.

Your decision depends critically on your outlook for interest rates. If you think rates will fall, you will want to lock in the current higher rates by investing in a relatively long-term CD. If you expect rates to rise, you will want to postpone committing any funds to long-term CDs.

Forecasting interest rates is one of the most notoriously difficult parts of applied macroeconomics. Nonetheless, we do have a good understanding of the fundamental factors that determine the level of interest rates:

1. The supply of funds from savers, primarily households.
2. The demand for funds from businesses to be used to finance investments in plant, equipment, and inventories (real assets or capital formation).
3. The government's net supply of or demand for funds as modified by actions of the Federal Reserve Bank.

Before we elaborate on these forces and resultant interest rates, we need to distinguish real from nominal interest rates.

Real and Nominal Rates of Interest

An interest rate is a promised rate of return denominated in some unit of account (dollars, yen, euros, or even purchasing power units) over some time period (a month, a year, 20 years, or longer). Thus, when we say the interest rate is 5%, we must specify both the unit of account and the time period.

Assuming there is no default risk, we can refer to the promised rate of interest as a risk-free rate for that particular unit of account and time period. But if an interest rate is risk-free for one unit of account and time period, it will not be risk-free for other units or periods. For example, interest rates that are absolutely safe in dollar terms will be risky when evaluated in terms of purchasing power because of inflation uncertainty.

To illustrate, consider a 1-year dollar (nominal) risk-free interest rate. Suppose exactly 1 year ago you deposited $1,000 in a 1-year time deposit guaranteeing a rate of interest of 10%. You are about to collect $1,100 in cash. What is the real return on your investment? That depends on what money can buy these days, relative to what you *could* buy a year ago. The consumer price index (CPI) measures purchasing power by averaging the prices of goods and services in the consumption basket of an average urban family of four.

Suppose the rate of inflation (the percent change in the CPI, denoted by i) for the last year amounted to $i = 6\%$. This tells you that the purchasing power of money is reduced by 6% a year. The value of each dollar depreciates by 6% a year in terms of the goods it can buy. Therefore, part of your interest earnings are offset by the reduction in the purchasing power of the dollars you will receive at the end of the year. With a 10% interest rate, after you net out the 6% reduction in the purchasing power of money, you are left with a net increase in purchasing power of about 4%. Thus we need to distinguish between a **nominal interest rate**—the growth rate of your money—and a **real interest rate**—the growth rate of your purchasing power. If we call R the nominal rate, r the real rate, and i the inflation rate, then we conclude

$$r \approx R - i \tag{5.1}$$

In words, the real rate of interest is the nominal rate reduced by the loss of purchasing power resulting from inflation. If inflation turns out higher than 6%, your *realized* real return will be lower than 4%; if inflation is lower, your real rate will be higher.

In fact, the exact relationship between the real and nominal interest rate is given by

$$1 + r = \frac{1 + R}{1 + i} \tag{5.2}$$

This is because the growth factor of your purchasing power, $1 + r$, equals the growth factor of your money, $1 + R$, divided by the new price level, that is, $1 + i$ times its value in the previous period. The exact relationship can be rearranged to

$$r = \frac{R - i}{1 + i} \tag{5.3}$$

which shows that the approximation rule overstates the real rate by the factor $1 + i$.

EXAMPLE 5.1 Approximating the Real Rate

If the nominal interest rate on a 1-year CD is 8%, and you expect inflation to be 5% over the coming year, then using the approximation formula, you expect the real rate of interest to be $r = 8\% - 5\% = 3\%$. Using the exact formula, the real rate is $r = \frac{.08 - .05}{1 + .05} = .0286$, or 2.86%. Therefore, the approximation rule overstates the expected real rate by only .14% (14 basis points). The approximation rule is more exact for small inflation rates and is perfectly exact for continuously compounded rates. We discuss further details in the next section.

Before the decision to invest, you should realize that conventional certificates of deposit offer a guaranteed *nominal* rate of interest. Thus you can only infer the expected real rate on these investments by subtracting your expectation of the rate of inflation.

It is always possible to calculate the real rate after the fact. The inflation rate is published by the Bureau of Labor Statistics (BLS). The future real rate, however, is unknown, and one has to rely on expectations. In other words, because future inflation is risky, the real rate of return is risky even when the nominal rate is risk-free.

The Equilibrium Real Rate of Interest

Three basic factors—supply, demand, and government actions—determine the *real* interest rate. The nominal interest rate, which is the rate we actually observe, is the real rate plus the expected rate of inflation. So a fourth factor affecting the interest rate is the expected rate of inflation.

Although there are many different interest rates economywide (as many as there are types of securities), these rates tend to move together, so economists frequently talk as if there were a single representative rate. We can use this abstraction to gain some insights into the real rate of interest if we consider the supply and demand curves for funds.

Figure 5.1 shows a downward-sloping demand curve and an upward-sloping supply curve. On the horizontal axis, we measure the quantity of funds, and on the vertical axis, we measure the real rate of interest.

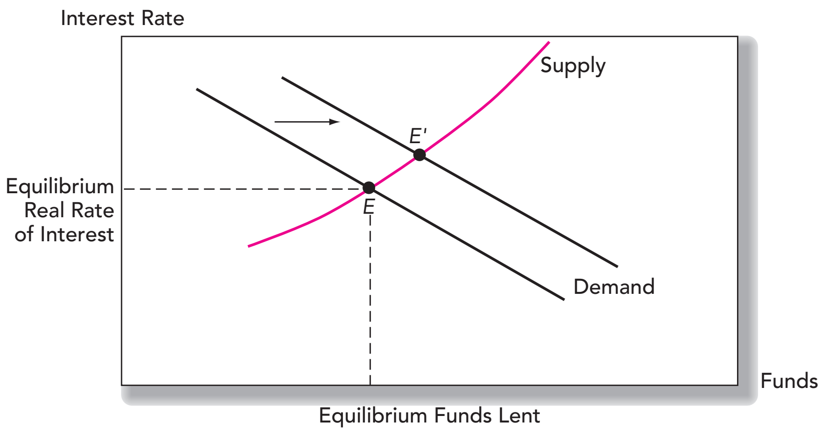

FIGURE 5.1 Determination of the equilibrium real rate of interest

The supply curve slopes up from left to right because the higher the real interest rate, the greater the supply of household savings. The assumption is that at higher real interest rates households will choose to postpone some current consumption and set aside or invest more of their disposable income for future use.[1]

The demand curve slopes down from left to right because the lower the real interest rate, the more businesses will want to invest in physical capital. Assuming that businesses rank projects by the expected real return on invested capital, firms will undertake more projects the lower the real interest rate on the funds needed to finance those projects.

Equilibrium is at the point of intersection of the supply and demand curves, point E in Figure 5.1.

The government and the central bank (the Federal Reserve) can shift these supply and demand curves either to the right or to the left through fiscal and monetary policies. For example, consider an increase in the government's budget deficit. This increases the government's borrowing demand and shifts the demand curve to the right, which causes the equilibrium real interest rate to rise to point E'. That is, a forecast that indicates higher than previously expected government borrowing increases expected future interest rates. The Fed can offset such a rise through an expansionary monetary policy, which will shift the supply curve to the right.

Thus, although the fundamental determinants of the real interest rate are the propensity of households to save and the expected productivity (or we could say profitability) of investment in physical capital, the real rate can be affected as well by government fiscal and monetary policies.

The Equilibrium Nominal Rate of Interest

We've seen that the real rate of return on an asset is approximately equal to the nominal rate minus the inflation rate. Because investors should be concerned with their real returns—the increase in their purchasing power—we would expect that as the inflation rate increases, investors will demand higher nominal rates of return on their investments. This higher rate is necessary to maintain the expected real return offered by an investment.

Irving Fisher (1930) argued that the nominal rate ought to increase one-for-one with increases in the expected inflation rate. If we use the notation $E(i)$ to denote the current

[1]There is considerable disagreement among experts on the extent to which household saving does increase in response to an increase in the real interest rate.

expectation of the inflation rate that will prevail over the coming period, then we can state the so-called Fisher equation formally as

$$R = r + E(i) \tag{5.4}$$

The equation implies that if real rates are reasonably stable, then increases in nominal rates ought to predict higher inflation rates. This relationship has been debated and empirically investigated. The results are mixed; although the data do not strongly support this relationship, nominal interest rates seem to predict inflation as well as alternative methods, in part because we are unable to forecast inflation well with any method.

One reason it is difficult to determine the empirical validity of the Fisher hypothesis that changes in nominal rates predict changes in future inflation rates is that the real rate also changes unpredictably over time. Nominal interest rates can be viewed as the sum of the required real rate on nominally risk-free assets, plus a "noisy" forecast of inflation.

In Part Four we discuss the relationship between short- and long-term interest rates. Longer rates incorporate forecasts for long-term inflation. For this reason alone, interest rates on bonds of different maturity may diverge. In addition, we will see that prices of longer-term bonds are more volatile than those of short-term bonds. This implies that expected returns on longer-term bonds may include a risk premium, so that the expected real rate offered by bonds of varying maturity also may vary.

CONCEPT CHECK 1

a. Suppose the real interest rate is 3% per year and the expected inflation rate is 8%. What is the nominal interest rate?

b. Suppose the expected inflation rate rises to 10%, but the real rate is unchanged. What happens to the nominal interest rate?

Taxes and the Real Rate of Interest

Tax liabilities are based on *nominal* income and the tax rate determined by the investor's tax bracket. Congress recognized the resultant "bracket creep" (when nominal income grows due to inflation and pushes taxpayers into higher brackets) and mandated index-linked tax brackets in the Tax Reform Act of 1986.

Index-linked tax brackets do not provide relief from the effect of inflation on the taxation of savings, however. Given a tax rate (t) and a nominal interest rate (R), the after-tax interest rate is $R(1 - t)$. The real after-tax rate is approximately the after-tax nominal rate minus the inflation rate:

$$R(1 - t) - i = (r + i)(1 - t) - i = r(1 - t) - it \tag{5.5}$$

Thus the after-tax real rate of return falls as the inflation rate rises. Investors suffer an inflation penalty equal to the tax rate times the inflation rate. If, for example, you are in a 30% tax bracket and your investments yield 12%, while inflation runs at the rate of 8%, then your before-tax real rate is approximately 4%, and you *should,* in an inflation-protected tax system, net after taxes a real return of 4%(1 − .3) = 2.8%. But the tax code does not recognize that the first 8% of your return is no more than compensation for inflation—not real income—and hence your after-tax return is reduced by 8% × .3 = 2.4%, so that your after-tax real interest rate, at .4%, is almost wiped out.

5.2 COMPARING RATES OF RETURN FOR DIFFERENT HOLDING PERIODS

Consider an investor who seeks a safe investment, for example, in U.S. Treasury securities.[2] Suppose we observe zero-coupon Treasury securities with several different maturities. Zero-coupon bonds, discussed more fully in Chapter 14, are bonds that are sold at a discount from par value and provide their entire return from the difference between the purchase price and the ultimate repayment of par value.[3] Given the price, $P(T)$, of a Treasury bond with \$100 par value and maturity of T years, we calculate the total risk-free return as the percentage increase in the value of the investment over the life of the bond.

$$r_f(T) = \frac{100}{P(T)} - 1 \tag{5.6}$$

For $T = 1$, Equation 5.6 provides the risk-free rate for an investment horizon of 1 year.

EXAMPLE 5.2 Annualized Rates of Return

Suppose prices of zero-coupon Treasuries with \$100 face value and various maturities are as follows. We find the total return of each security by using Equation 5.6:

Horizon, T	Price, $P(T)$	$[100/P(T)] - 1$	Risk-Free Return for Given Horizon
Half-year	\$97.36	100/97.36 − 1 = .0271	$r_f(.5) = 2.71\%$
1 year	\$95.52	100/95.52 − 1 = .0469	$r_f(1) = 4.69\%$
25 years	\$23.30	100/23.30 − 1 = 3.2918	$r_f(25) = 329.18\%$

Not surprisingly, longer horizons in Example 5.2 provide greater total returns. How should we compare the returns on investments with differing horizons? This requires that we re-express each *total* return as a *rate* of return for a common period. We typically express all investment returns as an **effective annual rate (EAR),** defined as the percentage increase in funds invested over a 1-year horizon.

For a 1-year investment, the EAR equals the total return, $r_f(1)$, and the gross return, (1 + EAR), is the terminal value of a \$1 investment. For investments that last less than 1 year, we compound the per-period return for a full year. For example, for the 6-month bill in Example 5.2, we compound the 2.71% half-year return for two semiannual periods to obtain a terminal value of $1 + \text{EAR} = (1.0271)^2 = 1.0549$, implying that EAR = 5.49%.

For investments longer than a year, the convention is to express the EAR as the annual rate that would compound to the same value as the actual investment. For example, the

[2]Yields on Treasury bills and bonds of various maturities are widely available on the Web, for example at Yahoo! Finance, MSN Money, or directly from the Federal Reserve.

[3]The U.S. Treasury issues T-bills, which are pure discount (or zero-coupon) securities with maturities of up to 1 year. However, financial institutions create zero-coupon Treasury bonds called Treasury strips with maturities up to 30 years by buying coupon-paying T-bonds, “stripping” off the coupon payments, and selling claims to the coupon payments and final payment of face value separately. See Chapter 14 for further details.

investment in the 25-year bond in Example 5.2 grows by its maturity date by a factor of 4.2918 (i.e., 1 + 3.2918), so its EAR is

$$(1 + \text{EAR})^{25} = 4.2918$$

$$1 + \text{EAR} = 4.2918^{1/25} = 1.0600$$

In general, we can relate EAR to the total return, $r_f(T)$, over a holding period of length T by using the following equation:

$$1 + \text{EAR} = [1 + r_f(T)]^{1/T} \quad \textbf{(5.7)}$$

We can illustrate with an example.

EXAMPLE 5.3 Equivalent Annual Return versus Total Return

For the 6-month Treasury in Example 5.2, $T = ½$, and $1/T = 2$. Therefore,

$$1 + \text{EAR} = (1.0271)^2 = 1.0549 \text{ and EAR} = 5.49\%$$

For the 25-year Treasury in Example 5.2, $T = 25$. Therefore,

$$1 + \text{EAR} = 4.2918^{1/25} = 1.060 \text{ and EAR} = 6.0\%$$

Annual Percentage Rates

Rates on short-term investments (by convention, $T < 1$ year) often are annualized using simple rather than compound interest. These are called **annual percentage rates,** or **APRs.** For example, the APR corresponding to a monthly rate such as that charged on a credit card is calculated by multiplying the monthly rate by 12. More generally, if there are n compounding periods in a year, and the per-period rate is $r_f(T)$, then the APR $= n \times r_f(T)$. Conversely, you can find the true per-period rate from the APR as $r_f(T) = T \times \text{APR}$.

Using this procedure, the APR of the 6-month bond in Example 5.2 (which had a 6-month rate of 2.71%) is $2 \times 2.71 = 5.42\%$. To generalize, note that for short-term investments of length T, there are $n = 1/T$ compounding periods in a year. Therefore, the relationship among the compounding period, the EAR, and the APR is

$$1 + \text{EAR} = [1 + r_f(T)]^n = [1 + r_f(T)]^{1/T} = [1 + T \times \text{APR}]^{1/T} \quad \textbf{(5.8)}$$

Equivalently,

$$\text{APR} = \frac{(1 + \text{EAR})^T - 1}{T}$$

EXAMPLE 5.4 EAR versus APR

We use Equation 5.8 to find the APR corresponding to an EAR of 5.8% with various common compounding periods, and, conversely, the values of EAR implied by an APR of 5.8%. The results appear in Table 5.1.

Continuous Compounding

It is evident from Table 5.1 (and Equation 5.8) that the difference between APR and EAR grows with the frequency of compounding. This raises the question, How far will these

Compounding Period	T	EAR = $[1 + r_f(T)]^{1/T} - 1 = .058$		APR = $r_f(T)$*(1/T) = .058	
		$r_f(T)$	APR = [(1 + EAR)^T − 1]/T	$r_f(T)$	EAR = (1 + APR*T)^(1/T) −1
1 year	1.0000	.0580	.05800	.0580	.05800
6 months	0.5000	.0286	.05718	.0290	.05884
1 quarter	0.2500	.0142	.05678	.0145	.05927
1 month	0.0833	.0047	.05651	.0048	.05957
1 week	0.0192	.0011	.05641	.0011	.05968
1 day	0.0027	.0002	.05638	.0002	.05971
Continuous			r_{cc} = ln(1 + EAR) = .05638		EAR = exp (r_{cc}) − 1 = .05971

TABLE 5.1

Annual percentage rate (APR) and effective annual rates (EAR). In the first set of columns, we hold the equivalent annual rate (EAR) fixed at 5.8%, and find APR for each holding period. In the second set of columns, we hold APR fixed and solve for EAR.

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two rates diverge as the compounding frequency continues to grow? Put differently, what is the limit of $[1 + T \times \text{APR}]^{1/T}$, as T gets ever smaller? As T approaches zero, we effectively approach *continuous compounding (CC)*, and the relation of EAR to the annual percentage rate, denoted by r_{cc} for the continuously compounded case, is given by the exponential function

$$1 + \text{EAR} = \exp(r_{cc}) = e^{r_{cc}} \tag{5.9}$$

where e is approximately 2.71828.

To find r_{cc} from the effective annual rate, we solve Equation 5.9 for r_{cc} as follows:

$$\ln(1 + \text{EAR}) = r_{cc}$$

where ln (•) is the natural logarithm function, the inverse of exp (•). Both the exponential and logarithmic functions are available in Excel, and are called LN() and EXP(), respectively.

EXAMPLE 5.5 Continuously Compounded Rates

The continuously compounded annual percentage rate, r_{cc}, that provides an EAR of 5.8% is 5.638% (see Table 5.1). This is virtually the same as the APR for daily compounding. But for less frequent compounding, for example, semiannually, the APR necessary to provide the same EAR is noticeably higher, 5.718%. With less frequent compounding, a higher APR is necessary to provide an equivalent effective return.

While continuous compounding may at first seem to be a mathematical nuisance, working with such rates in many cases can actually simplify calculations of expected return and risk. For example, given a continuously compounded rate, the total return for any period T, $r_{cc}(T)$, is simply $\exp(T \times r_{cc})$.[4] In other words, the total return scales up in direct

[4]This follows from Equation 5.9. If $1 + \text{EAR} = e^{r_{cc}}$, then $(1 + \text{EAR})^T = e^{r_{cc}T}$.

proportion to the time period, T. This is far simpler than working with the exponents that arise using discrete period compounding. As another example, look again at Equation 5.1. There, the relationship between the real rate, r, the nominal rate R, and the inflation rate i, $r \approx R - i$, was only an approximation, as demonstrated by Equation 5.3. But if we express all rates as continuously compounded, then Equation 5.1 is exact,[5] that is, $r_{cc}(\text{real}) = r_{cc}(\text{nominal}) - i_{cc}$.

CONCEPT CHECK 2

A bank offers you two alternative interest schedules for a savings account of $100,000 locked in for 3 years: (*a*) a monthly rate of 1%; (*b*) an annually, continuously compounded rate (r_{cc}) of 12%. Which alternative should you choose?

5.3 BILLS AND INFLATION, 1926–2005

In this chapter we will often work with a history that begins in 1926, and it is fair to ask why. The reason is simply that January 1, 1926, is the starting date of the most widely available accurate return database.

Table 5.2 summarizes the history of short-term interest rates in the U.S., the inflation rate, and the resultant real rate. You can find the entire post-1926 history of the annual rates of these series on the text's Web site, **www.mhhe.com/bkm** (link to the student material for Chapter 5). The annual rates on T-bills are computed from rolling over twelve 1-month bills during each year. The real rate is computed from the annual T-bill rate and the percent change in the CPI according to Equation 5.2.

Table 5.2 shows the averages, standard deviations, and the first-order serial correlations for the full 80-year history (1926–2005) as well as for various subperiods. The first-order serial correlation measures the relationship between the interest rate in one year with the rate in the preceding year. If this correlation is positive, then a high rate tends to be followed by another high rate, whereas if it is negative, a high rate tends to be followed by a low rate.

The discussion of equilibrium real rates of interest in Section 5.1 suggests that we should start with the series of real rates. The average real rate for the full 80-year period, .72%, is quite different from the average over the 40-year period 1966–2005, which is 1.25%. We see that the real rate has been steadily rising, reaching a level of 2.28% for the generation of 1981–2005. The standard deviation of the real rate over the whole period, 3.97%, was driven by much higher variability in the early years. The real rate was far more stable in the period of 1981–2005, with a standard deviation of only 2.35%.

We can attribute a good part of these trends to policies of the Federal Reserve Board. Since the early 1980s, the Fed has adopted a policy of maintaining a low rate of inflation and a stable real rate. Some believe that the higher level of real rates in recent years may also be attributable to increased productivity of capital, particularly investments in information technology when applied to a better educated labor force.

[5] $1 + r(\text{real}) = \dfrac{1 + r(\text{nominal})}{1 + \text{inflation}}$

$$\Rightarrow \ln[1 + r(\text{real})] = \ln\left(\frac{1 + r(\text{nominal})}{1 + \text{inflation}}\right) = \ln[1 + r(\text{nominal})] - \ln(1 + \text{inflation})$$

$$\Rightarrow r_{cc}(\text{real}) = r_{cc}(\text{nominal}) - i_{cc}$$

Portfolio	Statistic	1926–2005	1966–2005	1981–2005	1971–1995	1961–1985	1951–1975	1941–1965	1931–1955	1926–1950
U.S. T-bills	Average	3.75	5.98	5.73	7.04	6.55	3.66	1.62	0.63	1.02
	Standard Deviation	3.15	2.84	3.15	2.87	3.15	1.97	1.16	0.57	1.33
	Serial Correlation	0.91	0.81	0.87	0.76	0.82	0.82	0.85	0.85	0.88
U.S. CPI inflation	Average	3.13	4.70	3.36	5.60	5.39	3.28	3.39	2.21	1.51
	Standard Deviation	4.29	3.02	1.62	3.38	3.63	2.99	4.28	5.75	6.02
	Serial Correlation	0.64	0.73	0.32	0.68	0.74	0.69	0.35	0.46	0.52
U.S. real rate	Average	0.72	1.25	2.28	1.41	1.14	0.40	−1.54	−1.25	−0.13
	Standard Deviation	3.97	2.35	2.18	2.76	2.60	1.61	4.40	5.68	6.35
	Serial Correlation	0.66	0.71	0.72	0.71	0.76	0.49	0.53	0.51	0.64

TABLE 5.2

History of T-bill rates, inflation, and real rates for generations, 1926–2005
Sources: *T-bills:* Salomon Smith Barney 3-month U.S. T-bill index; *inflation data:* Bureau of Labor Statistics.

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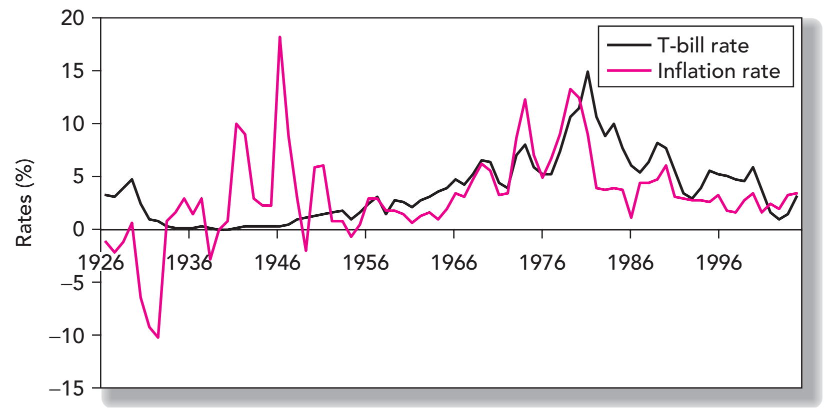

FIGURE 5.2 Interest and inflation rates, 1926–2005

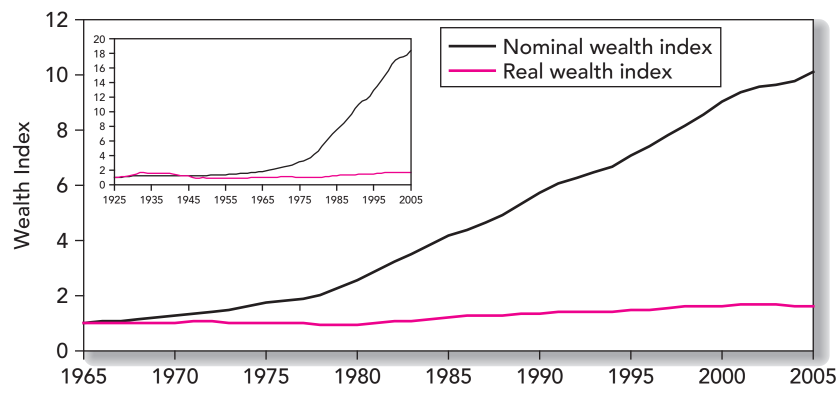

FIGURE 5.3 Nominal and real wealth indexes for investments in Treasury bills, 1966–2005 (inset figure is for 1925–2005)

In the same vein, we observe that average rates of inflation in the years 1966 through 2005 were higher than in the early twentieth century because of deflation in the early period. In line with modern Fed policies, the standard deviation of the rate of inflation moderated significantly to a level of 1.62% from 1981 through 2005. Of course, no one can rule out more extreme temporary fluctuations as a result of possible severe shocks to the economy.

We have seen that fluctuations in short-term interest rates are determined by variation in real interest rates and the expected short-term rate of inflation. In recent years, for which there has been less variability in the real rate, inflation has been the driving force. This is clear in Figure 5.2, where we see that short-term interest rates have tracked inflation quite closely since the 1950s. Indeed, the correlation between the T-bill rate and the inflation rate is .41 for the full 80-year history, .69 for the later 40 years, and 0.72 for the most recent generation, 1981–2005.

Figure 5.3 shows the progression of the nominal and real value of $1 invested in T-bills at the beginning of 1926, accumulated to 2005. The progression of the value of a $1 investment is called a *wealth index*. The wealth index in a current year is obtained by compounding the portfolio value from the end of the previous year by $1 + r$, the gross rate of return in the current year. Deviations of the curve of the nominal wealth index in Figure 5.3 from a smooth exponential line are due to variation over time in the rate of return. The lines in Figure 5.3, which grow quite smoothly, clearly demonstrate that short-term interest rate risk (real as well as nominal) is small even for long-term horizons. It certainly is less risky by an order of magnitude than investments in stocks, as we will soon see.

One important lesson from this history is the effect of inflation when compounded over long periods. The average inflation rate was 3.02% between 1926 and 2005, and 4.29%

between 1966 and 2005. These rates may not seem impressive, but are sufficient to reduce the terminal value of \$1 invested in 1966 from a nominal value of \$10.08 in 2005 to a real (constant purchasing power) value of only \$1.63.

5.4 RISK AND RISK PREMIUMS

Holding-Period Returns

You are considering investing in a stock-index fund. The fund currently sells for \$100 per share. With an investment horizon of 1 year, the realized rate of return on your investment will depend on (*a*) the price per share at year's end and (*b*) the cash dividends you will collect over the year.

Suppose the price per share at year's end is \$110 and cash dividends over the year amount to \$4. The realized return, called the *holding-period return,* HPR (in this case, the holding period is 1 year), is defined as

$$\text{HPR} = \frac{\text{Ending price of a share} - \text{Beginning price} + \text{Cash dividend}}{\text{Beginning price}} \tag{5.10}$$

In our case we have

$$\text{HPR} = \frac{\$110 - \$100 + \$4}{\$100} = .14, \text{ or } 14\%$$

This definition of the HPR assumes the dividend is paid at the end of the holding period. To the extent that dividends are received earlier, the HPR ignores reinvestment income between the receipt of the payment and the end of the holding period. The percent return from dividends is called the **dividend yield,** and so the dividend yield plus the capital gains yield equals the HPR.

Expected Return and Standard Deviation

There is considerable uncertainty about the price of a share plus dividend income 1 year from now, however, so you cannot be sure about your eventual HPR. We can quantify our beliefs about the state of the economy and the stock market in terms of three possible scenarios with probabilities as presented in columns A through E of Spreadsheet 5.1.

How can we evaluate this probability distribution? Throughout this book we will characterize probability distributions of rates of return in terms of their expected or mean return, $E(r)$, and their standard deviation, σ. The expected rate of return is a probability-weighted average of the rates of return in each scenario. Calling $p(s)$ the probability of each scenario and $r(s)$ the HPR in each scenario, where scenarios are labeled or "indexed" by s, we may write the expected return as

$$E(r) = \sum_s p(s)r(s) \tag{5.11}$$

Applying this formula to the data in Spreadsheet 5.1, we find that the expected rate of return on the index fund is

$$E(r) = (0.30 \times 34\%) + (.5 \times 14\%) + [0.20 \times (-16\%)] = 14\%$$

Spreadsheet 5.1 shows that this sum can be evaluated easily in Excel, using the SUMPRODUCT function, which first calculates the products of a series of number pairs, and

	A	B	C	D	E	F	G	H
1								
2								
3	Rates of return expressed as decimals							
4	Purchase Price =		$100		T-bill Rate =	0.06		
5								
6						Squared		Squared
7	State of the		Year-end	Cash		Deviations	Excess	Deviations
8	Economy	Probability	Price	Dividends	HPR	from Mean	Returns	from Mean
9	Boom	0.3	129.50	4.50	0.34	0.040	0.28	0.040
10	Normal growth	0.5	110.00	4.00	0.14	0.000	0.08	0.000
11	Recession	0.2	80.50	3.50	−0.16	0.090	−0.22	0.090
12	Expected value (mean)		SUMPRODUCT(B9:B11, E9:E11) =		0.14			
13	Standard deviation of HPR			SUMPRODUCT(B9:B11, F9:F11)^.5 =		0.1732		
14	Risk premium				SUMPRODUCT(B9:B11, G9:G11) =		0.08	
15	Standard deviation of excess return					SUMPRODUCT(B9:B11, H9:H11)^0.5 =		0.1732

SPREADSHEET 5.1

Distribution of HPR on the stock-index fund

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then sums the products. Here, the number pair is the probability of each scenario and the rate of return.

The standard deviation of the rate of return (σ) is a measure of risk. It is defined as the square root of the variance, which in turn is the expected value of the squared deviations from the expected return. The higher the volatility in outcomes, the higher will be the average value of these squared deviations. Therefore, variance and standard deviation measure the uncertainty of outcomes. Symbolically,

$$\sigma^2 = \sum_s p(s)\,[r(s) - E(r)]^2 \qquad \textbf{(5.12)}$$

Therefore, in our example

$$\sigma^2 = 0.3(34 - 14)^2 + .5(14 - 14)^2 + 0.2(-16 - 14)^2 = 300,$$

and

$$\sigma = \sqrt{300} = 17.32\%$$

Clearly, what would trouble potential investors in the index fund is the downside risk of a −16% rate of return, not the upside potential of a 34% rate of return. The standard deviation of the rate of return does not distinguish between these two; it treats both simply as deviations from the mean. As long as the probability distribution is more or less symmetric about the mean, σ is an adequate measure of risk. In the special case where we can assume that the probability distribution is normal—represented by the well-known bell-shaped curve—$E(r)$ and σ are perfectly adequate to characterize the distribution.

Excess Returns and Risk Premiums

How much, if anything, should you invest in the index fund? First, you must ask how much of an expected reward is offered for the risk involved in investing money in stocks.

We measure the reward as the difference between the *expected* HPR on the index stock fund and the **risk-free rate,** that is, the rate you can earn by leaving money in risk-free assets such as T-bills, money market funds, or the bank. We call this difference the **risk premium** on common stocks. If the risk-free rate in the example is 6% per year, and the expected index fund return is 14%, then the risk premium on stocks is 8% per year. The difference in any particular period between the *actual* rate of return on a risky asset and the risk-free rate is called **excess return.** Therefore, the risk premium is the expected value of the excess return, and the standard deviation of the excess return is an appropriate measure of its risk. (See Spreadsheet 5.1 for these calculations.)

The degree to which investors are willing to commit funds to stocks depends on **risk aversion.** Financial analysts generally assume investors are risk averse in the sense that, if the risk premium were zero, people would not be willing to invest any money in stocks. In theory, then, there must always be a positive risk premium on stocks in order to induce risk-averse investors to hold the existing supply of stocks instead of placing all their money in risk-free assets.

Although this sample scenario analysis illustrates the concepts behind the quantification of risk and return, you may still wonder how to get a more realistic estimate of $E(r)$ and σ for common stocks and other types of securities. Here, history has insights to offer. Analysis of the historical record of portfolio returns, however, makes use of a variety of important statistical tools and concepts, and so we first turn to a preparatory discussion.

CONCEPT CHECK 3

You invest $27,000 in a corporate bond selling for $900 per $1,000 par value. Over the coming year, the bond will pay interest of $75 per $1,000 of par value. The price of the bond at year's end will depend on the level of interest rates that will prevail at that time. You construct the following scenario analysis:

Interest Rates	Probability	Year-End Bond Price
High	.2	$850
Unchanged	.5	915
Low	.3	985

Your alternative investment is a T-bill that yields a sure rate of return of 5%. Calculate the HPR for each scenario, the expected rate of return, and the risk premium on your investment. What is the expected end-of-year dollar value of your investment?

5.5 TIME SERIES ANALYSIS OF PAST RATES OF RETURN

Time Series versus Scenario Analysis

In a forward-looking scenario analysis we determine a set of relevant scenarios and associated investment outcomes (rates of return), assign probabilities to each, and conclude by computing the risk premium (the reward) and standard deviation (the risk) of the proposed investment. In contrast, asset and portfolio return histories come in the form of time series of past realized returns that do not explicitly provide investors' original assessments of the probabilities of those observed returns; we observe only dates and associated HPRs. We must infer from this limited data the probability distributions from which these returns might have been drawn or, at least, some of its characteristics such as expected return and standard deviation.

Expected Returns and the Arithmetic Average

When we use historical data, we treat each observation as an equally likely "scenario." So if there are n observations, we substitute equal probabilities of magnitude $1/n$ for each $p(s)$

in Equation 5.11. The expected return, $E(r)$, is then estimated by the arithmetic average of the sample rates of return:

$$E(r) = \sum_{s=1}^{n} p(s)r(s) = \frac{1}{n}\sum_{s=1}^{n} r(s) \tag{5.13}$$

$$= \text{arithmetic average of rates of return}$$

EXAMPLE 5.6 Arithmetic Average and Expected Return

Spreadsheet 5.2 presents a (short) time series of annual holding-period returns for the S&P 500 index over the period 2001–2005. We treat each HPR of the $n = 5$ observations in the time series as an equally likely annual outcome during the sample years and assign it an equal probability of 1/5, or .2. Column B in Spreadsheet 5.2 therefore uses .2 as probabilities, and Column C shows the annual HPRs. Applying Equation 5.13 (using Excel's SUMPRODUCT function) to the time series in Spreadsheet 5.2 demonstrates that adding up the products of probability times HPR amounts to taking the arithmetic average of the HPRs (compare cells C10 and C11).

Example 5.6 illustrates the logic for the wide use of the arithmetic average in investments. If the time series of historical returns fairly represents the true underlying probability distribution, then the arithmetic average return from a historical period provides a good forecast of the investment's expected HPR.

The Geometric (Time-Weighted) Average Return

We saw that the arithmetic average provides an unbiased estimate of the *expected* rate of return. But what does the time series tell us about the *actual* performance of the portfolio over the full sample period? Column F in Spreadsheet 5.2 shows the wealth index from investing $1 in an S&P 500 index fund at the beginning of 2001. The value of the wealth index at the end of 2005, $1.0275, is the terminal value of the $1 investment, which implies a *5-year* holding-period return (HPR) of 2.75%.

	A	B	C	D	E	F
1						
2						
3		Implicitly Assumed		Squared	Gross HPR =	Wealth
4	Period	Probability = 1/5	HPR (decimal)	Deviation	1 + HPR	Index*
5	2001	.2	–0.1189	0.0196	0.8811	0.8811
6	2002	.2	–0.2210	0.0586	0.7790	0.6864
7	2003	.2	0.2869	0.0707	1.2869	0.8833
8	2004	.2	0.1088	0.0077	1.1088	0.9794
9	2005	.2	0.0491	0.0008	1.0491	1.0275
10	Arithmetic average	AVERAGE(C5:C9) =	0.0210			
11	Expected HPR	SUMPRODUCT(B5:B9, C5:C9) =	0.0210			
12		Standard deviation	SUMPRODUCT(B5:B9, D5:D9)^.5 =	0.1774		Check:
13			STDEV(C5:C9) =	0.1983		1.0054^5=
14			Geometric average return	GEOMEAN(E5:E9) – 1 =	0.0054	1.0275
15	*The value of $1 invested at the beginning of the sample period (1/1/2001).					

SPREADSHEET 5.2

Time series of HPR for the S&P 500

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An intuitive measure of performance over the sample period is the (fixed) annual HPR that would compound over the period to the same terminal value as obtained from the sequence of actual returns in the time series. Denote this rate by g, so that

$$\text{Terminal value} = (1 + r_1) \times (1 + r_2) \times \cdots \times (1 + r_5) = 1.0275$$
$$(1 + g)^n = \text{Terminal value} = 1.0275 \quad \text{(cell F9 in Spreadsheet 5.2)} \tag{5.14}$$
$$g = \text{Terminal value}^{1/n} - 1 = 1.0275^{1/5} - 1 = .0054 = .54\% \quad \text{(cell E14)}$$

where $1 + g$ is the geometric average of the gross returns $(1 + r)$ from the time series (which can be computed with Excel's GEOMEAN function) and g is the annual HPR that would replicate the final value of our investment.

Practitioners of investments also call g the *time-weighted* (as opposed to dollar-weighted) average return, to emphasize that each past return receives an equal weight in the process of averaging. This distinction is important because investment managers often experience significant changes in funds under management as investors purchase or redeem shares. Rates of return obtained during periods when the fund is large produce larger dollar profits than rates obtained when the fund is small. We discuss this distinction further in the chapter on performance evaluation.

EXAMPLE 5.7 Geometric versus Arithmetic Average

The geometric average in Example 5.6 (.54%) is substantially less than the arithmetic average (2.10%). This discrepancy sometimes is a source of confusion. It arises from the asymmetric effect of positive and negative rates of returns on the terminal value of the portfolio.

Observe the returns in years 2002 (−.2210) and 2003 (.2869). The arithmetic average return over the 2 years is $(-.2210 + .2869)/2 = .03295$ (3.295%). However, if you had invested \$100 at the beginning of 2002, you would have only \$77.90 at the end of the year. In order to simply break even, you would then have needed to earn \$21.10 in 2003, which would amount to a whopping return of 27.09% (21.10/77.90). Why is such a high rate necessary to break even, rather than the 22.10% you lost in 2002? Because your base for 2003 was much smaller than \$100; the lower base means that it takes a greater subsequent percentage gain to just break even. Even a rate as high as the 28.69% realized in 2003 yields a portfolio value in 2003 of $\$77.90 \times 1.2869 = \$100.25$, barely greater than \$100. This implies a 2-year annually compounded rate (the geometric average) of only .12%, significantly less than the arithmetic average of 3.295%.

The larger the swings in rates of return, the greater the discrepancy between the arithmetic and geometric averages, that is, between the compound rate earned over the sample period and the average of the annual returns. If returns come from a normal distribution, the difference exactly equals half the variance of the distribution, that is,

$$\text{Geometric average} = \text{Arithmetic average} - \tfrac{1}{2}\sigma^2 \tag{5.15}$$

(A warning: to use Equation 5.15, you must express returns as decimals, not percentages.)

Variance and Standard Deviation

When thinking about risk, we are interested in the likelihood of deviations from the *expected* return. In practice, we usually cannot directly observe expectations, so we estimate the variance by averaging squared deviations from our *estimate* of the expected return, the arithmetic average, $\bar{r}$. Adapting Equation 5.12 for historic data, we again use equal probabilities for each observation, and use the sample average in place of the unobservable $E(r)$.

$$\text{Variance} = \text{expected value of squared deviations}$$

$$\sigma^2 = \sum p(s)\,[r(s) - E(r)]^2$$

Using historical data with n observations, we *estimate* variance as

$$\sigma^2 = \frac{1}{n}\sum_{s=1}^{n}[r(s) - \bar{r}]^2 \qquad \textbf{(5.16)}$$

EXAMPLE 5.8 Variance and Standard Deviation

Take another look at Spreadsheet 5.2. Column D shows the square deviations from the arithmetic average, and cell D12 gives the standard deviation as the square root of the sum of products of the (equal) probabilities times the squared deviations (.1774).

The variance estimate from Equation 5.16 is downward biased, however. The reason is that we have taken deviations from the sample arithmetic average, $\bar{r}$, instead of the unknown, true expected value, $E(r)$, and so have introduced a bit of estimation error. This is sometimes called a *degrees of freedom* bias. We can eliminate the bias by multiplying the arithmetic average of squared deviations by the factor $n/(n - 1)$. The variance and standard deviation then become

$$\sigma^2 = \left(\frac{n}{n-1}\right) \times \frac{1}{n}\sum_{j=1}^{n}[r(s) - \bar{r}]^2 = \frac{1}{n-1}\sum_{j=1}^{n}[r(s) - \bar{r}]^2$$

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}[r(s) - \bar{r}]^2} \qquad \textbf{(5.17)}$$

Cell D13 shows that the unbiased estimate of the standard deviation is .1983, which is a bit higher than the .1774 value obtained in cell D12.

The Reward-to-Volatility (Sharpe) Ratio

Finally, it is worth noting that investors presumably are interested in the expected *excess* return they can earn over the T-bill rate by replacing T-bills with a risky portfolio as well as the risk they would thereby incur. While the T-bill rate is not fixed each period, we still know with certainty what rate we will earn if we purchase a bill and hold it to maturity. Other investments typically entail accepting some risk in return for the prospect of earning more than the safe T-bill rate. Investors price risky assets so that the risk premium will be commensurate with the risk of that expected *excess* return, and hence it's best to measure risk by the standard deviation of excess, not total, returns.

The importance of the trade-off between reward (the risk premium) and risk (as measured by standard deviation or SD) suggests that we measure the attraction of an investment portfolio by the ratio of its risk premium to the SD of its excess returns.

$$\text{Sharpe ratio (for portfolios)} = \frac{\text{Risk premium}}{\text{SD of excess return}} \qquad \textbf{(5.18)}$$

This reward-to-volatility measure (first proposed by William Sharpe and hence called the *Sharpe ratio*) is widely used to evaluate the performance of investment managers.

EXAMPLE 5.9 Sharpe Ratio

Take another look at Spreadsheet 5.1. The scenario analysis for the proposed investment in the stock-index fund resulted in a risk premium of 8%, and standard deviation of excess returns of 17.32%. This implies a Sharpe ratio of .46, a value that is pretty much in line with past performance of stock-index funds. We elaborate on this important measure in future chapters and show that while it is an adequate measure of the risk–return trade-off for diversified portfolios (the subject of this chapter), it is inadequate when applied to individual assets such as shares of stock that may be held as part of larger diversified portfolios.

CONCEPT CHECK 4

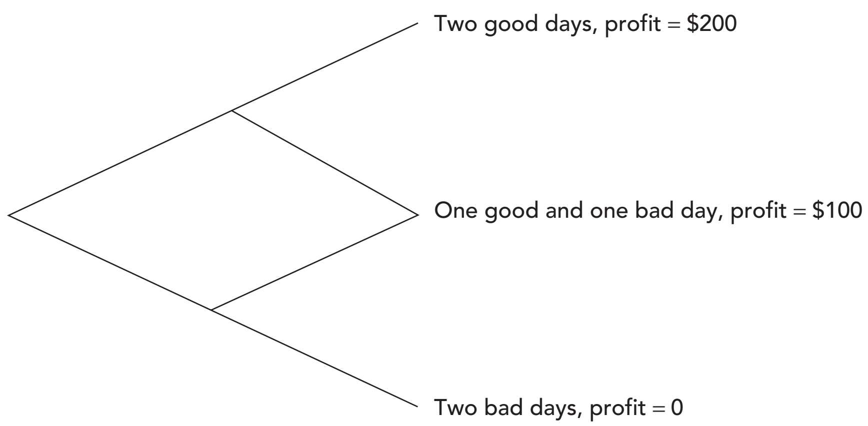
Using the annual returns for years 2003–2005 in Spreadsheet 5.2,

a. Compute the arithmetic average return.
b. Compute the geometric average return.
c. Compute the standard deviation of returns.
d. Compute the Sharpe ratio assuming the risk-free rate was 6% per year.

5.6 THE NORMAL DISTRIBUTION

The bell-shaped **normal distribution** appears naturally in many applications. For example, heights and weights of the population are well described by the normal distribution. In fact, many variables that are the end result of multiple random influences will exhibit a normal distribution. By the same logic, if return expectations implicit in asset prices are rational, actual rates of return realized should be normally distributed around these expectations.

To see why the normal curve is "normal," consider a newspaper stand that turns a profit of $100 on a good day and breaks even on a bad day, with equal probabilities of .5. Thus, the mean daily profit is $50 dollars. We can build a tree that compiles all the possible outcomes at the end of any period. Here is an **event tree** showing outcomes after 2 days:



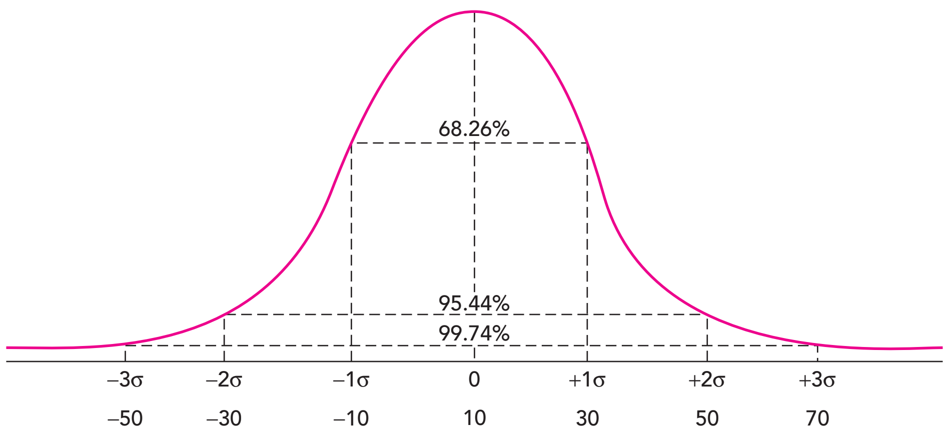


FIGURE 5.4 The normal distribution

Notice that 2 days can produce three different outcomes and, in general, n days can produce $n + 1$ possible outcomes. The most likely 2-day outcome is "one good and one bad day," which can happen in two ways (first a good day, or first a bad day). The probability of this outcome is .5. Less likely are the two extreme outcomes (both good days or both bad days) with probability .25 each.

What is the distribution of profits at the end of many business days? For example, after 200 days, there are 201 possible outcomes and, again, the midrange outcomes are the more likely because there are more sequences that lead to them. For example, while there is only one sequence that results in 200 consecutive bad days, there are an enormous number of sequences that result in 100 good days and 100 bad days. The probability distribution will eventually take on the appearance of the bell-shaped normal distribution, with midrange outcomes most likely, and extreme outcomes least likely.[6]

Figure 5.4 is a graph of the normal curve with mean of 10% and standard deviation of 20%. The graph shows the theoretical probability of rates of return within various ranges given these parameters. A smaller SD means that possible outcomes cluster more tightly around the mean, while a higher SD implies more diffuse distributions. The likelihood of realizing any particular outcome when sampling from a normal distribution is fully determined by the number of standard deviations that separate that outcome from the mean. Put differently, the normal distribution is completely characterized by two parameters, the mean and SD.

Investment management is far more tractable when rates of return can be well approximated by the normal distribution. First, the normal distribution is symmetric, that is, the probability of any positive deviation above the mean is equal to that of a negative deviation of the same magnitude. Absent symmetry, measuring risk as the standard deviation of returns is inadequate. Second, the normal distribution belongs to a special family of

[6]As a historical footnote, early descriptions of the normal distribution in the eighteenth century were based on the outcomes of a "binomial tree" like the one we have drawn for the newspaper stand, extended out to many periods. This representation is used in practice to price many option contracts, as we will see in Chapter 21. For a nice demonstration of how the binomial distribution quickly approximates the normal, go to **www.jcu.edu/math/isep/Quincunx/Quincunx.html.**

distributions characterized as "stable," because of the following property: When assets with normally distributed returns are mixed to construct a portfolio, the portfolio return also is normally distributed. Third, scenario analysis is greatly simplified when only two parameters (mean and SD) need to be estimated to obtain the probabilities of future scenarios.

How closely must actual return distributions fit the normal curve to justify its use in investment management? Clearly, the normal curve cannot be a perfect description of reality. For example, actual returns cannot be less than −100%, which the normal distribution would not rule out. But this does not mean that the normal curve cannot still be useful. A similar issue arises in many other contexts. For example, shortly after birth, a baby's weight is typically evaluated by comparing it to a normal curve of newborn weights. This may seem surprising, because a normal distribution admits values from minus to plus infinity, and surely no baby is born with a negative weight. The normal distribution still is useful in this application because the SD of the weight is small relative to its mean, and the likelihood of a negative weight would be too trivial to matter.[7] In a similar spirit, we must identify criteria to determine the adequacy of the normality assumption for rates of return.

EXAMPLE 5.10 Normal Distribution Function in Excel

Suppose the monthly rate of return on the S&P 500 is approximately normally distributed with a mean of 1% and standard deviation of 6%. What is the probability that the return on the index in any month will be negative? We can use Excel's built-in functions to quickly answer this question. The probability of observing an outcome less than some cutoff according to the normal distribution function is given as NORMDIST(cutoff, mean, standard deviation, TRUE). In this case, we want to know the probability of an outcome below zero, when the mean is 1% and the standard deviation is 6%, so we compute NORMDIST(0, 1, 6, TRUE) = .4338. We could also use Excel's built-in *standard* normal function and ask for the probability of an outcome 1/6 of a standard deviation below the mean. This would be the same: NORMSDIST(−1/6) = .4338.

CONCEPT CHECK 5

What is the probability that the return on the index in Example 5.10 will be below −15%?

5.7 DEVIATIONS FROM NORMALITY

To assess the adequacy of the assumption of normality we focus on deviations from normality that would invalidate the use of standard deviation as an adequate measure of risk. Our first criterion is symmetry. A measure of asymmetry called **skew** uses the ratio of the

[7]In fact, the standard deviation is 511 grams while the mean is 3,958 grams. A negative weight would therefore be 7.74 standard deviations below the mean, and according to the normal distribution would have probability of only 4.97×10^{-15}. The issue of negative birth weight clearly isn't a *practical* concern.

average *cubed* deviations from the mean, called the third moment, to the cubed standard deviation to measure any asymmetry or "skewness" of a distribution.

$$\text{Skew} = \frac{E[r(s) - E(r)]^3}{\sigma^3} \tag{5.19}$$

Cubing deviations maintains their sign (for example, the cube of a negative number is negative). Thus, if the distribution is "skewed to the right," as is the dark curve in Figure 5.5A, the extreme positive values, when cubed, will dominate the third moment, resulting in a positive measure of skew. If the distribution is "skewed to the left," the cubed extreme negative values will dominate, and the skew will be negative.

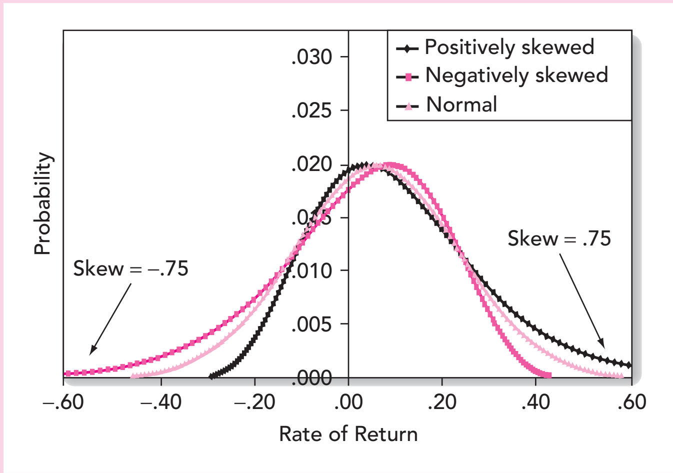


FIGURE 5.5A Normal and skewed distributions (mean = 6%, SD = 17%)

When the distribution is positively skewed (the skew is greater than zero), the standard deviation overestimates risk, because extreme positive deviations from expectation (which are not a source of concern to the investor) nevertheless increase the estimate of volatility. Conversely, and more importantly, when the distribution is negatively skewed, the SD will underestimate risk.

Another potentially important deviation from normality concerns the likelihood of extreme values on either side of the mean at the expense of a smaller fraction of moderate deviations. Graphically speaking, when the tails of a distribution are "fat," there is more probability mass in the tails of the distribution than predicted by the normal distribution, at the expense of "slender shoulders," that is, less probability mass near the center of the distribution. Figure 5.5B superimposes a "fat-tailed" distribution on a normal with the same mean and SD. Although symmetry is still preserved, the SD will underestimate the likelihood of extreme events: large losses as well as large gains.

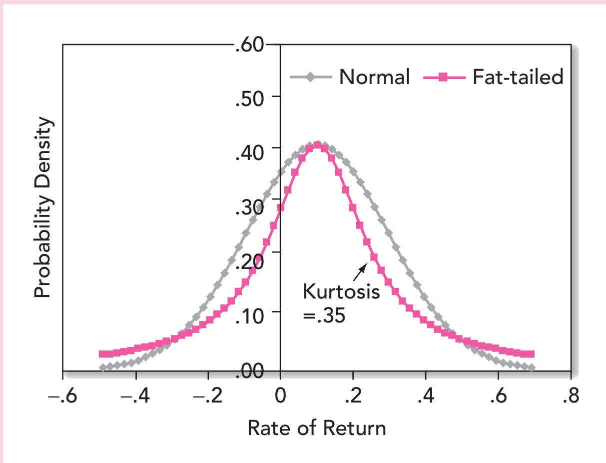


FIGURE 5.5B Normal and fat-tailed distributions (mean = .1, SD = .2)

Kurtosis is a measure of the degree of fat tails. In this case, we use the expectation of deviations from the mean raised to the *fourth* power and standardize by dividing by the fourth power of the SD, that is,

$$\text{Kurtosis} = \frac{E[r(s) - E(r)]^4}{\sigma^4} - 3 \tag{5.20}$$

We subtract 3 from the ratio in Equation 5.20, because the ratio for a normal distribution would be 3. Thus, the kurtosis of a normal distribution is defined as zero, and any kurtosis above zero is a sign of fatter tails than would be observed in a normal distribution. The kurtosis of the distribution in Figure 5.5B, which has visible fat tails, is .36.

CONCEPT CHECK 6

Estimate the skew and kurtosis of the five rates in Spreadsheet 5.2.

5.8 THE HISTORICAL RECORD OF RETURNS ON EQUITIES AND LONG-TERM BONDS

We took a long road to reach this section, but now we are in a position to derive useful insights from the historical record. We examine the time series of five broadly diversified risky portfolios. The World portfolio of large stocks includes the market-index portfolios of large stocks in 40 countries, weighted by the market capitalization (total market value) of the country indexes. The rates of return on this (and the World bond) portfolio are based on *dollar* wealth indexes, that is, they include gains/losses from changes in the value of the foreign currencies relative to the U.S. dollar. Thus, the picture we present is from the standpoint of a U.S. investor.

U.S. large stocks make up a significant part, approximately 40%, of the World portfolio of large stocks. Along with the World large equities, we show results for a portfolio of large U.S. stocks, specifically, the S&P 500 index. The riskier portfolio composed of smaller U.S. stocks shows up next. Finally, we present statistics for two long-term bond portfolios. "World bonds" averages the return on long-term government bond indexes of 16 countries, weighted by the GDP of these countries. Here, too, U.S. Treasury bonds make up a significant, although somewhat smaller, fraction of the portfolio returns.

Average Returns and Standard Deviations

Table 5.3 compiles the average rates of return and their standard deviations over generational periods of 25 years, as well as summaries for the overall period of 80 years and the recent 40 years since 1966. Figure 5.6 presents frequency distributions of those returns. As we have seen, averages and standard deviations of raw annual returns should be interpreted with caution. First, standard deviations of total returns are affected by variation in the risk-free rate and thus do not measure the true source of risk, namely, the uncertainty surrounding *excess* returns. Second, annual rates that compound over a whole year exhibit meaningful amounts of skewness, and estimates of kurtosis also may be misleading.

Nevertheless, these simple statistics still reveal much about the nature of returns for these asset classes. For example, the asset classes with higher volatility (standard deviation) have provided higher average returns, supporting the idea that investors demand a risk premium to bear risk. Observe, for example, the consistently larger average return as well as standard deviations of small compared with large stocks, or stock compared to bond portfolios. In fact, for every generation, the average returns on the stock portfolios were higher than the T-bill rate.

Another feature (also observed for T-bill and inflation rates) is that the nature of returns around the world and in the U.S. seems to have changed since the 1960s. Standard deviations of stock portfolios have fallen, particularly for small stocks, but have remained about the same for bonds.

Other Statistics of the Risky Portfolios

Table 5.4 summarizes the essential statistics of the annual *excess* returns of the five risky portfolios. The statistics from which we can make inferences about the nature of the return distributions—skew, kurtosis, and serial correlation—are computed from the excess continuously compounded rates, that is, the difference between the continuously compounded rates on the risky portfolios and the continuously compounded T-bill rate.

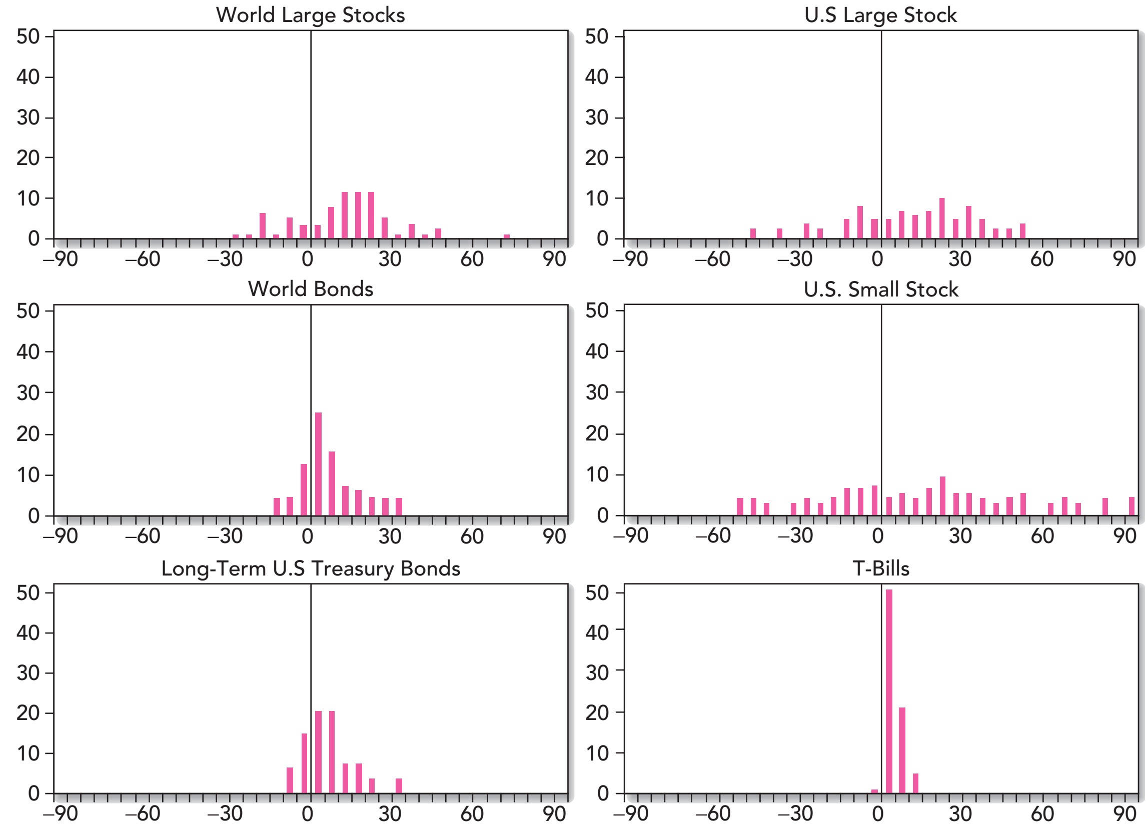


FIGURE 5.6 Frequency distributions of rates of return for 1926–2005

Sharpe Ratios

The reward-to-volatility (Sharpe) ratios of the five risky portfolios are of the same order of magnitude. The Sharpe ratios of the more recent 40 years, 1966–2005, are somewhat lower and generally more uniform across portfolios, in the range of .30 to .34. Notice, however, that the portfolio of U.S. long-term T-bonds has a significantly lower Sharpe measure (.21) than the other four, possibly for a good reason. Although the year-to-year rate of return on these bonds will vary, these bonds may serve as “the” risk-free choice for investors with long-term horizons. Consider a pension fund that must provide a known future cash flow to pay beneficiaries. The only risk-free vehicle to accomplish this objective would be to invest in a portfolio of U.S. T-bonds providing cash flows that match the pension fund’s obligations. Hence, investors with a long horizon may not demand a risk premium commensurate with the risk as measured by the standard deviation of short-term returns.

Serial Correlation

In well-functioning capital markets, we would expect excess returns from successive years to be uncorrelated, that is, the serial correlation of excess returns should be nearly zero. Suppose, for example, that the serial correlation of the annual rate of return on a stock

Portfolio	Statistic	1926–2005	1966–2005	1981–2005	1971–1995	1961–1985	1951–1975	1941–1965	1931–1955	1926–1950
World large stocks	Arithmetic avg.	11.46	12.12	13.45	14.20	11.17	12.28	13.01	10.80	7.70
	SD	18.57	17.72	17.84	17.59	16.10	17.64	14.18	21.42	21.61
	Geometric avg.	9.85	10.67	12.03	12.79	9.99	10.89	12.15	8.77	5.59
U.S. large stocks	Arithmetic avg.	12.15	11.64	13.65	13.51	10.92	11.90	15.70	13.16	10.34
	SD	20.26	16.97	16.02	16.62	16.74	19.28	17.17	25.40	25.98
	Geometric avg.	10.17	10.31	12.50	12.26	9.63	10.27	14.47	10.04	7.05
U.S. small stocks	Arithmetic avg.	17.95	14.98	12.27	16.01	18.37	14.64	23.09	28.41	23.40
	SD	38.71	29.58	20.24	27.21	33.65	35.68	33.00	51.80	55.46
	Geometric avg.	12.01	11.27	10.44	12.61	13.62	9.59	19.15	19.03	11.85
World bonds	Arithmetic avg.	6.14	9.40	11.22	11.48	7.10	3.92	1.69	2.23	2.74
	SD	9.09	9.56	10.89	9.96	8.39	4.58	5.16	8.76	8.89
	Geometric avg.	5.77	9.00	10.71	11.07	6.80	3.82	1.56	1.88	2.38
Long-term U.S. treasury bonds	Arithmetic avg.	5.68	8.17	10.28	9.94	5.52	2.75	2.31	3.34	3.94
	SD	8.09	9.97	10.80	10.20	8.59	6.37	4.45	3.96	3.90
	Geometric avg.	5.38	7.73	9.78	9.50	5.20	2.56	2.22	3.27	3.87
U.S. T-bills	Arithmetic avg.	3.75	5.98	5.73	7.04	6.55	3.66	1.62	0.63	1.02
	SD	3.15	2.84	3.15	2.87	3.15	1.97	1.16	0.57	1.33
	Geometric avg.	3.70	5.95	5.68	7.00	6.50	3.64	1.62	0.62	1.01

TABLE 5.3

History of rates of return of asset classes for generations, 1926–2005

Sources: World portfolio: Datastream (16 countries index returns weighted by market capitalization).
U.S. stock returns for 1926–1995: Center for Research in Security Prices (CRSP).
U.S. stock returns since 1996: Returns on appropriate index portfolios: Large stocks, S&P 500; Small stocks, Russell 2000.
World bonds: Elroy Dimson, Paul Marsh, and Mike Staunton (16 countries weighted by GDP).
Long-term Government bonds: Lehman Bros. LT Treasury index.
T-bills: Salomon Smith Barney 3-month U.S. T-bill index.

Portfolio	Statistic	1926–2005	1966–2005	1981–2005	1971–1995	1961–1985	1951–1975	1941–1965	1931–1955	1926–1950
World large stocks	Average excess return	7.71	6.14	7.73	7.16	4.63	8.62	11.39	10.18	6.68
	SD of excess return	18.90	18.21	18.33	18.33	16.67	18.87	14.30	21.38	21.66
	Sharpe ratio	0.41	0.34	0.42	0.39	0.28	0.46	0.80	0.48	0.31
	Skew	−0.61	−0.62	−0.53	−0.93	−0.78	−0.65	−0.12	−0.70	−0.57
	Kurtosis	0.98	−0.38	−0.57	0.48	0.32	0.38	0.55	3.05	1.88
	Serial correlation	0.14	0.05	0.13	−0.01	−0.16	0.03	0.04	0.03	0.23
U.S. large stocks	Average excess return	8.39	5.66	7.92	6.47	4.38	8.24	14.08	12.54	9.32
	SD of excess return	20.54	17.10	16.12	16.97	17.22	20.47	17.43	25.39	26.01
	Sharpe ratio	0.41	0.33	0.49	0.38	0.25	0.40	0.81	0.49	0.36
	Skew	−0.80	−0.70	−0.65	−1.00	−0.79	−0.39	−0.14	−1.15	−0.91
	Kurtosis	1.03	−0.20	−0.46	0.91	−0.02	0.03	−0.67	1.62	0.62
	Serial correlation	0.08	0.02	0.07	−0.15	−0.18	−0.08	−0.20	−0.05	0.16
U.S. small stocks	Average excess return	14.20	9.00	6.54	8.97	11.82	10.98	21.47	27.78	22.38
	SD of excess return	39.31	29.89	20.70	27.50	34.06	36.38	33.40	51.92	55.86
	Sharpe ratio	0.36	0.30	0.32	0.33	0.35	0.30	0.64	0.54	0.40
	Skew	−0.22	−0.30	−0.42	−0.86	−0.41	−0.07	0.45	−0.28	−0.31
	Kurtosis	0.86	0.20	−0.20	0.41	−0.03	−0.26	−0.84	0.75	−0.17
	Serial correlation	0.16	0.07	−0.26	0.12	0.12	0.01	0.11	0.05	0.26
World bonds	Average excess return	2.39	3.42	5.49	4.44	0.55	0.26	0.07	1.61	1.72
	SD of excess return	8.97	10.36	11.58	11.20	9.02	4.71	4.92	8.78	8.81
	Sharpe ratio	0.27	0.33	0.47	0.40	0.06	0.05	0.01	0.18	0.20
	Skew	0.48	0.23	−0.06	0.02	0.36	0.14	−1.13	0.69	0.65
	Kurtosis	0.70	−0.42	−0.63	−0.36	1.29	0.09	2.26	2.50	2.39
	Serial correlation	0.13	0.11	−0.14	0.16	0.07	0.16	0.13	0.19	0.16
Long-term U.S. Treasury bonds	Average excess return	1.93	2.18	4.55	2.90	−1.02	−0.91	0.69	2.72	2.92
	SD of excess return	7.91	10.18	11.01	10.70	8.44	6.36	4.60	4.21	4.19
	Sharpe ratio	0.24	0.21	0.41	0.27	−0.12	−0.14	0.15	0.64	0.70
	Skew	0.23	0.23	−0.04	0.45	0.87	0.17	0.08	−0.21	−0.38
	Kurtosis	0.28	−0.57	−0.78	−0.49	1.60	−0.01	−0.51	−0.20	0.09
	Serial correlation	−0.07	−0.05	−0.31	0.01	0.23	0.02	−0.19	−0.14	−0.25

TABLE 5.4

History of excess rates of return of asset classes for generations, 1926–2005

* Skew, kurtosis, and serial correlation are estimated from continuously compounded excess rates of return.

Sources: World portfolio: Datastream (16 countries index returns weighted by market capitalization).
U.S. stock returns for 1926–1995: Center for Research in Security Prices (CRSP).
U.S. stock returns since 1996: Returns on appropriate index portfolios: Large stocks, S&P 500; Small stocks, Russell 2000.
World bonds: Elroy Dimson, Paul Marsh, and Mike Staunton (16 countries weighted by GDP).
Long-term Government bonds: Lehman Bros. LT Treasury index.
T-bills: Salomon Smith Barney 3-month U.S. T-bill index.

index were negative and that the index fell last year. Investors therefore could predict that stock prices are more likely than usual to rise in the coming year. But armed with this insight, they would *immediately* buy shares and bid up stock prices, thereby eliminating the prospect of an above-normal return in the coming year. We elaborate on this mechanism in the chapter on market efficiency.

Such a consideration does not apply to the T-bill rate, whose return is known in advance. The positive serial correlation of T-bill rates (.83 for the last 40 years) indicates that the short-term rate follows periods in which it predictably tends to rise or fall. However, this predictability in the baseline risk-free rate is not a source of abnormal profits (i.e., excessive profits relative to risk borne). This is a reason why the serial correlation of the *total* return on risky assets will be "contaminated" by that of the risk-free rate, and why we instead prefer to measure serial correlation from excess rates. Indeed, we find that the serial correlation is practically zero for four of the five portfolios. The serial correlation for World bond portfolio returns is somewhat high, but the fact that it was negative for the most recent years 1981–2005 suggests it is not economically significant.

Skewness and Kurtosis

Skewness and kurtosis are computed from the continuously compounded rate. Therefore, if the true underlying distribution of continuously compounded returns is normal, both should be zero. In fact, the skews of the large stock portfolios are significantly negative, −.62 for the World and −.70 to −.80 for the U.S. This negative skew may result from "lumpiness" of bad news (compared with good news) that produces occasional but large negative "jumps" in prices. It appears that the much larger standard deviation of the small stock portfolio reduces the relative impact of such negative jumps, and so the negative skew of the distribution is less pronounced (in the range of −.22 to −.30). Returns on the World and U.S. government bond portfolios are slightly positively skewed.

Negative skews imply that the standard deviation underestimates the actual level of risk. Take another look at Figure 5.5A; it shows two distributions with identical annual means (6%) and standard deviations (17%), similar to those of the excess returns of U.S. large stocks. But the skews of −.75 and .75 suggest a significant difference in risk, as is evident from the magnitude of possible losses. The probability of an annual loss greater than 40% is significantly higher for the negatively skewed distribution than for the normal distribution with the same mean and standard deviation.

Concern expressed in the literature about the presence of fat tails in stock return distributions does not manifest itself in this history. It appears that observed fat tails are largely due to older history. The most recent 40 years show no kurtosis for the large stock index, and only a small value for small stocks.

Estimates of Historical Risk Premiums

The striking observation here, again, is that the average excess return was positive for every generation over the entire 80-year history. In fact, research shows that this pattern characterizes periods as short as decades. Average excess returns of large stocks are somewhat lower in the more recent 40-year history and, overall, suggest a risk premium of 6–8%. Average excess returns for small U.S. stocks, as well as their standard deviation, were much lower over the recent 40-year history than over the full 80-year period.

An often-overlooked fact about the precision of estimates of expected returns and standard deviation needs to be clarified. Suppose we observe the time series of a stock price over 10 years. We compute the 10-year HPR from the price at the beginning, $P(0)$, and at the end of the 10 years, $P(10)$, by $r(10) = P(10)/P(1) - 1$. We can then annualize the 10-year return. Notice from this calculation that we obtain the average return *solely* from the start and ending prices. Prices from more frequent observations *during* the 10-year period would change neither the final value of the stock nor, therefore, our estimate of its expected return. The only way to increase the precision of this estimate of the expected annual return would be to obtain a sample longer than 10 years. But as we dig deeper into the past to obtain a longer sample, we have to ask whether the return distribution of more-distant history is representative of more-recent periods. This is precisely the dilemma we face when we observe a large difference between 80-year and 40-year historical averages.

Interestingly, this limitation does *not* apply to estimates of variance and standard deviation. Increasing the number of observations by slicing a 10-year sample into progressively shorter intervals does increase the accuracy of the estimate of the standard deviation of annual returns, even if the overall sample period remains 10 years. This is because we learn about volatility by observing fluctuations of returns within the sample period. (In contrast, intra-period fluctuations do not teach us about the general trend of stock prices, which is the basis of the estimate of expected return.) For this reason, estimates of risk (standard deviation) can be made more reliable than estimates of expected returns by sampling more frequently.[8]

Our estimate of risk may also sharpen our estimates of expected return. For example, when we observe that broadly diversified portfolios show similar Sharpe ratios, we have more confidence in the estimates of their expected returns from historical averages. Similarly, when we observe that the average return of small stocks fell in tandem with their standard deviation (the latter was 39% from 1926 to 2005 but only 29% between 1966 and 2005), we have more confidence that the more recent averages better estimate expected returns for the near future.

A Global View of the Historical Record

As financial markets around the world grow and become more transparent, U.S. investors look to improve diversification by investing internationally. Foreign investors that traditionally used U.S. financial markets as a safe haven to supplement home-country investments also seek international diversification to reduce risk. The question arises as to how historical U.S. experience compares with that of stock markets around the world.

Figure 5.7 shows a century-long history (1900–2000) of average nominal and real returns in stock markets of 16 developed countries. We find the United States in fourth place in terms of average real returns, behind Sweden, Australia, and South Africa. Figure 5.8 shows the standard deviations of real stock and bond returns for these same countries. We find the United States tied with four other countries for third place in terms of lowest standard deviation of real stock returns. So the United States has done well, but not abnormally so, compared with these countries.

One interesting feature of these figures is that the countries with the worst results, measured by the ratio of average real returns to standard deviation, are Italy, Belgium, Germany, and Japan—the countries most devastated by World War II. The top-performing countries are Australia, Canada, and the United States, the countries least devastated by the wars of the twentieth century. Another, perhaps more telling feature, is the insignificant difference between the real returns in the different countries. The difference between the highest average real rate (Sweden, at 7.6%) from the average return across the 16 countries (5.1%) is 2.5%. Similarly, the difference between the average and the lowest country return (Belgium, at 2.5%) is 2.6%. Using the average standard deviation of 23%, the *t*-statistic for a difference of 2.6% with 100 observations is

$$t\text{-Statistic} = \frac{\text{Difference in mean}}{\text{Standard deviation}/\sqrt{n}} = \frac{2.6}{23/\sqrt{100}} = 1.3$$

[8]The 10-year average return $r(10)$ is a geometric average. We know from Equation 5.15 that the arithmetic average is greater by ½ σ^2. Any improved accuracy in estimating σ^2 will still leave us with the original imprecision in the geometric average return.

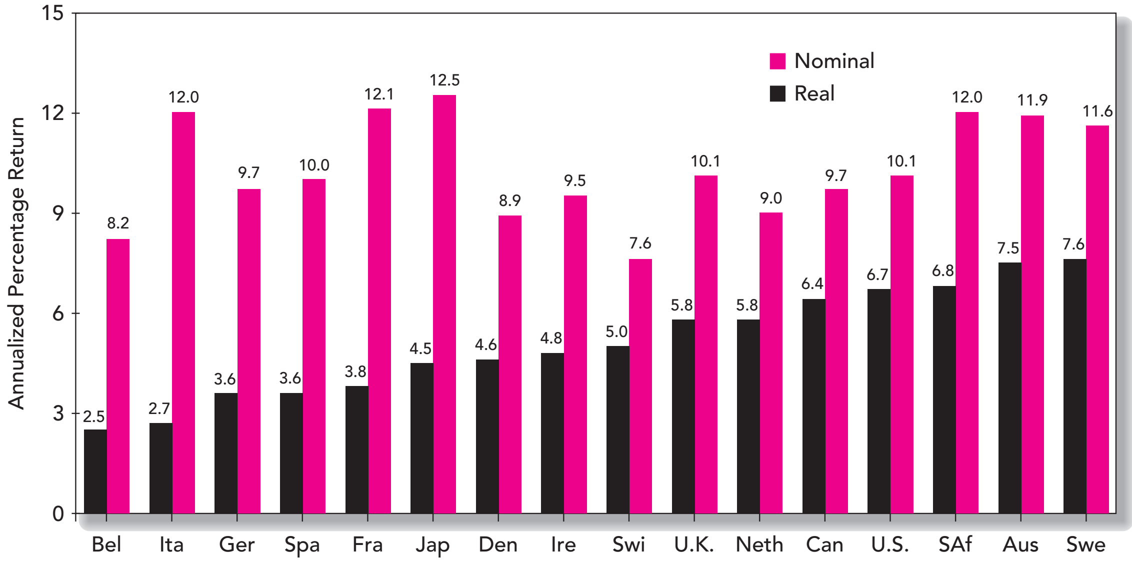


FIGURE 5.7 Nominal and real equity returns around the world, 1900–2000

Source: Elroy Dimson, Paul Marsh, and Mike Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns* (Princeton University Press, 2002), p. 50. Reprinted by permission of the Princeton University Press.

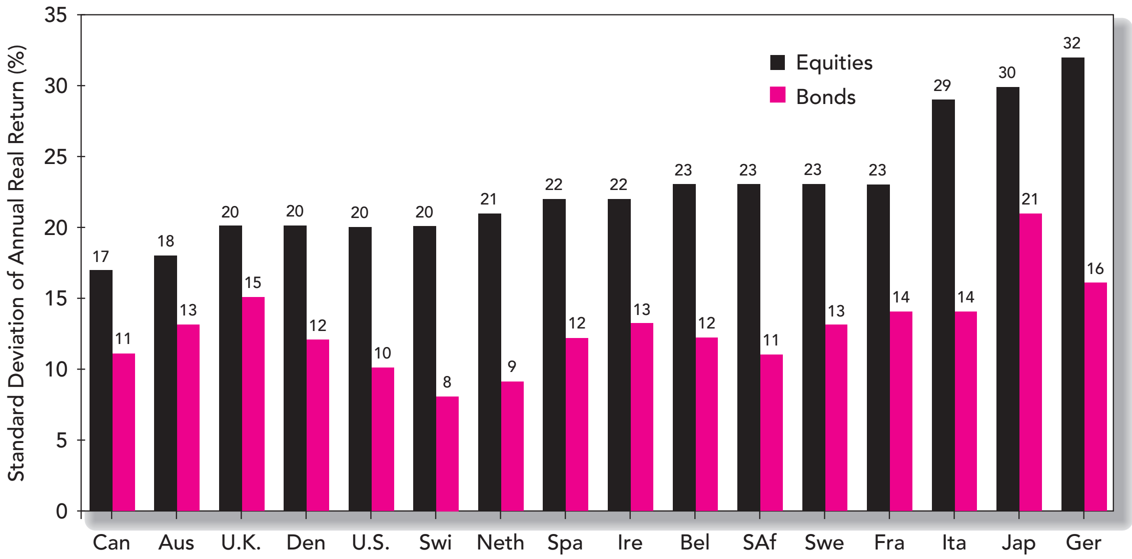


FIGURE 5.8 Standard deviations of real equity and bond returns around the world, 1900–2000

Source: Elroy Dimson, Paul Marsh, and Mike Staunton, *Triumph of the Optimists: 101 Years of Global Investment Returns* (Princeton University Press, 2002), p. 61. Reprinted by permission of the Princeton University Press.

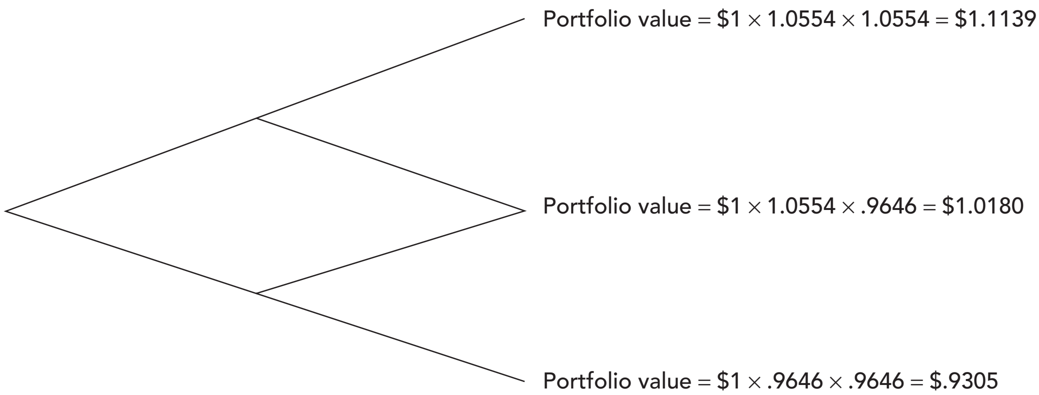
which is far below conventional levels of statistical significance. We conclude that the U.S. experience cannot be dismissed as an outlier case. Hence, using the U.S. stock market as a yardstick for return characteristics may be reasonable.

These days, practitioners and scholars are debating whether the historical U.S. average risk-premium of large stocks over T-bills of 8.39% (Table 5.4) is a reasonable forecast for the long term. This debate centers around two questions: First, do economic factors that prevailed over that historic period (1926–2005) adequately represent those that may prevail over the forecasting horizon? Second, is the arithmetic average from the available history a good yardstick for long-term forecasts?

5.9 LONG-TERM INVESTMENTS*

Consider an investor saving $1 today toward retirement in 25 years, or 300 months. Investing the dollar in a risky stock portfolio (reinvesting dividends until retirement) with an expected rate of return of 1% per month, this retirement "fund" is expected to grow almost 20-fold to a terminal value of $(1 + .01)^{300}$ = $19.79 (providing total growth of 1,879%). Compare this impressive result to an investment in a 25-year Treasury bond with a risk-free EAR of 6% (.407% per month) that yields a retirement fund of 1.06^{25} = $4.29. We see that a monthly risk premium of just .593% produces a retirement fund that is more than four times that of the risk-free alternative. Such is the power of compound interest. Why, then, would anyone invest in Treasuries? Obviously, this is an issue of trading excess return for risk. What is the nature of this return-to-risk trade-off? The risk of an investment that compounds at fluctuating rates over the long run is widely misunderstood, and it is important to figure it out.

We can construct the probability distribution of the stock-fund terminal value from a binomial tree just as we did earlier for the newspaper stand, except that instead of *adding* monthly profits, the portfolio value *compounds* monthly by a rate drawn from a given distribution. For example, suppose we can approximate the portfolio monthly distribution as follows: Each month the rate of return is either 5.54% or –3.54%, with equal probabilities of .5. This configuration generates an expected return of 1% per month. The portfolio risk is measured as the monthly standard deviation: $\sqrt{.5\times(5.54-1)^2+.5\times(-3.54-1)^2} = 4.54\%$. After 2 months, the event tree looks like this:



*The material in this and the next subsection addresses important and ongoing debates about risk and return, but is more challenging. It may be skipped in shorter courses without impairing the ability to understand later chapters.

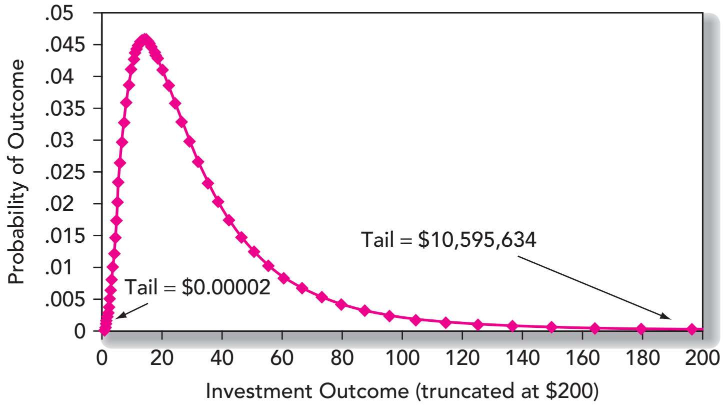


FIGURE 5.9 Probability of investment outcomes after 25 years with a lognormal distribution (approximated from a binomial tree)

"Growing" the tree for 300 months will result in 301 different possible outcomes. The probability of each outcome can be obtained from Excel's BINOMDIST function. From the 301 possible outcomes and associated probabilities we compute the mean ($19.79) and the standard deviation ($18.09) of the terminal value. Can we use this standard deviation as a measure of risk to be weighed against the risk premium of 19.79 − 4.29 = 15.5 (1,550%)? Recalling the effect of asymmetry on the validity of standard deviation as a measure of risk, we must first view the shape of the probability distribution at the end of the tree.

Figure 5.9 plots the probability of possible outcomes against the terminal value. The asymmetry of the distribution is striking. The highly positive skewness suggests the standard deviation of terminal value will not be useful in this case. Indeed, the binomial distribution, when period outcomes compound, converges to a **lognormal,** rather than a normal, **distribution.** The lognormal describes the distribution of a variable whose *logarithm* is normally distributed.

Risk in the Long Run and the Lognormal Distribution

When the continuously compounded rate of return on an asset is normally distributed at every instant, the effective rate of return, the actual HPR, will be lognormally distributed. We should say at the outset that for short periods of up to 1 month, the difference between the normal and lognormal distribution is sufficiently small to be safely ignored. This is so because for low rates of return (either negative or positive), $r_{cc} = \ln(1 + r) \approx r$, that is, r_{cc} is very close to r. But when concerned with longer periods, it is important to take account of the fact that it is the continuously compounded rates that are normally distributed, while the observed HPR is lognormally distributed.

Suppose that the annually, continuously compounded rate, r_{cc}, is normally distributed with an annual geometric mean of g and standard deviation σ. Remember that the geometric mean is the annual rate that will compound to the observed terminal value of a portfolio.

If the continuously compounded rate is normally distributed, the arithmetic mean, which gives the expected annual return, will be larger than the geometric mean by exactly half the variance. Thus, the expected return of the continuously compounded rate will be (restating Equation 5.15)

$$m = g + \tfrac{1}{2}\sigma^2 \quad \textbf{(5.21)}$$

Therefore, we can write the expected EAR as

$$1 + E(r) = e^{g + \frac{1}{2}\sigma^2} \quad \textbf{(5.22)}$$

The convenience of working with continuously compounded rates now becomes evident. Because the rate of return on an investment compounds at the expected annual rate of $E(r)$, the terminal value after T years will be $[1 + E(r)]^T$. We can write the terminal value in terms of the continuously compounded rate with an annual mean, m, and standard deviation, σ, as

$$[1 + E(r)]^T = [e^{g+\frac{1}{2}\sigma^2}]^T = e^{gT+\frac{1}{2}\sigma^2 T} \tag{5.23}$$

Notice that the mean of the continuously compounded rate (mT) and the variance ($\sigma^2 T$) both grow in direct proportion to the investment horizon T. It follows that the standard deviation grows in time at the rate of $\sqrt{T}$. This is the source of what *appears* to be a mitigation of investment risk in the long run: Because the expected return increases with horizon at a faster rate than the standard deviation, the expected return of a long-term, risky investment becomes ever larger relative to its standard deviation. This applies to the long-term investment we have examined with the binomial tree.

EXAMPLE 5.11 Shortfall Risk in the Short Run and the Long Run

Suppose we wish to estimate the probability that an indexed stock portfolio provides a rate of return less than that on risk-free T-bills. This is called a return shortfall. In line with historical experience, we will assume the monthly HPR on the investment is drawn from a lognormal distribution with an expected continuously compounded rate of $r_{cc} = .96\%$ and monthly standard deviation of $\sigma = 4.5\%$. The monthly risk-free rate is taken to be .5%. The index underperforms bills if its return during the month is less than .5%, which is $(.96 - .50)/4.5 = .102$ standard deviations below its mean. The probability of this event if returns are normally distributed is .46.

Now consider the probability of shortfall for a 25-year (300-month) horizon. The mean 25-year continuously compounded total return is $.96 \times 300 = 2.88$ (i.e., 288%), and the standard deviation is $.045 \times \sqrt{300} = .779\ (77.9\%)$. At the same time, the monthly risk-free rate of .5% is equivalent to a 25-year continuously compounded total return of $300 \times .5\% = 150\%$.

Because the 25-year continuously compounded rate is also normally distributed, we can easily find the probability that the terminal value of the risky portfolio will be below that of the risk-free investment. The expected total return on the index portfolio exceeds that on bills by $288\% - 150\% = 138\%$, and the standard deviation of the 25-year return is 77.9%. Therefore, stocks would have to fall short of their expected return by $138/77.9 = 1.722$ standard deviations before they would underperform bills. The probability of this outcome is only 3.8%. The far lower probability of a shortfall appears to vindicate those who advocate that investment in the stock market is less risky in the long run. After all, the argument goes, 96.2% of the time, the stock fund will outperform the safe investment, while its expected terminal value is almost four times higher.

A warning: The probability of a shortfall is an incomplete measure of investment risk. Such probability does not take into account the *size* of potential losses, which for some of the possible outcomes (however unlikely) amount to complete ruin. The worst-case scenarios for the 25-year investment are *far* worse than for the 1-month investment. We demonstrate the build-up of risk over the long run graphically in Figures 5.10 and 5.11.

A better way to quantify the risk of a long-term investment would be the market price of insuring it against a shortfall. An insurance premium must take into account both the probability of possible losses and the magnitude of these losses. We show in later chapters how the fair market price of portfolio insurance can be estimated from option-pricing models.

Despite the low probability that a portfolio insurance policy would have to pay up (only 3.8% for the 25-year policy), the magnitude and timing[9] of possible losses would make such long-term insurance surprisingly costly. For example, standard option-pricing models suggest that the value of insurance against shortfall risk over a 10-year horizon would cost nearly 20% of the initial value of the portfolio. And contrary to any intuition that a longer horizon reduces shortfall risk, the value of portfolio insurance increases dramatically with the maturity of the contract. For example, a 25-year policy would be about 50% more costly, or about 30% of the initial portfolio value.

The Sharpe Ratio Revisited

The Sharpe ratio (the reward-to-volatility ratio) divides average excess return by its standard deviation. You should be aware, however, that the Sharpe ratio has a time dimension, in that the Sharpe ratio for any given portfolio will vary systematically with the assumed investment holding period.

We have seen that as the holding period grows longer, the average continuously compounded return grows proportionally to the investment horizon (this is approximately true as well for short-term effective rates). The standard deviation, however, grows at a slower pace, the square root of time. Therefore, the Sharpe ratio *grows* with the length of the holding period at the rate of the square root of time. Hence, when comparing Sharpe ratios from a series of monthly rates to those from a series of annual rates, we must first multiply the monthly Sharpe ratio by the square root of 12.

EXAMPLE 5.12 Sharpe Ratios

For the long-term risky portfolio (with a monthly expected return of 1% and standard deviation of 5%), given a risk-free rate of .5%, the Sharpe ratio is $(1 - .5)/5 = .10$. The expected annual return would be 12% and annual standard deviation would be $5\% \times \sqrt{12} = 16.6\%$ so the Sharpe ratio using annual returns would be $(12 - 6)/16.6 = .36$, similar to values we find in the historical record of well-diversified portfolios.

Simulation of Long-Term Future Rates of Return

The frequency distributions in Figure 5.6 provide only rough descriptions of the nature of the return distributions and are even harder to interpret for long-term investments. A good way to use history to learn about the distribution of long-term future returns is to simulate these future returns from the available sample. A popular method to accomplish this task is called *bootstrapping.*

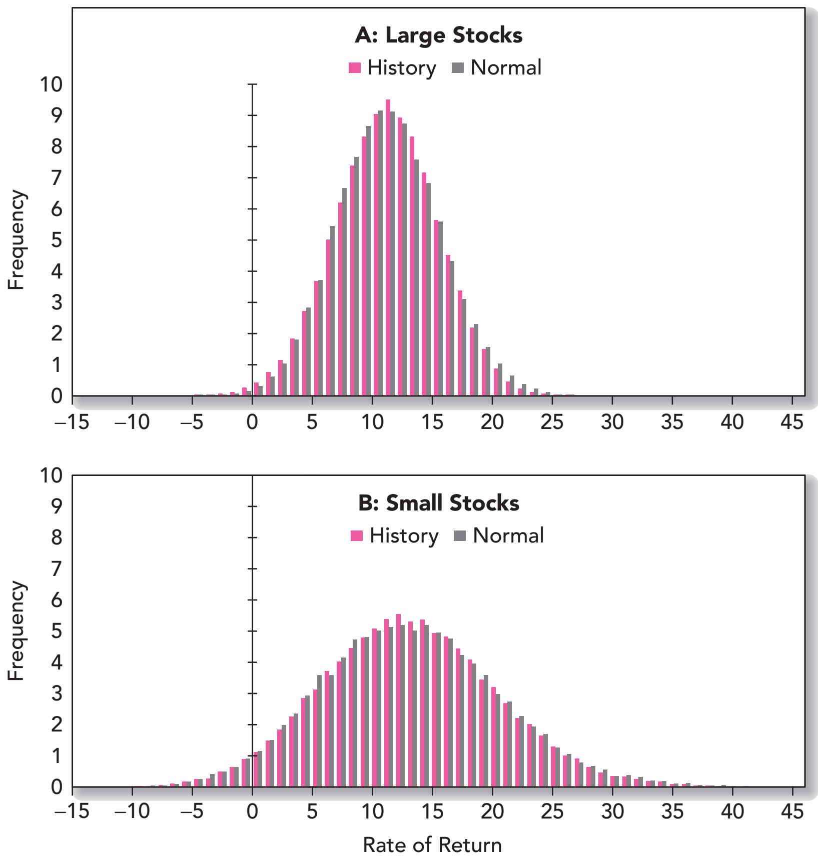
Bootstrapping is a procedure that avoids any assumptions about the return distribution, except that all rates of return in the sample history are equally likely. For example, we could simulate a 25-year sample of possible future returns by sampling (with replacement) 25 randomly selected returns from our available 80-year history. We compound those 25 returns to obtain one possible 25-year holding-period return. This procedure is repeated thousands of times to generate a probability distribution of long-term total returns that is anchored in the historical frequency distribution.

[9]By "timing," we mean that a decline in stock prices is associated with a bad economy when extra income would be most important to an investor. The fact that the insurance policy would pay off in these scenarios contributes to its market value.

The cardinal decision when embarking on a bootstrapping exercise is the choice of how far into the past we should go to draw observations for "future" return sequences. We will use our entire 80-year sample so that we are more likely to include low probability events of extreme value.

One important objective of this exercise is to assess the potential effect of deviations from the normality assumption on the probability distribution of a long-term investment in U.S. stocks. For this purpose, we simulate a 25-year distribution of annual returns for large and small stocks and contrast these samples to similar samples drawn from normal distributions that (due to compounding) result in lognormally distributed long-term total returns. Results are shown in Figure 5.10. Panel A shows the frequency distributions of the paired samples of large U.S. stocks, constructed by sampling both from actual returns and from the normal distribution. Panel B shows the same frequency distributions for small U.S. stocks. The boxes inside Figure 5.10 show the statistics of the distributions.

We first review the results for large stocks in panel A. Viewing the frequency distributions, we see that the difference between the simulated history and the normal draw is small but distinct. Despite the very small differences between the averages of 1-year and



	Actual*	Normal
Avg. 1-year HPR	12.13	12.15
Avg. 25-year HPR	10.24	10.29
SD	4.27	4.32
Min	−11.53	−6.67
Terminal loss (%)	95	82
Max	28.88	29.32
Probability (loss)	.0095	.0064
Probability (shortfall)	.1044	.0603
Skew	−.0854	.1135
Kurtosis	.0040	−.0121

*Bootstrapped history

	Actual*	Normal
Avg. 1-year HPR	17.97	17.95
Avg. 25-year HPR	12.28	12.21
SD	7.28	7.41
Min	−17	−14.9
Terminal loss (%)	99	98
Max	47.03	48.34
Probability (loss)	.0415	.0428
Probability (shortfall)	.1178	.1232
Skew	.1362	.2000
Kurtosis	.0678	.0598

*Bootstrapped history

FIGURE 5.10 Annually compounded, 25-year HPRs from bootstrapped history and a normal distribution (50,000 observations)

25-year annual returns, as well as between the standard deviations, the small differences in skewness and kurtosis combine to produce significant differences in the probabilities of shortfalls and losses, as well as in the potential terminal loss. For small stocks, shown in panel B, the smaller differences in skewness and kurtosis lead to almost identical figures for the probability and magnitude of losses.

What about risk for investors with other long-term horizons? Figure 5.11 compares 25-year to 10-year investments in large and small stocks. For an appropriate comparison, we must account for the fact that the 10-year investment will be supplemented with a 15-year investment in T-bills. To accomplish this comparison, we bootstrap 15-year samples from the 80-year history of T-bill rates and augment each sample with 10 annual rates drawn from the history of the risky investment. Panels A1 and A2 in Figure 5.11 show the comparison for large stocks. The frequency distributions reveal a substantial difference in the risks of the terminal portfolio. This difference is clearly manifested in the portfolio performance statistics. The same picture arises in panels B1 and B2 for small stocks.

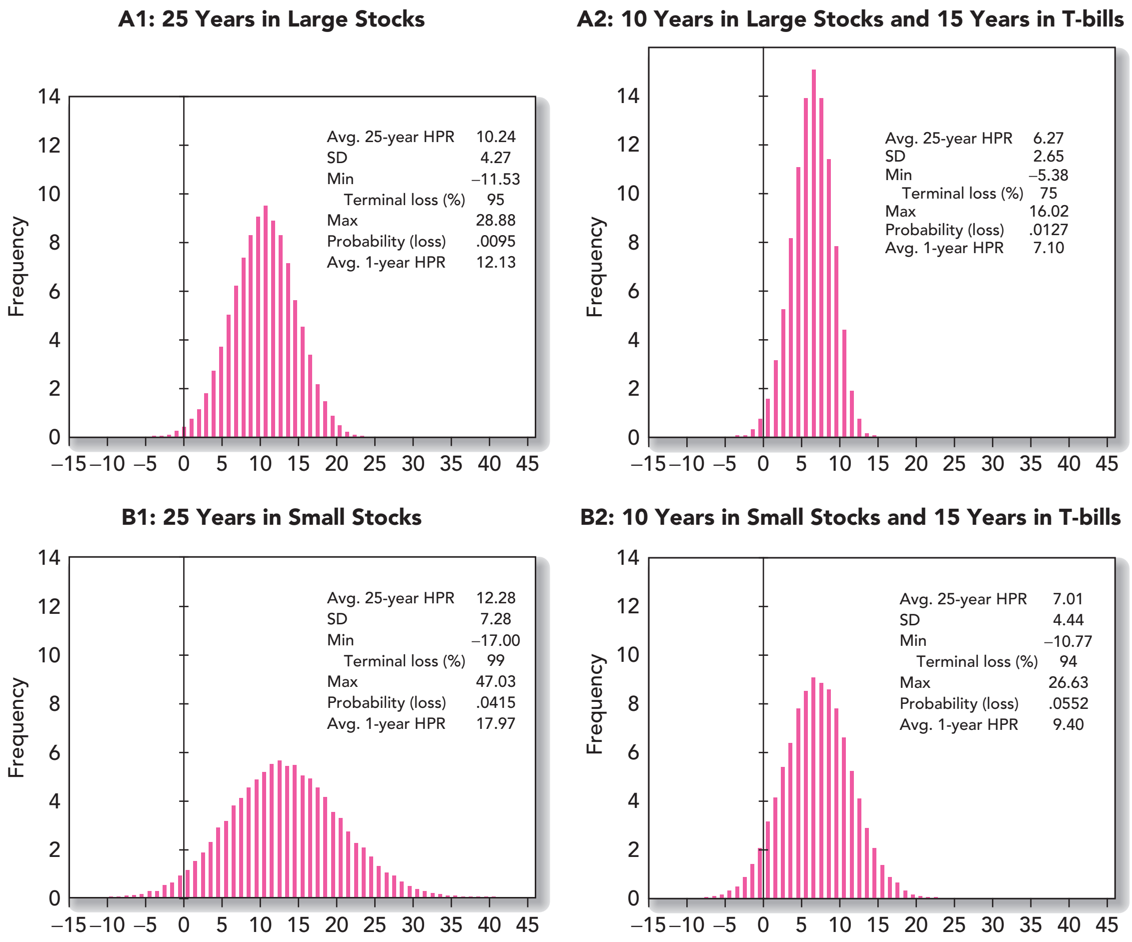


FIGURE 5.11 Annually compounded, 25-year HPRs from bootstrapped history (50,000 observations)

Figure 5.12 shows the trajectories of the wealth indexes of possible outcomes of a 25-year investment in large stocks, compared with the wealth index of the average outcome of a T-bill portfolio. The outcomes of the stock portfolio in Figure 5.12 range from the worst, through the bottom 1% and 5% of terminal value, and up to the mean and median terminal values. The bottom 5% still results in a significant shortfall relative to the T-bill portfolio. In sum, the analysis clearly demonstrates that the notion that investments in stocks become less risky in the long run must be rejected.

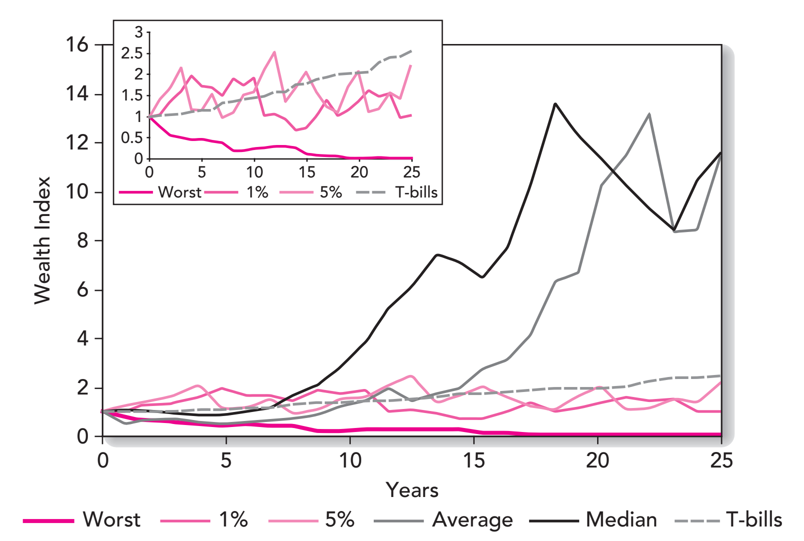


FIGURE 5.12 Wealth indexes of selected outcomes of large stock portfolios and the average T-bill portfolio. Inset: Focus on worst, 1%, and 5% outcomes versus bills.

Yet many practitioners hold on to the view that investment risk is less pertinent to long-term investors. A typical demonstration shown in the nearby box relies on the fact that the standard deviation (or range of likely outcomes) of *annualized* returns is lower for longer-term horizons. But the demonstration is silent on the range of *total* returns.

Forecasts for the Long Haul

We use arithmetic averages to forecast future rates of return because they are unbiased estimates of expected rates over equivalent holding periods. But the arithmetic average of short-term returns can be misleading when used to forecast long-term cumulative returns. This is because sampling errors in the estimate of expected return will have asymmetric impact when compounded over long periods. Positive sampling variation will compound to greater upward errors than negative variation.

Jacquier, Kane, and Marcus[10] show that an unbiased forecast of total return over long horizons requires compounding at a weighted average of the arithmetic and geometric historical averages. The proper weight applied to the geometric average equals the ratio of the length of the forecast horizon to the length of the estimation period. For example, if we wish to forecast the cumulative return for a 25-year horizon from a 80-year history, an unbiased estimate would be to compound at a rate of

$$\text{Geometric average} \times \frac{25}{80} + \text{Arithmetic average} \times \frac{(80-25)}{80}$$

This correction would take about .6% off the historical arithmetic average risk premium on large stocks and about 2% off the arithmetic average of small stocks. A forecast for the next 80 years would require compounding at only the geometric average, and for longer horizons at an even lower number. The forecast horizons that are relevant for current middle-aged investors would depend on their life expectancies.

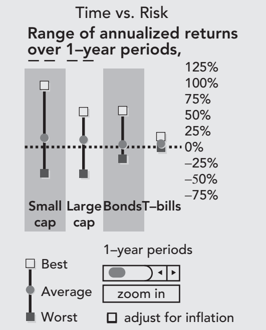
[10]Eric Jacquier, Alex Kane, and Alan J. Marcus, "Geometric or Arithmetic Means: A Reconsideration," *Financial Analysts Journal*, November/December 2003.

WORDS FROM THE STREET

TIME VS. RISK

MANY BEGINNING INVESTORS eye the stock market with a bit of suspicion. They view equity investing as an anxious game of Russian roulette: The longer they stay in, the greater their chance of experiencing more losses. In fact, history shows that the opposite is true. The easiest way to reduce the risk of investing in equities—and improve the gain—is to increase the time you hang on to your portfolio.

See for yourself. The demonstration below uses historical data from 1950 through 2005 to compare investment returns over different lengths of time for small-cap stocks, large caps, long-term bonds and T-bills.



Source: CRSP, Federal Reserve

The graph starts out showing results for investments held over one-year periods. There's no doubt about it: Over such short intervals, small-cap stocks are definitely the riskiest bet.

But what about investing for more than a year? If you move the slider at the bottom right of the graph, you can see the range of returns for longer time periods. Even investing for two years instead of one cuts your risk significantly. As the length of time increases, the volatility of equities decreases sharply—so much so that you may need to click the "zoom in" button to get a closer view. Over 10-year periods, government bonds look safer than large-cap equities on the downside. Click the "adjust for inflation" box, however, and you'll see that bond "safety" can be illusory. Inflation has an uncanny ability to erode the value of securities that don't grow fast enough.

Now move the slider all the way to the right to see the results of investing for 20-year intervals. Adjusting for inflation, the best 20-year gain a portfolio of long-term Treasury bonds could muster is much lower than that achieved by small- and large-cap stocks. And contrary to popular belief, over their worst 20-year period, long-term bonds actually *lost* money when adjusted for inflation. Meanwhile, small-cap investors still had gains over a 20-year-period, even when stocks were at their worst.

Source: Abridged from **www.smartmoney.com/university/Investing101/RiskvsReward/index.cfm?story=timevsrisk,** accessed October 15, 2007.

5.10 MEASUREMENT OF RISK WITH NON-NORMAL DISTRIBUTIONS

The realization that rates of return on stock portfolios are not quite normally distributed, and that as a result, standard deviations may not adequately measure risk, has preoccupied practitioners for quite some time. As we have seen, this concern is indeed well placed. Three methods to augment the measurement of risk are common in the industry: Value at Risk (VaR), Conditional Tail Expectations (CTE), and Lower Partial Standard Deviation (LPSD). We show these statistics for the bootstrapped distributions, contrasted with those for the normal distribution in Table 5.5.

Value at Risk (VaR)

Professional investors extensively use a risk measure that highlights the potential loss from extreme negative returns, called **value at risk,** denoted by **VaR** (to distinguish it from VAR or Var, commonly used to denote variance). The VaR is another name for the *quantile* of a distribution. The quantile (q) of a distribution is the value below which lie q% of the values. Thus the median of the distribution is the 50% quantile. Practitioners commonly use the 5% quantile as the VaR of the distribution. It tells us that, with a probability of 5%, we

TABLE 5.5

Risk measures for non-normal distributions

	Large U.S. Stocks		Small U.S. Stocks	
	History	Normal	History	Normal
Value at Risk				
VaR 1%	0.02%	0.18%	–0.63%	–0.64%
VaR 5%	1.16	1.27	0.17	0.13
VaR 10%	2.17	2.26	1.13	1.04
VaR 50%	10.58	10.29	16.41	15.99
Conditional Tail Expectation				
CTE 1%	–0.28%	–0.14%	–0.77%	–0.76%
CTE 5%	0.46	0.62	–0.33	–0.35
CTE 10%	1.07	1.20	0.16	0.12
CTE 50%	5.07	4.99	5.80	5.49
Lower Partial Standard Deviation				
LPSD of 25-year HPR	4.34%	4.23%	7.09%	7.14%
LPSD of 1-year HPR	21.71	21.16	35.45	35.72
Average 1-year HPR	**12.13**	**12.15**	**17.97**	**17.95**

can expect a loss equal to or greater than the VaR. For a normal distribution, which is completely described by its mean and standard deviation, the 5% VaR always lies 1.65 standard deviations below the mean, and thus, while it may be a convenient benchmark, it adds no information about risk. But if the distribution is not adequately described by the normal, the VaR does give useful information about the magnitude of loss we can expect in a "bad" (e.g., 5% quantile) scenario.

The first four lines in Table 5.5 show the VaR from the bootstrapped distributions and the paired normal samples. The VaR values provide important input for investments in large stocks. The commonly used 5% VaR for large stocks is a 25-year annual holding-period return of 1.16%, compared with 1.27% for the paired normal distribution. The distribution of the portfolio of small stocks is more reasonably approximated by the normal, as is evident in the similarity of the VaR values.

Conditional Tail Expectation (CTE)

The 5% **conditional tail expectation (CTE)** provides the answer to the question, "Assuming the terminal value of the portfolio falls in the bottom 5% of possible outcomes, what is its expected value?" This value for large stocks is a 25-year holding-period return of .46%. Notice the difference from the 5% VaR (1.16%). The 5% VaR is in fact the outcome at the upper boundary of these worst-case outcomes. This is of course the highest holding-period return among the 5% worst-case scenarios, and by construction is higher than the CTE. CTE improves on VaR, as it is more like an expected value that accounts for the entire tail of the distribution, in particular worst-case scenarios, and thus provides a fuller sense of potential losses from low-probability events.

Lower Partial Standard Deviation (LPSD)

An appropriate measure of risk for non-normal distributions is the standard deviation computed solely from values below the expected return. This is a measure of "downside risk" and is called the **lower partial standard deviation (LPSD).** Some practitioners even go as far as using the LPSD in place of the regular standard deviation to compute the Sharpe ratio. The LPSD for the large and small stock portfolios are not very different from values from the normal distribution because the skews are similar to those from the normal (see Table 5.5). For large stocks, for example, assuming a T-bill rate of 6%, the Sharpe ratio from the LPSD would be $(12.13 - 6)/21.71 = 0.28$, compared with 0.29 from the normal distribution. Therefore, the Sharpe ratios calculated from the LPSD are not economically different from the conventional Sharpe ratio.

SUMMARY

1. The economy's equilibrium level of real interest rates depends on the willingness of households to save, as reflected in the supply curve of funds, and on the expected profitability of business investment in plant, equipment, and inventories, as reflected in the demand curve for funds. It depends also on government fiscal and monetary policy.
2. The nominal rate of interest is the equilibrium real rate plus the expected rate of inflation. In general, we can directly observe only nominal interest rates; from them, we must infer expected real rates, using inflation forecasts.
3. The equilibrium expected rate of return on any security is the sum of the equilibrium real rate of interest, the expected rate of inflation, and a security-specific risk premium.
4. Investors face a trade-off between risk and expected return. Historical data confirm our intuition that assets with low degrees of risk provide lower returns on average than do those of higher risk.
5. Assets with guaranteed nominal interest rates are risky in real terms because the future inflation rate is uncertain.
6. Historical rates of return over the twentieth century in developed capital markets suggest the U.S. history of stock returns is not an outlier compared to other countries.
7. Investments in risky portfolios *do not* become safer in the long run. On the contrary, the longer a risky investment is held, the greater the risk. The basis of the argument that stocks are safe in the long run is the fact that the probability of a shortfall becomes smaller. However, probability of shortfall is a poor measure of the safety of an investment. It ignores the magnitude of possible losses.
8. Historical returns on stocks exhibit more frequent large negative deviations from the mean than would be predicted from a normal distribution. The lower partial standard deviation (LPSD) and the skewness of the actual distribution quantify the deviation from normality. The LPSD, instead of the standard deviation, is sometimes used by practitioners as a measure of risk.
9. Widely used measures of risk are value at risk (VaR) and conditional tail expectations (CTE). VaR measures the loss that will be exceeded with a specified probability such as 5%. The VaR does not add new information when returns are normally distributed. When negative deviations from the average are larger and more frequent than the normal distribution, the 5% VaR will be more than 1.65 standard deviations below the average return. Conditional tail expectations (CTE) measure the expected rate of return conditional on the portfolio falling below a certain value. Thus, 1% CTE is the expected return of all possible outcomes in the bottom 1% of the distribution.

Related Web sites for this chapter are available at **www.mhhe.com/bkm**

KEY TERMS

nominal interest rate
real interest rate
effective annual rate (EAR)
annual percentage rate (APR)
dividend yield
risk-free rate
risk premium
excess return
risk aversion
normal distribution
event tree
skew
kurtosis
lognormal distribution
value at risk (VaR)
conditional tail expectation (CTE)
lower partial standard deviation (LPSD)

PROBLEM SETS

Quiz

1. The Fisher equation tells us that the real interest rate approximately equals the nominal rate minus the inflation rate. Suppose the inflation rate increases from 3% to 5%. Does the Fisher equation imply that this increase will result in a fall in the real rate of interest? Explain.
2. You've just stumbled on a new dataset that enables you to compute historical rates of return on U.S. stocks all the way back to 1880. What are the advantages and disadvantages in using these data to help estimate the expected rate of return on U.S. stocks over the coming year?
3. You are considering two alternative 2-year investments: You can invest in a risky asset with a positive risk premium and returns in each of the 2 years that will be identically distributed and uncorrelated, or you can invest in the risky asset for only 1 year and then invest the proceeds in a risk-free asset. Which of the following statements about the first investment alternative (compared with the second) are true?
   a. Its 2-year risk premium is the same as the second alternative.
   b. The standard deviation of its 2-year return is the same.
   c. Its annualized standard deviation is lower.
   d. Its Sharpe ratio is higher.
   e. It is relatively more attractive to investors who have lower degrees of risk aversion.

Problems

4. You have $5,000 to invest for the next year and are considering three alternatives:
   a. A money market fund with an average maturity of 30 days offering a current yield of 6% per year.
   b. A 1-year savings deposit at a bank offering an interest rate of 7.5%.
   c. A 20-year U.S. Treasury bond offering a yield to maturity of 9% per year.

   What role does your forecast of future interest rates play in your decisions?
5. Use Figure 5.1 in the text to analyze the effect of the following on the level of real interest rates:
   a. Businesses become more pessimistic about future demand for their products and decide to reduce their capital spending.
   b. Households are induced to save more because of increased uncertainty about their future Social Security benefits.
   c. The Federal Reserve Board undertakes open-market purchases of U.S. Treasury securities in order to increase the supply of money.
6. You are considering the choice between investing $50,000 in a conventional 1-year bank CD offering an interest rate of 5% and a 1-year "Inflation-Plus" CD offering 1.5% per year plus the rate of inflation.
   a. Which is the safer investment?
   b. Which offers the higher expected return?
   c. If you expect the rate of inflation to be 3% over the next year, which is the better investment? Why?
   d. If we observe a risk-free nominal interest rate of 5% per year and a risk-free real rate of 1.5% on inflation-indexed bonds, can we infer that the market's expected rate of inflation is 3.5% per year?
7. Look at Spreadsheet 5.1 in the text. Suppose you now revise your expectations regarding the stock price as follows:

State of the Economy	Probability	Ending Price	HPR (including dividends)
Boom	.35	$140	44.5%
Normal growth	.30	110	14.0
Recession	.35	80	−16.5

   Use Equations 5.11 and 5.12 to compute the mean and standard deviation of the HPR on stocks. Compare your revised parameters with the ones in the spreadsheet.

8. Derive the probability distribution of the 1-year HPR on a 30-year U.S. Treasury bond with an 8% coupon if it is currently selling at par and the probability distribution of its yield to maturity a year from now is as follows:

State of the Economy	Probability	YTM
Boom	.20	11.0%
Normal growth	.50	8.0
Recession	.30	7.0

For simplicity, assume the entire 8% coupon is paid at the end of the year rather than every 6 months.

9. What is the standard deviation of a random variable *q* with the following probability distribution:

Value of *q*	Probability
0	.25
1	.25
2	.50

10. The continuously compounded annual return on a stock is normally distributed with a mean of 20% and standard deviation of 30%. With 95.44% confidence, we should expect its actual return in any particular year to be between which pair of values? *Hint:* look again at Figure 5.4.
    a. −40.0% and 80.0%
    b. −30.0% and 80.0%
    c. −20.6% and 60.6%
    d. −10.4% and 50.4%
11. Using historical risk premiums over the 1926–1995 period as your guide, what would be your estimate of the expected annual HPR on the S&P 500 stock portfolio if the current risk-free interest rate is 6%?

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12. You can find annual holding-period returns for several asset classes at our Web site (**www.mhhe.com/bkm**); look for links to Chapter 5. Compute the means, standard deviations, skewness, and kurtosis of the annual HPR of large stocks and long-term Treasury bonds using only the 30 years of data between 1976 and 2005. How do these statistics compare with those computed from the data for the period 1926–1941? Which do you think are the most relevant statistics to use for projecting into the future?
13. During a period of severe inflation, a bond offered a nominal HPR of 80% per year. The inflation rate was 70% per year.
    a. What was the real HPR on the bond over the year?
    b. Compare this real HPR to the approximation $r \approx R - i$.
14. Suppose that the inflation rate is expected to be 3% in the near future. Using the historical data provided in this chapter, what would be your predictions for:
    a. The T-bill rate?
    b. The expected rate of return on large stocks?
    c. The risk premium on the stock market?
15. An economy is making a rapid recovery from steep recession, and businesses foresee a need for large amounts of capital investment. Why would this development affect real interest rates?

Challenge Problems

Challenge Problems 16 and 17 are more difficult. You may need to review the definitions of call and put options in Chapter 2.

16. You are faced with the probability distribution of the HPR on the stock market index fund given in Spreadsheet 5.1 of the text. Suppose the price of a put option on a share of the index fund with exercise price of $110 and time to expiration of 1 year is $12.
    a. What is the probability distribution of the HPR on the put option?
    b. What is the probability distribution of the HPR on a portfolio consisting of one share of the index fund and a put option?

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c. In what sense does buying the put option constitute a purchase of insurance in this case?

17. Take as given the conditions described in the previous problem, and suppose the risk-free interest rate is 6% per year. You are contemplating investing $107.55 in a 1-year CD and simultaneously buying a call option on the stock market index fund with an exercise price of $110 and expiration of 1 year. What is the probability distribution of your dollar return at the end of the year?

CFA® PROBLEMS

1. Given $100,000 to invest, what is the expected risk premium in dollars of investing in equities versus risk-free T-bills (U.S. Treasury bills) based on the following table?

Action	Probability	Expected Return
Invest in equities	.6	$ 50,000
	.4	–$ 30,000
Invest in risk-free T-bill	1.0	$ 5,000

2. Based on the scenarios below, what is the expected return for a portfolio with the following return profile?

	Market Condition		
	Bear	Normal	Bull
Probability	.2	.3	.5
Rate of return	–25%	10%	24%

Use the following scenario analysis for Stocks X and Y to answer CFA Problems 3 through 6 (round to the nearest percent).

	Bear Market	Normal Market	Bull Market
Probability	0.2	0.5	0.3
Stock X	–20%	18%	50%
Stock Y	–15%	20%	10%

3. What are the expected rates of return for Stocks X and Y?
4. What are the standard deviations of returns on Stocks X and Y?
5. Assume that of your $10,000 portfolio, you invest $9,000 in Stock X and $1,000 in Stock Y. What is the expected return on your portfolio?
6. Probabilities for three states of the economy and probabilities for the returns on a particular stock in each state are shown in the table below.

State of Economy	Probability of Economic State	Stock Performance	Probability of Stock Performance in Given Economic State
Good	.3	Good	.6
		Neutral	.3
		Poor	.1
Neutral	.5	Good	.4
		Neutral	.3
		Poor	.3
Poor	.2	Good	.2
		Neutral	.3
		Poor	.5

What is the probability that the economy will be neutral *and* the stock will experience poor performance?

7. An analyst estimates that a stock has the following probabilities of return depending on the state of the economy:

State of Economy	Probability	Return
Good	.1	15%
Normal	.6	13
Poor	.3	7

What is the expected return of the stock?

STANDARD &POOR'S

Go to **www.mhhe.com/edumarketinsight** (bookmark this page!) and link to *Company*. Choose a few companies of interest and record their ticker symbols. Under Excel Analytics, go to *Market Data* and find Monthly Adjusted Prices for each firm, which you should download into a spreadsheet. Calculate the standard deviation, skew, and kurtosis of the recent history of returns for each firm. How do they compare to the values for the S&P 500? Try repeating the exercise for other firms. Can you reach any conclusions about the pattern of these statistics for individual firms versus the diversified market index? Do returns for the index appear to be better described by the normal distribution than the returns of the individual firms?

E-Investments

Inflation and Rates

The Federal Reserve Bank of St. Louis has information available on interest rates and economic conditions. A publication called *Monetary Trends* contains graphs and tables with information about current conditions in the capital markets. Go to the Web site **www.stls.frb.org** and click on *Economic Research* on the menu at the top of the page. Find the most recent issue of *Monetary Trends* in the *Recent Data Publications* section and answer these questions.

1. What is the professionals' consensus forecast for inflation for the next 2 years? (Use the *Federal Reserve Bank of Philadelphia* line on the graph to answer this.)
2. What do consumers expect to happen to inflation over the next 2 years? (Use the *University of Michigan* line on the graph to answer this.)
3. Have real interest rates increased, decreased, or remained the same over the last 2 years?
4. What has happened to short-term nominal interest rates over the last 2 years? What about long-term nominal interest rates?
5. How do recent U.S. inflation and long-term interest rates compare with those of the other countries listed?
6. What are the most recently available levels of 3-month and 10-year yields on Treasury securities?

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SOLUTIONS TO CONCEPT CHECKS

1. *a.* $1 + R = (1 + r)(1 + i) = (1.03)(1.08) = 1.1124$

   $R = 11.24\%$

   b. $1 + R = (1.03)(1.10) = 1.133$

   $R = 13.3\%$

2. *a.* $\text{EAR} = (1 + .01)^{12} - 1 = .1268 = 12.68\%$

   b. $\text{EAR} = e^{.12} - 1 = .1275 = 12.75\%$

   Choose the continuously compounded rate for its higher EAR.

3. Number of bonds bought is 27,000/900 = 30

Interest Rates	Probability	Year-end Bond Price	HPR	End-of-Year Value
High	.2	$850	(75 + 850)/900 – 1 = .0278	(75 + 850)30 = $27,750
Unchanged	.5	915	.1000	$29,700
Low	.3	985	.1778	$31,800
Expected rate of return			.1089	
Expected end-of-year dollar value				$29,940
Risk premium			.0589	

4. *a.* Arithmetic return = (1/3)(.2869) + (1/3)(.1088) + (1/3)(0.0491) = .1483 = 14.83%

   b. Geometric average $= \sqrt[3]{1.2869 \times 1.1088 \times 1.0491} - 1 = .1439 = 14.39\%$

   c. Standard deviation = 12.37%

   d. Sharpe ratio = (14.83 − 6.0)/12.37 = .71

5. The probability of a more extreme bad month, with return below −15%, is much lower: NORMDIST(−15,1,6,TRUE) = .00383. Alternatively, we can note that −15% is 16/6 standard deviations below the mean return, and use the standard normal function to compute NORMSDIST(−16/6) = .00383.

6. If the probabilities in Spreadsheet 5.2 represented the true return distribution, we would use Equations 5.19 and 5.20 to obtain: Skew = 0.0931; Kurtosis = −1.2081. However, in this case, the data in the table represent a (short) historical sample, and correction for degrees-of-freedom bias is required (in a similar manner to our calculations for standard deviation). You can use Excel functions to obtain: SKEW(C5:C9) = 0.1387; KURT(C5:C9) = −0.2832.