CHAPTER NINE

PART III

THE CAPITAL ASSET PRICING MODEL

THE CAPITAL ASSET pricing model, almost always referred to as the CAPM, is a centerpiece of modern financial economics. The model gives us a precise prediction of the relationship that we should observe between the risk of an asset and its expected return. This relationship serves two vital functions. First, it provides a benchmark rate of return for evaluating possible investments. For example, if we are analyzing securities, we might be interested in whether the expected return we forecast for a stock is more or less than its "fair" return given its risk. Second, the model helps us to make an educated guess as to the expected return on assets that have not yet been traded in the marketplace. For example, how do we price an initial public offering of stock? How will a major new investment project affect the return investors require on a company's stock? Although the CAPM does not fully withstand empirical tests, it is widely used because of the insight it offers and because its accuracy is deemed acceptable for important applications.

9.1 THE CAPITAL ASSET PRICING MODEL

The capital asset pricing model is a set of predictions concerning equilibrium expected returns on risky assets. Harry Markowitz laid down the foundation of modern portfolio management in 1952. The CAPM was developed 12 years later in articles by William Sharpe,[1] John Lintner,[2] and Jan Mossin.[3] The time for this gestation indicates that the leap from Markowitz's portfolio selection model to the CAPM is not trivial.

We will approach the CAPM by posing the question "what if," where the "if" part refers to a simplified world. Positing an admittedly unrealistic world allows a relatively easy leap to the "then" part. Once we accomplish this, we can add complexity to the hypothesized

[1] William Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium," *Journal of Finance*, September 1964.

[2] John Lintner, "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets," *Review of Economics and Statistics*, February 1965.

[3] Jan Mossin, "Equilibrium in a Capital Asset Market," *Econometrica*, October 1966.

environment one step at a time and see how the conclusions must be amended. This process allows us to derive a reasonably realistic and comprehensible model.

We summarize the simplifying assumptions that lead to the basic version of the CAPM in the following list. The thrust of these assumptions is that we try to ensure that individuals are as alike as possible, with the notable exceptions of initial wealth and risk aversion. We will see that conformity of investor behavior vastly simplifies our analysis.

1. There are many investors, each with an endowment (wealth) that is small compared to the total endowment of all investors. Investors are price-takers, in that they act as though security prices are unaffected by their own trades. This is the usual perfect competition assumption of microeconomics.
2. All investors plan for one identical holding period. This behavior is myopic (short-sighted) in that it ignores everything that might happen after the end of the single-period horizon. Myopic behavior is, in general, suboptimal.
3. Investments are limited to a universe of publicly traded financial assets, such as stocks and bonds, and to risk-free borrowing or lending arrangements. This assumption rules out investment in nontraded assets such as education (human capital), private enterprises, and governmentally funded assets such as town halls and international airports. It is assumed also that investors may borrow or lend any amount at a fixed, risk-free rate.
4. Investors pay no taxes on returns and no transaction costs (commissions and service charges) on trades in securities. In reality, of course, we know that investors are in different tax brackets and that this may govern the type of assets in which they invest. For example, tax implications may differ depending on whether the income is from interest, dividends, or capital gains. Furthermore, actual trading is costly, and commissions and fees depend on the size of the trade and the good standing of the individual investor.
5. All investors are rational mean-variance optimizers, meaning that they all use the Markowitz portfolio selection model.
6. All investors analyze securities in the same way and share the same economic view of the world. The result is identical estimates of the probability distribution of future cash flows from investing in the available securities; that is, for any set of security prices, they all derive the same input list to feed into the Markowitz model. Given a set of security prices and the risk-free interest rate, all investors use the same expected returns and covariance matrix of security returns to generate the efficient frontier and the unique optimal risky portfolio. This assumption is often referred to as **homogeneous expectations** or beliefs.

These assumptions represent the “if” of our “what if” analysis. Obviously, they ignore many real-world complexities. With these assumptions, however, we can gain some powerful insights into the nature of equilibrium in security markets.

We can summarize the equilibrium that will prevail in this hypothetical world of securities and investors briefly. The rest of the chapter explains and elaborates on these implications.

1. All investors will choose to hold a portfolio of risky assets in proportions that duplicate representation of the assets in the **market portfolio** (*M*), which includes all traded assets. For simplicity, we generally refer to all risky assets as *stocks.* The proportion of each stock in the market portfolio equals the market value of the stock

(price per share multiplied by the number of shares outstanding) divided by the total market value of all stocks.

2. Not only will the market portfolio be on the efficient frontier, but it also will be the tangency portfolio to the optimal capital allocation line (CAL) derived by each and every investor. As a result, the *capital market line* (CML), the line from the risk-free rate through the market portfolio, *M*, is also the best attainable capital allocation line. All investors hold *M* as their optimal risky portfolio, differing only in the amount invested in it versus in the risk-free asset.
3. The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the representative investor. Mathematically,

$$E(r_M) - r_f = \overline{A}\sigma_M^2$$

where σ_M^2 is the variance of the market portfolio and $\overline{A}$ is the average degree of risk aversion across investors. Note that because *M* is the optimal portfolio, which is efficiently diversified across all stocks, σ_M^2 is the systematic risk of this universe.
4. The risk premium on *individual* assets will be proportional to the risk premium on the market portfolio, *M*, and the *beta coefficient* of the security relative to the market portfolio. Beta measures the extent to which returns on the stock and the market move together. Formally, beta is defined as

$$\beta_i = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2}$$

and the risk premium on individual securities is

$$E(r_i) - r_f = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2}[E(r_M) - r_f] = \beta_i[E(r_M) - r_f]$$

Why Do All Investors Hold the Market Portfolio?

What is the market portfolio? When we sum over, or aggregate, the portfolios of all individual investors, lending and borrowing will cancel out (because each lender has a corresponding borrower), and the value of the aggregate risky portfolio will equal the entire wealth of the economy. This is the market portfolio, *M*. The proportion of each stock in this portfolio equals the market value of the stock (price per share times number of shares outstanding) divided by the sum of the market values of all stocks.[4] The CAPM implies that as individuals attempt to optimize their personal portfolios, they each arrive at the same portfolio, with weights on each asset equal to those of the market portfolio.

Given the assumptions of the previous section, it is easy to see that all investors will desire to hold identical risky portfolios. If all investors use identical Markowitz analysis (Assumption 5) applied to the same universe of securities (Assumption 3) for the same time horizon (Assumption 2) and use the same input list (Assumption 6), they all must arrive at the same composition of the optimal risky portfolio, the portfolio on the efficient frontier identified by the tangency line from T-bills to that frontier, as in Figure 9.1. This

[4]As noted previously, we use the term "stock" for convenience; the market portfolio properly includes all assets in the economy.

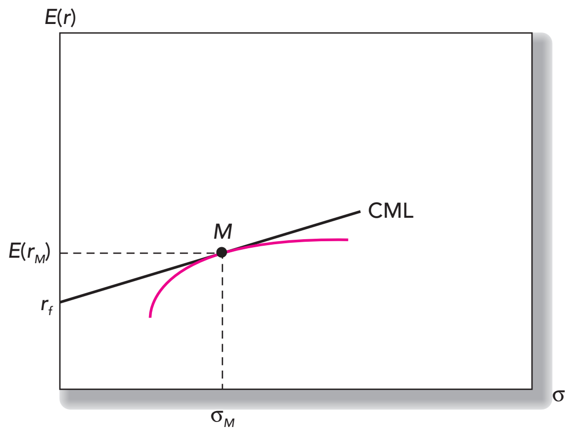


FIGURE 9.1 The efficient frontier and the capital market line

implies that if the weight of GE stock, for example, in each common risky portfolio is 1%, then GE also will comprise 1% of the market portfolio. The same principle applies to the proportion of any stock in each investor's risky portfolio. As a result, the optimal risky portfolio of all investors is simply a share of the market portfolio in Figure 9.1.

Now suppose that the optimal portfolio of our investors does not include the stock of some company, such as Delta Airlines. When all investors avoid Delta stock, the demand is zero, and Delta's price takes a free fall. As Delta stock gets progressively cheaper, it becomes ever more attractive and other stocks look relatively less attractive. Ultimately, Delta reaches a price where it is attractive enough to include in the optimal stock portfolio.

Such a price adjustment process guarantees that all stocks will be included in the optimal portfolio. It shows that *all* assets have to be included in the market portfolio. The only issue is the price at which investors will be willing to include a stock in their optimal risky portfolio.

This may seem a roundabout way to derive a simple result: If all investors hold an identical risky portfolio, this portfolio has to be *M*, the market portfolio. Our intention, however, is to demonstrate a connection between this result and its underpinnings, the equilibrating process that is fundamental to security market operation.

The Passive Strategy Is Efficient

In Chapter 6 we defined the CML (capital market line) as the CAL (capital allocation line) that is constructed from a money market account (or T-bills) and the market portfolio. Perhaps now you can fully appreciate why the CML is an interesting CAL. In the simple world of the CAPM, *M* is the optimal tangency portfolio on the efficient frontier, as shown in Figure 9.1.

In this scenario, the market portfolio held by all investors is based on the common input list, thereby incorporating all relevant information about the universe of securities. This means that investors can skip the trouble of doing security analysis and obtain an efficient portfolio simply by holding the market portfolio. (Of course, if everyone were to follow this strategy, no one would perform security analysis and this result would no longer hold. We discuss this issue in greater depth in Chapter 11 on market efficiency.)

Thus the passive strategy of investing in a market index portfolio is efficient. For this reason, we sometimes call this result a **mutual fund theorem.** The mutual fund theorem is another incarnation of the separation property discussed in Chapter 7. Assuming that all investors choose to hold a market index mutual fund, we can separate portfolio selection into two components—a technical problem, creation of mutual funds by professional managers—and a personal problem that depends on an investor's risk aversion, allocation of the *complete* portfolio between the mutual fund and risk-free assets.

In reality, different investment managers do create risky portfolios that differ from the market index. We attribute this in part to the use of different input lists in the formation of the optimal risky portfolio. Nevertheless, the practical significance of the mutual fund theorem is that a passive investor may view the market index as a reasonable first approximation to an efficient risky portfolio.

WORDS FROM THE STREET

THE PARABLE OF THE MONEY MANAGERS

Some years ago, in a land called Indicia, revolution led to the overthrow of a socialist regime and the restoration of a system of private property. Former government enterprises were reformed as corporations, which then issued stocks and bonds. These securities were given to a central agency, which offered them for sale to individuals, pension funds, and the like (all armed with newly printed money).

Almost immediately a group of money managers came forth to assist these investors. Recalling the words of a venerated elder, uttered before the previous revolution ("Invest in Corporate Indicia"), they invited clients to give them money, with which they would buy a cross-section of all the newly issued securities. Investors considered this a reasonable idea, and soon everyone held a piece of Corporate Indicia.

Before long the money managers became bored because there was little for them to do. Soon they fell into the habit of gathering at a beachfront casino where they passed the time playing roulette, craps, and similar games, for low stakes, with their own money.

After a while, the owner of the casino suggested a new idea. He would furnish an impressive set of rooms which would be designated the Money Managers' Club. There the members could place bets with one another about the fortunes of various corporations, industries, the level of the Gross National Product, foreign trade, etc. To make the betting more exciting, the casino owner suggested that the managers use their clients' money for this purpose.

The offer was immediately accepted, and soon the money managers were betting eagerly with one another. At the end of each week, some found that they had won money for their clients, while others found that they had lost. But the losses always exceeded the gains, for a certain amount was deducted from each bet to cover the costs of the elegant surroundings in which the gambling took place.

Before long a group of professors from Indicia U. suggested that investors were not well served by the activities being conducted at the Money Managers' Club. "Why pay people to gamble with your money? Why not just hold your own piece of Corporate Indicia?" they said.

This argument seemed sensible to some of the investors, and they raised the issue with their money managers. A few capitulated, announcing that they would henceforth stay away from the casino and use their clients' money only to buy proportionate shares of all the stocks and bonds issued by corporations.

The converts, who became known as managers of Indicia funds, were initially shunned by those who continued to frequent the Money Managers' Club, but in time, grudging acceptance replaced outright hostility. The wave of puritan reform some had predicted failed to materialize, and gambling remained legal. Many managers continued to make their daily pilgrimage to the casino. But they exercised more restraint than before, placed smaller bets, and generally behaved in a manner consonant with their responsibilities. Even the members of the Lawyers' Club found it difficult to object to the small amount of gambling that still went on.

And everyone but the casino owner lived happily ever after.

Source: William F. Sharpe, "The Parable of the Money Managers," *The Financial Analysts' Journal* 32 (July/August 1976), p. 4.

The nearby box contains a parable illustrating the argument for indexing. If the passive strategy is efficient, then attempts to beat it simply generate trading and research costs with no offsetting benefit, and ultimately inferior results.

CONCEPT CHECK 1

If there are only a few investors who perform security analysis, and all others hold the market portfolio, *M,* would the CML still be the efficient CAL for investors who do not engage in security analysis? Why or why not?

The Risk Premium of the Market Portfolio

In Chapter 6 we discussed how individual investors go about deciding how much to invest in the risky portfolio. Returning now to the decision of how much to invest in portfolio *M* versus in the risk-free asset, what can we deduce about the equilibrium risk premium of portfolio *M?*

We asserted earlier that the equilibrium risk premium on the market portfolio, $E(r_M) - r_f$, will be proportional to the average degree of risk aversion of the investor population and the risk of the market portfolio, σ_M^2. Now we can explain this result.

Recall that each individual investor chooses a proportion y, allocated to the optimal portfolio M, such that

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2} \quad \textbf{(9.1)}$$

In the simplified CAPM economy, risk-free investments involve borrowing and lending among investors. Any borrowing position must be offset by the lending position of the creditor. This means that net borrowing and lending across all investors must be zero, and in consequence, substituting the representative investor's risk aversion, $\overline{A}$, for A, the average position in the risky portfolio is 100%, or $\overline{y} = 1$. Setting $y = 1$ in Equation 9.1 and rearranging, we find that the risk premium on the market portfolio is related to its variance by the average degree of risk aversion:

$$E(r_M) - r_f = \overline{A}\sigma_M^2 \quad \textbf{(9.2)}$$

CONCEPT CHECK 2

Data from the last eight decades (see Table 5.3) for the S&P 500 index yield the following statistics: average excess return, 8.4%; standard deviation, 20.3%.

a. To the extent that these averages approximated investor expectations for the period, what must have been the average coefficient of risk aversion?

b. If the coefficient of risk aversion were actually 3.5, what risk premium would have been consistent with the market's historical standard deviation?

Expected Returns on Individual Securities

The CAPM is built on the insight that the appropriate risk premium on an asset will be determined by its contribution to the risk of investors' overall portfolios. Portfolio risk is what matters to investors and is what governs the risk premiums they demand.

Remember that all investors use the same input list, that is, the same estimates of expected returns, variances, and covariances. We saw in Chapter 7 that these covariances can be arranged in a covariance matrix, so that the entry in the fifth row and third column, for example, would be the covariance between the rates of return on the fifth and third securities. Each diagonal entry of the matrix is the covariance of one security's return with itself, which is simply the variance of that security.

Suppose, for example, that we want to gauge the portfolio risk of GE stock. We measure the contribution to the risk of the overall portfolio from holding GE stock by its covariance with the market portfolio. To see why this is so, let us look again at the way the variance of the market portfolio is calculated. To calculate the variance of the market portfolio, we use the bordered covariance matrix with the market portfolio weights, as discussed in Chapter 7. We highlight GE in this depiction of the n stocks in the market portfolio.

Portfolio Weights	w_1	w_2	...	w_{GE}	...	w_n
w_1	$\text{Cov}(r_1, r_1)$	$\text{Cov}(r_1, r_2)$	...	$\text{Cov}(r_1, r_{GE})$	...	$\text{Cov}(r_1, r_n)$
w_2	$\text{Cov}(r_2, r_1)$	$\text{Cov}(r_2, r_2)$	...	$\text{Cov}(r_2, r_{GE})$	...	$\text{Cov}(r_2, r_n)$
⋮	⋮	⋮		⋮		⋮
w_{GE}	$\text{Cov}(r_{GE}, r_1)$	$\text{Cov}(r_{GE}, r_2)$	...	$\text{Cov}(r_{GE}, r_{GE})$	...	$\text{Cov}(r_{GE}, r_n)$
⋮	⋮	⋮		⋮		⋮
w_n	$\text{Cov}(r_n, r_1)$	$\text{Cov}(r_n, r_2)$	...	$\text{Cov}(r_n, r_{GE})$	...	$\text{Cov}(r_n, r_n)$

Recall that we calculate the variance of the portfolio by summing over all the elements of the covariance matrix, first multiplying each element by the portfolio weights from the row and the column. The contribution of one stock to portfolio variance therefore can be expressed as the sum of all the covariance terms in the column corresponding to the stock, where each covariance is first multiplied by both the stock's weight from its row and the weight from its column.[5]

For example, the contribution of GE's stock to the variance of the market portfolio is

$$w_{GE}[w_1\text{Cov}(r_1, r_{GE}) + w_2\text{Cov}(r_2, r_{GE}) + \ldots + w_{GE}\text{Cov}(r_{GE}, r_{GE}) + \ldots + w_n\text{Cov}(r_n, r_{GE})] \tag{9.3}$$

Equation 9.3 provides a clue about the respective roles of variance and covariance in determining asset risk. When there are many stocks in the economy, there will be many more covariance terms than variance terms. Consequently, the covariance of a particular stock with all other stocks will dominate that stock's contribution to total portfolio risk. Notice that the sum inside the square brackets in Equation 9.3 is the covariance of GE with the market portfolio. In other words, we can best measure the stock's contribution to the risk of the market portfolio by its covariance with that portfolio:

$$\text{GE's contribution to variance} = w_{GE}\text{Cov}(r_{GE}, r_M)$$

This should not surprise us. For example, if the covariance between GE and the rest of the market is negative, then GE makes a "negative contribution" to portfolio risk: By providing returns that move inversely with the rest of the market, GE stabilizes the return on the overall portfolio. If the covariance is positive, GE makes a positive contribution to overall portfolio risk because its returns reinforce swings in the rest of the portfolio.

To demonstrate this more rigorously, note that the rate of return on the market portfolio may be written as

$$r_M = \sum_{k=1}^{n} w_k r_k$$

[5]An alternative approach would be to measure GE's contribution to market variance as the sum of the elements in the row *and* the column corresponding to GE. In this case, GE's contribution would be twice the sum in Equation 9.3. The approach that we take in the text allocates contributions to portfolio risk among securities in a convenient manner in that the sum of the contributions of each stock equals the total portfolio variance, whereas the alternative measure of contribution would sum to twice the portfolio variance. This results from a type of double-counting, because adding both the rows and the columns for each stock would result in each entry in the matrix being added twice.

Therefore, the covariance of the return on GE with the market portfolio is

$$\mathrm{Cov}(r_{\mathrm{GE}}, r_M) = \mathrm{Cov}\left(r_{\mathrm{GE}}, \sum_{k=1}^{n} w_k r_k\right) = \sum_{k=1}^{n} w_k \mathrm{Cov}(r_k, r_{\mathrm{GE}}) \quad \textbf{(9.4)}$$

Notice that the last term of Equation 9.4 is precisely the same as the term in brackets in Equation 9.3. Therefore, Equation 9.3, which is the contribution of GE to the variance of the market portfolio, may be simplified to $w_{\mathrm{GE}}\,\mathrm{Cov}(r_{\mathrm{GE}}, r_M)$. We also observe that the contribution of our holding of GE to the risk premium of the market portfolio is $w_{\mathrm{GE}}\,[E(r_{\mathrm{GE}}) - r_f]$.

Therefore, the reward-to-risk ratio for investments in GE can be expressed as

$$\frac{\text{GE's contribution to risk premium}}{\text{GE's contribution to variance}} = \frac{w_{\mathrm{GE}}[E(r_{\mathrm{GE}}) - r_f]}{w_{\mathrm{GE}}\mathrm{Cov}(r_{\mathrm{GE}}, r_M)} = \frac{E(r_{\mathrm{GE}}) - r_f}{\mathrm{Cov}(r_{\mathrm{GE}}, r_M)}$$

The market portfolio is the tangency (efficient mean-variance) portfolio. The reward-to-risk ratio for investment in the market portfolio is

$$\frac{\text{Market risk premium}}{\text{Market variance}} = \frac{E(r_M) - r_f}{\sigma_M^2} \quad \textbf{(9.5)}$$

The ratio in Equation 9.5 is often called the **market price of risk**[6] because it quantifies the extra return that investors demand to bear portfolio risk. Notice that for *components* of the efficient portfolio, such as shares of GE, we measure risk as the *contribution* to portfolio variance (which depends on its *covariance* with the market). In contrast, for the efficient portfolio itself, its variance is the appropriate measure of risk.

A basic principle of equilibrium is that all investments should offer the same reward-to-risk ratio. If the ratio were better for one investment than another, investors would rearrange their portfolios, tilting toward the alternative with the better trade-off and shying away from the other. Such activity would impart pressure on security prices until the ratios were equalized. Therefore we conclude that the reward-to-risk ratios of GE and the market portfolio should be equal:

$$\frac{E(r_{\mathrm{GE}}) - r_f}{\mathrm{Cov}(r_{\mathrm{GE}}, r_M)} = \frac{E(r_M) - r_f}{\sigma_M^2} \quad \textbf{(9.6)}$$

To determine the fair risk premium of GE stock, we rearrange Equation 9.6 slightly to obtain

$$E(r_{\mathrm{GE}}) - r_f = \frac{\mathrm{Cov}(r_{\mathrm{GE}}, r_M)}{\sigma_M^2}[E(r_M) - r_f] \quad \textbf{(9.7)}$$

[6]We open ourselves to ambiguity in using this term, because the market portfolio's reward-to-volatility ratio

$$\frac{E(r_M) - r_f}{\sigma_M}$$

sometimes is referred to as the market price of risk. Note that because the appropriate risk measure of GE is its covariance with the market portfolio (its contribution to the variance of the market portfolio), this risk is measured in percent squared. Accordingly, the price of this risk, $[E(r_M) - r_f]/\sigma^2$, is defined as the percentage expected return per percent square of variance.

The ratio $\text{Cov}(r_{GE}, r_M)/\sigma_M^2$ measures the contribution of GE stock to the variance of the market portfolio as a fraction of the total variance of the market portfolio. The ratio is called **beta** and is denoted by β. Using this measure, we can restate Equation 9.7 as

$$E(r_{GE}) = r_f + \beta_{GE}[E(r_M) - r_f] \tag{9.8}$$

This **expected return–beta relationship** is the most familiar expression of the CAPM to practitioners. We will have a lot more to say about the expected return–beta relationship shortly.

We see now why the assumptions that made individuals act similarly are so useful. If everyone holds an identical risky portfolio, then everyone will find that the beta of each asset with the market portfolio equals the asset's beta with his or her own risky portfolio. Hence everyone will agree on the appropriate risk premium for each asset.

Does the fact that few real-life investors actually hold the market portfolio imply that the CAPM is of no practical importance? Not necessarily. Recall from Chapter 7 that reasonably well-diversified portfolios shed firm-specific risk and are left with mostly systematic or market risk. Even if one does not hold the precise market portfolio, a well-diversified portfolio will be so very highly correlated with the market that a stock's beta relative to the market will still be a useful risk measure.

In fact, several authors have shown that modified versions of the CAPM will hold true even if we consider differences among individuals leading them to hold different portfolios. For example, Brennan[7] examined the impact of differences in investors' personal tax rates on market equilibrium, and Mayers[8] looked at the impact of nontraded assets such as human capital (earning power). Both found that although the market portfolio is no longer each investor's optimal risky portfolio, the expected return–beta relationship should still hold in a somewhat modified form.

If the expected return–beta relationship holds for any individual asset, it must hold for any combination of assets. Suppose that some portfolio *P* has weight w_k for stock *k*, where *k* takes on values 1, . . . , *n*. Writing out the CAPM Equation 9.8 for each stock, and multiplying each equation by the weight of the stock in the portfolio, we obtain these equations, one for each stock:

$$\begin{array}{rcl}
w_1E(r_1) & = & w_1r_f + w_1\beta_1[E(r_M) - r_f] \\
+w_2E(r_2) & = & w_2r_f + w_2\beta_2[E(r_M) - r_f] \\
+ \quad \cdots & = & \cdots \\
\underline{+w_nE(r_n)} & \underline{=} & \underline{w_nr_f + w_n\beta_n[E(r_M) - r_f]} \\
E(r_P) & = & r_f + \beta_P[E(r_M) - r_f]
\end{array}$$

Summing each column shows that the CAPM holds for the overall portfolio because $E(r_P) = \sum_k w_k E(r_k)$ is the expected return on the portfolio, and $\beta_P = \sum_k w_k \beta_k$ is the portfolio beta. Incidentally, this result has to be true for the market portfolio itself,

$$E(r_M) = r_f + \beta_M[E(r_M) - r_f]$$

[7]Michael J. Brennan, "Taxes, Market Valuation, and Corporate Finance Policy," *National Tax Journal,* December 1973.

[8]David Mayers, "Nonmarketable Assets and Capital Market Equilibrium under Uncertainty," in *Studies in the Theory of Capital Markets,* ed. M. C. Jensen (New York: Praeger, 1972). We will look at this model more closely later in the chapter.

Indeed, this is a tautology because $\beta_M = 1$, as we can verify by noting that

$$\beta_M = \frac{\text{Cov}(r_M, r_M)}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_M^2}$$

This also establishes 1 as the weighted-average value of beta across all assets. If the market beta is 1, and the market is a portfolio of all assets in the economy, the weighted-average beta of all assets must be 1. Hence betas greater than 1 are considered aggressive in that investment in high-beta stocks entails above-average sensitivity to market swings. Betas below 1 can be described as defensive.

A word of caution: We are all accustomed to hearing that well-managed firms will provide high rates of return. We agree this is true if one measures the *firm's* return on investments in plant and equipment. The CAPM, however, predicts returns on investments in the *securities* of the firm.

Let us say that everyone knows a firm is well run. Its stock price will therefore be bid up, and consequently returns to stockholders who buy at those high prices will not be excessive. Security prices, in other words, already reflect public information about a firm's prospects; therefore only the risk of the company (as measured by beta in the context of the CAPM) should affect expected returns. In an efficient market investors receive high expected returns only if they are willing to bear risk.

Of course, investors do not directly observe or determine expected returns on securities. Rather, they observe security prices and bid those prices up or down. Expected rates of return are determined by the prices investors must pay compared to the cash flows those investments might garner.

CONCEPT CHECK 3

Suppose that the risk premium on the market portfolio is estimated at 8% with a standard deviation of 22%. What is the risk premium on a portfolio invested 25% in GM and 75% in Ford, if they have betas of 1.10 and 1.25, respectively?

The Security Market Line

We can view the expected return–beta relationship as a reward–risk equation. The beta of a security is the appropriate measure of its risk because beta is proportional to the risk that the security contributes to the optimal risky portfolio.

Risk-averse investors measure the risk of the optimal risky portfolio by its variance. In this world we would expect the reward, or the risk premium on individual assets, to depend on the *contribution* of the individual asset to the risk of the portfolio. The beta of a stock measures its contribution to the variance of the market portfolio. Hence we expect, for any asset or portfolio, the required risk premium to be a function of beta. The CAPM confirms this intuition, stating further that the security's risk premium is directly proportional to both the beta and the risk premium of the market portfolio; that is, the risk premium equals $\beta[E(r_M) - r_f]$.

The expected return–beta relationship can be portrayed graphically as the **security market line (SML)** in Figure 9.2. Because the market's beta is 1, the slope is the risk premium of the market portfolio. At the point on the horizontal axis where $\beta = 1$, we can read off the vertical axis the expected return on the market portfolio.

It is useful to compare the security market line to the capital market line. The CML graphs the risk premiums of *efficient* portfolios (i.e., portfolios composed of the market and the risk-free asset) as a function of portfolio standard deviation. This is appropriate because standard deviation is a valid measure of risk for efficiently diversified portfolios that are candidates for an investor's overall portfolio. The SML, in contrast, graphs *individual asset* risk premiums as a function of asset risk. The relevant measure of risk for individual assets held as parts of well-diversified portfolios is not the asset's standard deviation or variance; it is, instead, the contribution of the asset to the portfolio variance, which we measure by the asset's beta. The SML is valid for both efficient portfolios and individual assets.

The security market line provides a benchmark for the evaluation of investment performance. Given the risk of an investment, as measured by its beta, the SML provides the required rate of return necessary to compensate investors for both risk as well as the time value of money.

Because the security market line is the graphic representation of the expected return–beta relationship, "fairly priced" assets plot exactly on the SML; that is, their expected returns are commensurate with their risk. Given the assumptions we made at the start of this section, all securities must lie on the SML in market equilibrium. Nevertheless, we see here how the CAPM may be of use in the money-management industry. Suppose that the SML relation is used as a benchmark to assess the fair expected return on a risky asset. Then security analysis is performed to calculate the return actually expected. (Notice that we depart here from the simple CAPM world in that some investors now apply their own unique analysis to derive an "input list" that may differ from their competitors'.) If a stock is perceived to be a good buy, or underpriced, it will provide an expected return in excess of the fair return stipulated by the SML. Underpriced stocks therefore plot above the SML: Given their betas, their expected returns are greater than dictated by the CAPM. Overpriced stocks plot below the SML.

The difference between the fair and actually expected rates of return on a stock is called the stock's **alpha,** denoted by α. For example, if the market return is expected to be 14%, a stock has a beta of 1.2, and the T-bill rate is 6%, the SML would predict an expected return on the stock of $6 + 1.2(14 - 6) = 15.6\%$. If one believed the stock would provide an expected return of 17%, the implied alpha would be 1.4% (see Figure 9.3).

One might say that security analysis (which we treat in Part Five) is about uncovering securities with nonzero alphas. This analysis suggests that the starting point of

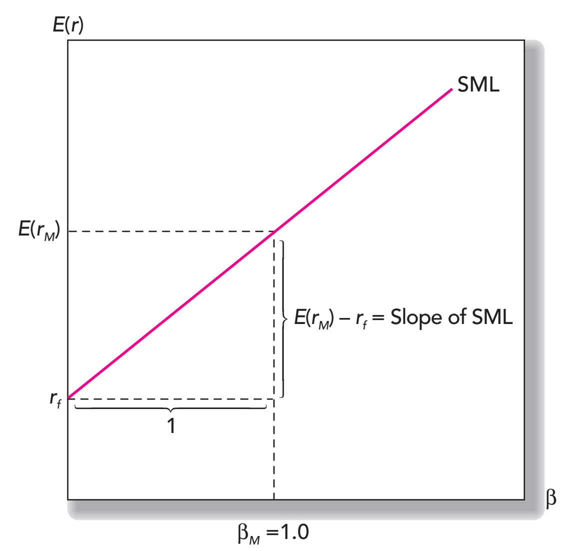


FIGURE 9.2 The security market line

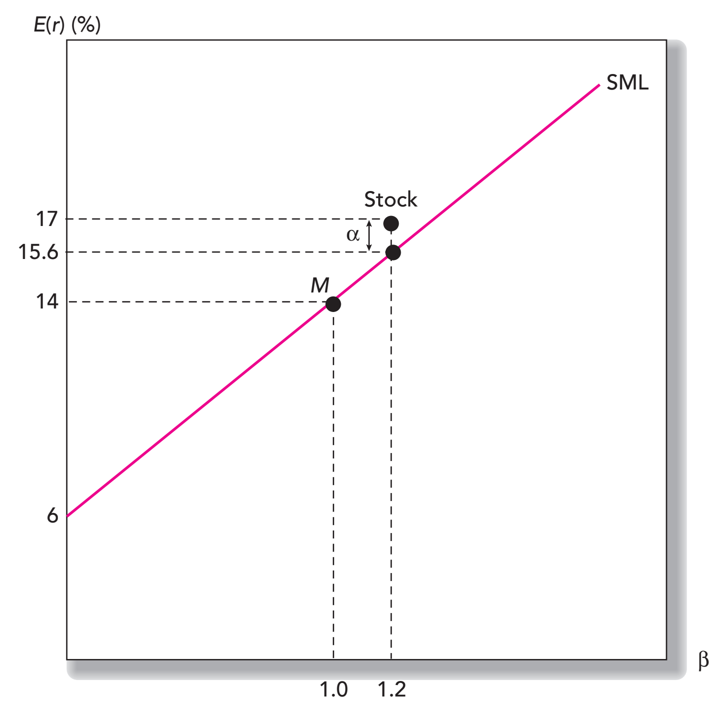


FIGURE 9.3 The SML and a positive-alpha stock

TALES FROM THE FAR SIDE

Financial markets' evaluation of risk determines the way firms invest. What if the markets are wrong?

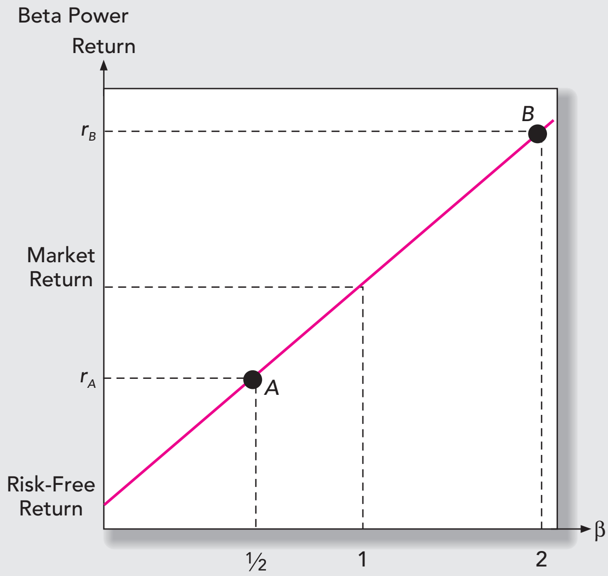
Investors are rarely praised for their good sense. But for the past two decades a growing number of firms have based their decisions on a model which assumes that people are perfectly rational. If they are irrational, are businesses making the wrong choices?

The model, known as the "capital-asset pricing model," or CAPM, has come to dominate modern finance. Almost any manager who wants to defend a project—be it a brand, a factory or a corporate merger—must justify his decision partly based on the CAPM. The reason is that the model tells a firm how to calculate the return that its investors demand. If shareholders are to benefit, the returns from any project must clear this "hurdle rate."

Although the CAPM is complicated, it can be reduced to five simple ideas:

- Investors can eliminate some risks—such as the risk that workers will strike, or that a firm's boss will quit—by diversifying across many regions and sectors.
- Some risks, such as that of a global recession, cannot be eliminated through diversification. So even a basket of all of the stocks in a stock market will still be risky.
- People must be rewarded for investing in such a risky basket by earning returns above those that they can get on safer assets, such as Treasury bills.
- The rewards on a specific investment depend only on the extent to which it affects the market basket's risk.
- Conveniently, that contribution to the market basket's risk can be captured by a single measure—dubbed "beta"—which expresses the relationship between the investment's risk and the market's.

Beta is what makes the CAPM so powerful. Although an investment may face many risks, diversified investors should care only about those that are related to the market basket. Beta not only tells managers how to measure those risks, but it also allows them to translate them directly into a hurdle rate. If the future profits from a project will not exceed that rate, it is not worth shareholders' money.



The diagram shows how the CAPM works. Safe investments, such as Treasury bills, have a beta of zero. Riskier investments should earn a premium over the risk-free rate which increases with beta. Those whose risks roughly match the market's have a beta of one, by definition, and should earn the market return.

So suppose that a firm is considering two projects, *A* and *B*. Project *A* has a beta of ½: when the market rises or falls by 10%, its returns tend to rise or fall by 5%. So its risk premium is only half that of the market. Project *B*'s risk premium is twice that of the

portfolio management can be a passive market-index portfolio. The portfolio manager will then increase the weights of securities with positive alphas and decrease the weights of securities with negative alphas. We showed one strategy for adjusting the portfolio weights in such a manner in Chapter 8.

The CAPM is also useful in capital budgeting decisions. For a firm considering a new project, the CAPM can provide the *required rate of return* that the project needs to yield, based on its beta, to be acceptable to investors. Managers can use the CAPM to obtain this cutoff internal rate of return (IRR), or "hurdle rate" for the project.

The nearby box describes how the CAPM can be used in capital budgeting. It also discusses some empirical anomalies concerning the model, which we address in detail in Chapters 11–13. The article asks whether the CAPM is useful for capital budgeting in light of these shortcomings; it concludes that even given the anomalies cited, the model still can be useful to managers who wish to increase the fundamental value of their firms.

market, so it must earn a higher return to justify the expenditure.

NEVER KNOWINGLY UNDERPRICED

But there is one small problem with the CAPM: Financial economists have found that beta is not much use for explaining rates of return on firms' shares. Worse, there appears to be another measure which explains these returns quite well.

That measure is the ratio of a firm's book value (the value of its assets at the time they entered the balance sheet) to its market value. Several studies have found that, on average, companies that have high book-to-market ratios tend to earn excess returns over long periods, even after adjusting for the risks that are associated with beta.

The discovery of this book-to-market effect has sparked a fierce debate among financial economists. All of them agree that some risks ought to carry greater rewards. But they are now deeply divided over how risk should be measured. Some argue that since investors are rational, the book-to-market effect must be capturing an extra risk factor. They conclude, therefore, that managers should incorporate the book-to-market effect into their hurdle rates. They have labeled this alternative hurdle rate the "new estimator of expected return," or NEER.

Other financial economists, however, dispute this approach. Since there is no obvious extra risk associated with a high book-to-market ratio, they say, investors must be mistaken. Put simply, they are underpricing high book-to-market stocks, causing them to earn abnormally high returns. If managers of such firms try to exceed those inflated hurdle rates, they will forgo many profitable investments. With economists now at odds, what is a conscientious manager to do?

Jeremy Stein, an economist at the Massachusetts Institute of Technology's business school, offers a paradoxical answer.* If investors are rational, then beta cannot be the only measure of risk, so managers should stop using it. Conversely, if investors are irrational, then beta is still the right measure in many cases. Mr. Stein argues that if beta captures an asset's fundamental risk—that is, its contribution to the market basket's risk—then it will often make sense for managers to pay attention to it, even if investors are somehow failing to.

Often, but not always. At the heart of Mr. Stein's argument lies a crucial distinction—that between (*a*) boosting a firm's long-term value and (*b*) trying to raise its share price. If investors are rational, these are the same thing: any decision that raises long-term value will instantly increase the share price as well. But if investors are making predictable mistakes, a manager must choose.

For instance, if he wants to increase today's share price—perhaps because he wants to sell his shares, or to fend off a takeover attempt—he must usually stick with the NEER approach, accommodating investors' misperceptions. But if he is interested in long-term value, he should usually continue to use beta. Showing a flair for marketing, Mr. Stein labels this far-sighted alternative to NEER the "fundamental asset risk"—or FAR—approach.

Mr. Stein's conclusions will no doubt irritate many company bosses, who are fond of denouncing their investors' myopia. They have resented the way in which CAPM—with its assumption of investor infallibility—has come to play an important role in boardroom decision-making. But it now appears that if they are right, and their investors are wrong, then those same far-sighted managers ought to be the CAPM's biggest fans.

*Jeremy Stein, "Rational Capital Budgeting in an Irrational World," *The Journal of Business,* October 1996.

Source: "Tales from the FAR Side," *The Economist Group,* Inc. November 16, 1996, p. 8.

EXAMPLE 9.1 Using the CAPM

Yet another use of the CAPM is in utility rate-making cases.[9] In this case the issue is the rate of return that a regulated utility should be allowed to earn on its investment in plant and equipment. Suppose that the equityholders have invested \$100 million in the firm and that the beta of the equity is .6. If the T-bill rate is 6% and the market risk premium is 8%, then the fair profits to the firm would be assessed as $6 + .6 \times 8 = 10.8\%$ of the \$100 million investment, or \$10.8 million. The firm would be allowed to set prices at a level expected to generate these profits.

[9]This application is fast disappearing, as many states are in the process of deregulating their public utilities and allowing a far greater degree of free market pricing. Nevertheless, a considerable amount of rate setting still takes place.

CONCEPT CHECK 4 and 5

Stock XYZ has an expected return of 12% and risk of $\beta = 1$. Stock ABC has expected return of 13% and $\beta = 1.5$. The market's expected return is 11%, and $r_f = 5\%$.

a. According to the CAPM, which stock is a better buy?

b. What is the alpha of each stock? Plot the SML and each stock's risk–return point on one graph. Show the alphas graphically.

The risk-free rate is 8% and the expected return on the market portfolio is 16%. A firm considers a project that is expected to have a beta of 1.3.

a. What is the required rate of return on the project?

b. If the expected IRR of the project is 19%, should it be accepted?

9.2 THE CAPM AND THE INDEX MODEL

Actual Returns versus Expected Returns

The CAPM is an elegant model. The question is whether it has real-world value—whether its implications are borne out by experience. Chapter 13 provides a range of empirical evidence on this point, but for now we focus briefly on a more basic issue: Is the CAPM testable even in principle?

For starters, one central prediction of the CAPM is that the market portfolio is a mean-variance efficient portfolio. Consider that the CAPM treats all traded risky assets. To test the efficiency of the CAPM market portfolio, we would need to construct a value-weighted portfolio of a huge size and test its efficiency. So far, this task has not been feasible. An even more difficult problem, however, is that the CAPM implies relationships among *expected* returns, whereas all we can observe are actual or realized holding-period returns, and these need not equal prior expectations. Even supposing we could construct a portfolio to represent the CAPM market portfolio satisfactorily, how would we test its mean-variance efficiency? We would have to show that the reward-to-volatility ratio of the market portfolio is higher than that of any other portfolio. However, this reward-to-volatility ratio is set in terms of expectations, and we have no way to observe these expectations directly.

The problem of measuring expectations haunts us as well when we try to establish the validity of the second central set of CAPM predictions, the expected return–beta relationship. This relationship is also defined in terms of expected returns $E(r_i)$ and $E(r_M)$:

$$E(r_i) = r_f + \beta_i[E(r_M) - r_f] \tag{9.9}$$

The upshot is that, as elegant and insightful as the CAPM is, we must make additional assumptions to make it implementable and testable.

The Index Model and Realized Returns

We have said that the CAPM is a statement about ex ante or expected returns, whereas in practice all anyone can observe directly are ex post or realized returns. To make the leap from expected to realized returns, we can employ the index model, which we will use in excess return form as

$$R_i = \alpha_i + \beta_i R_M + e_i \tag{9.10}$$

We saw in Chapter 8 how to apply standard regression analysis to estimate Equation 9.10 using observable realized returns over some sample period. Let us now see how this framework for statistically decomposing actual stock returns meshes with the CAPM.

We start by deriving the covariance between the returns on stock *i* and the market index. By definition, the firm-specific or nonsystematic component is independent of the market wide or systematic component, that is, $\text{Cov}(R_M, e_i) = 0$. From this relationship, it follows that the covariance of the excess rate of return on security *i* with that of the market index is

$$\begin{aligned}\text{Cov}(R_i, R_M) &= \text{Cov}(\beta_i R_M + e_i, R_M) \\ &= \beta_i \text{Cov}(R_M, R_M) + \text{Cov}(e_i, R_M) \\ &= \beta_i \sigma_M^2\end{aligned}$$

Note that we can drop α_i from the covariance terms because α_i is a constant and thus has zero covariance with all variables.

Because $\text{Cov}(R_i, R_M) = \beta_i \sigma_M^2$, the sensitivity coefficient, β_i, in Equation 9.10, which is the slope of the regression line representing the index model, equals

$$\beta_i = \frac{\text{Cov}(R_i, R_M)}{\sigma_M^2}$$

The index model beta coefficient turns out to be the same beta as that of the CAPM expected return–beta relationship, except that we replace the (theoretical) market portfolio of the CAPM with the well-specified and observable market index.

The Index Model and the Expected Return–Beta Relationship

Recall that the CAPM expected return–beta relationship is, for any asset *i* and the (theoretical) market portfolio,

$$E(r_i) - r_f = \beta_i [E(r_M) - r_f]$$

where $\beta_i = \text{Cov}(R_i, R_M)/\sigma_M^2$. This is a statement about the mean or expected excess returns of assets relative to the mean excess return of the (theoretical) market portfolio.

If the index *M* in Equation 9.10 represents the true market portfolio, we can take the expectation of each side of the equation to show that the index model specification is

$$E(r_i) - r_f = \alpha_i + \beta_i [E(r_M) - r_f]$$

A comparison of the index model relationship to the CAPM expected return–beta relationship (Equation 9.9) shows that the CAPM predicts that α_i should be zero for all assets. The alpha of a stock is its expected return in excess of (or below) the fair expected return as predicted by the CAPM. If the stock is fairly priced, its alpha must be zero.

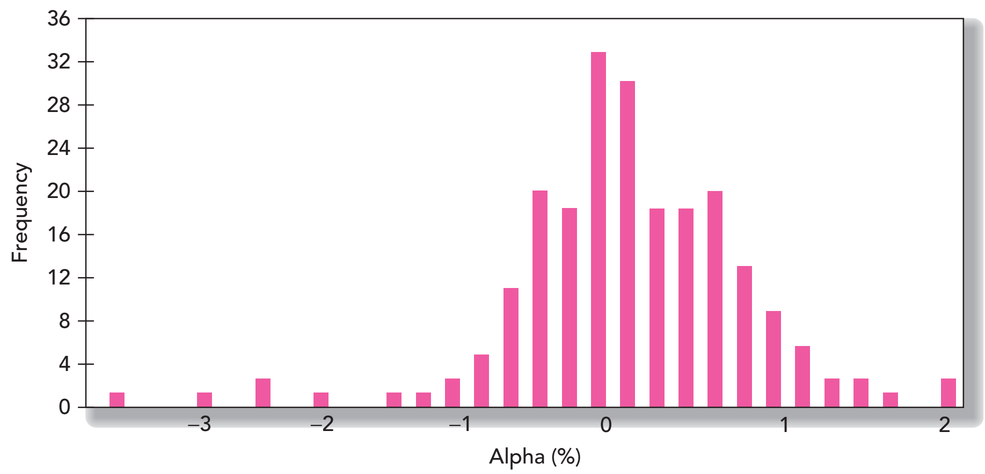


FIGURE 9.4 Estimates of individual mutual fund alphas, 1972–1991

This is a plot of the frequency distribution of estimated alphas for all-equity mutual funds with 10-year continuous records.

Source: Burton G. Malkiel, "Returns from Investing in Equity Mutual Funds 1971–1991," *Journal of Finance* 50 (June 1995), pp. 549–72. Reprinted by permission of the publisher, Blackwell Publishing, Inc.

We emphasize again that this is a statement about *expected* returns on a security. After the fact, of course, some securities will do better or worse than expected and will have returns higher or lower than predicted by the CAPM; that is, they will exhibit positive or negative alphas over a sample period. But this superior or inferior performance could not have been forecast in advance.

Therefore, if we estimate the index model for several firms, using Equation 9.10 as a regression equation, we should find that the ex post or realized alphas (the regression intercepts) for the firms in our sample center around zero. If the initial expectation for alpha were zero, as many firms would be expected to have a positive as a negative alpha for some sample period. The CAPM states that the *expected* value of alpha is zero for all securities, whereas the index model representation of the CAPM holds that the *realized* value of alpha should average out to zero for a sample of historical observed returns. Just as important, the sample alphas should be unpredictable, that is, independent from one sample period to the next.

Indirect evidence on the efficiency of the market portfolio can be found in a study by Burton Malkiel,[10] who estimates alpha values for a large sample of equity mutual funds. The results, which appear in Figure 9.4, show that the distribution of alphas is roughly bell shaped, with a mean that is slightly negative but statistically indistinguishable from zero. On average, it does not appear that mutual funds outperform the market index (the S&P 500) on a risk-adjusted basis.[11]

[10]Burton G. Malkiel, "Returns from Investing in Equity Mutual Funds 1971–1991," *Journal of Finance* 50 (June 1995), pp. 549–72.

[11]Notice that the study included all mutual funds with at least 10 years of continuous data. This suggests the average alpha from this sample would be upward biased because funds that failed after less than 10 years were ignored and omitted from the left tail of the distribution. This *survivorship bias* makes the finding that the average fund underperformed the index even more telling. We discuss survivorship bias further in Chapter 11.

This result is quite meaningful. While we might expect realized alpha values of individual securities to center around zero, professionally managed mutual funds might be expected to demonstrate average positive alphas. Funds with superior performance (and we do expect this set to be non-empty) should tilt the sample average to a positive value. The small impact of superior funds on this distribution suggests the difficulty in beating the passive strategy that the CAPM deems to be optimal.

There is yet another applicable variation on the intuition of the index model, the **market model.** Formally, the market model states that the return "surprise" of any security is proportional to the return surprise of the market, plus a firm-specific surprise:

$$r_i - E(r_i) = \beta_i[r_M - E(r_M)] + e_i$$

This equation divides returns into firm-specific and systematic components somewhat differently from the index model. If the CAPM is valid, however, you can confirm that, substituting for $E(r_i)$ from Equation 9.9, the market model equation becomes identical to the index model. For this reason the terms "index model" and "market model" often are used interchangeably.

CONCEPT CHECK 6

Can you sort out the nuances of the following maze of models?

a. CAPM
b. Single-factor model
c. Single-index model
d. Market model

9.3 IS THE CAPM PRACTICAL?

To discuss the role of the CAPM in real-life investments we have to answer two questions. First, even if we all agreed that the CAPM were the best available theoretical model to explain rates of return on risky assets, how would this affect practical investment policy? Second, how can we determine whether the CAPM is in fact the best available model to explain rates of return on risky assets?

Notice the wording of the first question. We don't pose it as: "Suppose the CAPM perfectly explains the rates of return on risky assets. . . ." All models, whether in economics or science, are based on simplifications that enable us to come to grips with a complicated reality, which means that perfection is an unreasonable and unusable standard. In our context, we must clarify what "perfectly explains" would mean. From the previous section we know that if the CAPM were valid, a single-index model in which the index includes all traded securities (i.e., all risky securities in the investable universe as in Assumption 3) also would be valid. In this case, "perfectly explains" would mean that all alpha values in security risk premiums would be identically zero.

The notion that all alpha values can be identically zero is feasible in principle, but such a configuration cannot be expected to emerge in real markets. This was demonstrated by Grossman and Stiglitz, who showed that such an equilibrium may be one that the real economy can approach, but not necessarily reach.[12] Their basic idea is that the

[12]Sanford J. Grossman and Joseph E. Stiglitz, "On the Impossibility of Informationally Efficient Markets," *American Economic Review* 70 (June 1981).

actions of security analysts are the forces that drive security prices to "proper" levels at which alpha is zero. But if all alphas were identically zero, there would be no incentive to engage in such security analysis. Instead, the market equilibrium will be characterized by prices hovering "near" their proper values, at which alphas are almost zero, but with enough slippage (and therefore reward for superior insight) to induce analysts to continue their efforts.

A more reasonable standard, that the CAPM is the "best available model to explain rates of return on risky assets," means that in the absence of security analysis, one should take security alphas as zero. A security is mispriced if and only if its alpha is nonzero—underpriced if alpha is positive and overpriced if alpha is negative—and positive or negative alphas are revealed only by superior security analysis. Absent the investment of significant resources in such analysis, an investor would obtain the best investment portfolio on the assumption that all alpha values are zero. This definition of the superiority of the CAPM over any other model also determines its role in real-life investments.

Under the assumption that the CAPM is the best available model, investors willing to expend resources to construct a superior portfolio must (1) identify a practical index to work with and (2) deploy macro analysis to obtain good forecasts for the index and security analysis to identify mispriced securities. This procedure was described in Chapter 8 and is further elaborated on in Part Five (Security Analysis) and Part Seven (Applied Portfolio Management).

We will examine several tests of the CAPM in Chapter 13. But it is important to explain the results of these tests and their implications.

Is the CAPM Testable?

Let us consider for a moment what testability means. A model consists of (i) a set of assumptions, (ii) logical/mathematical development of the model through manipulation of those assumptions, and (iii) a set of predictions. Assuming the logical/mathematical manipulations are free of errors, we can test a model in two ways, *normative* and *positive.* Normative tests examine the assumptions of the model, while positive tests examine the predictions.

If a model's assumptions are valid, and the development is error-free, then the predictions of the model must be true. In this case, testing the assumptions is synonymous with testing the model. But few, if any, models can pass the normative test. In most cases, as with the CAPM, the assumptions are admittedly invalid—we recognize that we have simplified reality, and therefore to this extent are relying on "untrue" assumptions. The motivation for invoking unrealistic assumptions is clear; we simply cannot solve a model that is perfectly consistent with the full complexity of real-life markets. As we've noted, the need to use simplifying assumptions is not peculiar to economics—it characterizes all of science.

Assumptions are chosen first and foremost to render the model solvable. But we prefer assumptions to which the model is "robust." A model is robust with respect to an assumption if its predictions are not highly sensitive to violation of the assumption. If we use only assumptions to which the model is robust, the model's predictions will be reasonably accurate despite its shortcomings. The upshot of all this is that tests of models are almost always positive—we judge a model on the success of its empirical predictions. This standard brings statistics into any science and requires us to take a stand on what are acceptable levels of significance and power.[13] Because the nonrealism of the assumptions precludes a normative test, the positive test is really a test of the robustness of the model to its assumptions.

The CAPM implications are embedded in two predictions: (1) the market portfolio is efficient, and (2) the security market line (the expected return–beta relationship) accurately describes the risk–return trade-off, that is, alpha values are zero. In fact, the second implication can be derived from the first, and therefore both stand or fall together in a test that the market portfolio is mean-variance efficient. The central problem in testing this prediction is that the hypothesized market portfolio is unobservable. The "market portfolio" includes *all* risky assets that can be held by investors. This is far more extensive than an equity index. It would include bonds, real estate, foreign assets, privately held businesses, and human capital. These assets are often traded thinly or (for example, in the case of human capital) not traded at all. It is difficult to test the efficiency of an observable portfolio, let alone an unobservable one. These problems alone make adequate testing of the model infeasible.[14] Moreover, even small departures from efficiency in the market portfolio can lead to large departures from the expected return–beta relationship of the SML, which would negate the practical usefulness of the model.

The CAPM Fails Empirical Tests

Because the market portfolio cannot be observed, tests of the CAPM revolve around the expected return–beta relationship. The tests use proxies such as the S&P 500 index to stand in for the true market portfolio. These tests therefore appeal to robustness of the assumption that the market proxy is sufficiently close to the true, unobservable market portfolio. The CAPM fails these tests, that is, the data reject the hypothesis that alpha values are uniformly zero at acceptable levels of significance. For example, we find that, on average, low-beta securities have positive alphas and high-beta securities have negative alphas.

It is possible that this is a result of a failure of our data, the validity of the market proxy, or statistical method. If so, we would conclude the following: There is no better model out there, but we measure beta and alpha values with unsatisfactory precision. This situation

[13] To illustrate the meanings of significance and power, consider a test of the efficacy of a new drug. The agency testing the drug may make two possible errors. The drug may be useless (or even harmful), but the agency may conclude that it is useful. This is called a "Type I" error. The *significance level* of a test is the probability of a Type I error. Typical practice is to fix the level of significance at some low level, for example, 5%. In the case of drug testing, for example, the first goal is to avoid introducing ineffective or harmful treatments. The other possible error is that the drug is actually useful, but the testing procedure concludes it is not. This mistake, called "Type II" error, would lead us to discard a useful treatment. The *power* of the test is the probability of avoiding Type II error (i.e., one minus the probability of making such an error), that is, the probability of accepting the drug if it is indeed useful. We want tests that, at a given level of significance, have the most power, so we will admit effective drugs with high probability. In social sciences in particular, available tests often have low power, in which case they are susceptible to Type II error and will reject a correct model (a "useful drug") with high frequency. "The drug is useful" is analogous in the CAPM to alphas being zero. When the test data reject the hypothesis that observed alphas are zero at the desired level of significance, the CAPM fails. However, if the test has low power, the probability that we accept the model when true is not all that high.

[14] The best-known discussion of the difficulty in testing the CAPM is now called "Roll's critique." See Richard Roll, "A Critique of the Asset Pricing Theory's Tests: Part I: On Past and Potential Testability of the Theory," *Journal of Financial Economics* 4 (1977). The issue is developed further in Richard Roll and Stephen A. Ross, "On the Cross-Sectional Relation between Expected Return and Betas," *Journal of Finance* 50 (1995); and Schmuel Kandel and Robert F. Stambaugh, "Portfolio Inefficiency and the Cross-Section of Expected Returns," *Journal of Finance* 50 (1995).

would call for improved technique. But if the rejection of the model is not an artifact of statistical problems, then we must search for extensions to the CAPM, or substitute models. We will consider several extensions of the model later in the chapter.

The Economy and the Validity of the CAPM

For better or worse, some industries are regulated, with rate commissions either setting or approving prices. Imagine a commission pondering a rate case for a regulated utility. The rate commission must decide whether the rates charged by the company are sufficient to grant shareholders a fair rate of return on their investments. The normative framework of the typical rate hearing is that shareholders, who have made an investment in the firm, are entitled to earn a "fair" rate of return on their equity investment. The firm is therefore allowed to charge prices that are expected to generate a profit consistent with that fair rate of return.

The question of fairness of the rate of return to the company shareholders cannot be divorced from the level of risk of these returns. The CAPM provides the commission a clear criterion: If the rates under current regulation are too low, then the rate of return to equity investors would be less than commensurate with risk, and alpha would be negative. As we pointed out in Example 9.1, the commissioner's problem may now be organized around arguments about estimates of risk and the security market line.

Similar applications arise in many legal settings. For example, contracts with payoffs that are contingent on a fair rate of return can be based on the index rate of return and the beta of appropriate assets. Many disputes involving damages require that a stream of losses be discounted to a present value. The proper discount rate depends on risk, and disputes about fair compensation to litigants can be (and often are) set on the basis of the SML, using past data that differentiate systematic from firm-specific risk.

It may be surprising to find that the CAPM is an accepted norm in the U.S. and many other developed countries, despite its empirical shortcomings. We can offer a twofold explanation. First, the logic of the decomposition to systematic and firm-specific risk is compelling. Absent a better model to assess nonmarket components of risk premiums, we must use the best method available. As improved methods of generating equilibrium security returns become empirically validated, they gradually will be incorporated into institutional decision making. Such improvements may come either from extensions of the CAPM and its companion, arbitrage pricing theory (discussed in the next chapter), or from a yet- undiscovered new model.

Second, there is impressive, albeit less-formal, evidence that the central conclusion of the CAPM—the efficiency of the market portfolio—may not be all that far from being valid. Thousands of mutual funds within hundreds of investment companies compete for investor money. These mutual funds employ professional analysts and portfolio managers and expend considerable resources to construct superior portfolios. But the number of funds that consistently outperform a simple strategy of investing in passive market index portfolios is extremely small, suggesting that the single-index model with ex ante zero alpha values may be a reasonable working approximation for most investors.

The Investments Industry and the Validity of the CAPM

More than other practitioners, investment firms must take a stand on the validity of the CAPM. If they judge the CAPM invalid, they must turn to a substitute framework to guide them in constructing optimal portfolios.

For example, the CAPM provides discount rates that help security analysts assess the intrinsic value of a firm. If an analyst believes that some actual prices differ from intrinsic values, then those securities have nonzero alphas, and there is an opportunity to construct an active portfolio with a superior risk–return profile. But if the discount rate used to assess intrinsic value is incorrect because of a failure in the CAPM, the estimate of alpha will be biased, and both the Markowitz model of Chapter 7 and the index model of Chapter 8 will actually lead to inferior portfolios. When constructing their presumed optimal risky portfolios, practitioners must be satisfied that the passive index they use for that purpose is satisfactory and that the ratios of alpha to residual variance are appropriate measures of investment attractiveness. This would not be the case if the CAPM is invalid. Yet it appears many practitioners do use index models (albeit often with additional indexes) when assessing security prices. The curriculum of the CFA Institute also suggests a widespread acceptance of the CAPM, at least as a starting point for thinking about the risk–return relationship. An explanation similar to the one we offered in the previous subsection is equally valid here.

The central conclusion from our discussion so far is that, explicitly or implicitly, practitioners do use a CAPM. If they use a single-index model and derive optimal portfolios from ratios of alpha forecasts to residual variance, they behave as if the CAPM is valid.[15] If they use a multi-index model, then they use one of the extensions of the CAPM (discussed later in this chapter) or arbitrage pricing theory (discussed in the next chapter). Thus, theory and evidence on the CAPM should be of interest to all sophisticated practitioners.

9.4 ECONOMETRICS AND THE EXPECTED RETURN–BETA RELATIONSHIP

When assessing the empirical success of the CAPM, we must also consider our econometric technique. If our tests are poorly designed, we may mistakenly reject the model. Similarly, some empirical tests implicitly introduce additional assumptions that are not part of the CAPM, for example, that various parameters of the model such as beta or residual variance are constant over time. If these extraneous additional assumptions are too restrictive, we also may mistakenly reject the model.

To begin, notice that all the coefficients of a regression equation are estimated simultaneously, and these estimates are not independent. In particular, the estimate of the intercept (alpha) of a single- (independent) variable regression depends on the estimate of the slope coefficient. Hence, if the beta estimate is inefficient and/or biased, so will be the estimate of the intercept. Unfortunately, statistical bias is easily introduced.

An example of this hazard was pointed out in an early paper by Miller and Scholes,[16] who demonstrated how econometric problems could lead one to reject the CAPM even if it were perfectly valid. They considered a checklist of difficulties encountered in testing the model and showed how these problems potentially could bias conclusions. To prove the point, they simulated rates of return that were *constructed* to satisfy the predictions of the CAPM and used these rates to “test” the model with standard statistical techniques of the day. The result of these tests was a rejection of the model that looks surprisingly similar to what we find in tests of returns from actual data—this despite the fact that the “data” were constructed to satisfy the CAPM. Miller and Scholes thus demonstrated that econometric technique alone could be responsible for the rejection of the model in actual tests.

[15]We need to be a bit careful here. On its face, the CAPM asserts that alpha values will equal zero in security market equilibrium. But as we argued earlier, consistent with the vast amount of security analysis that actually takes place, a better way to interpret the CAPM is that equilibrium really means that alphas should be taken to be zero in the absence of security analysis. With private information or superior insight one presumably would be able to identify stocks that are mispriced by the market and thus offer nonzero alphas.

[16]Merton H. Miller and Myron Scholes, “Rates of Return in Relations to Risk: A Re-examination of Some Recent Findings,” in *Studies in the Theory of Capital Markets,* Michael C. Jensen, ed. (New York: Praeger, 1972).

There are several potential problems with the estimation of beta coefficients. First, when residuals are correlated (as is common for firms in the same industry), standard beta estimates are not efficient. A simple approach to this problem would be to use statistical techniques designed for these complications. For example, we might replace OLS (ordinary least squares) regressions with GLS (generalized least squares) regressions, which account for correlation across residuals. Moreover, both coefficients, alpha and beta, as well as residual variance, are likely time varying. There is nothing in the CAPM that precludes such time variation, but standard regression techniques rule it out and thus may lead to false rejection of the model. There are now well-known techniques to account for time-varying parameters. In fact, Robert Engle won the Nobel Prize for his pioneering work on econometric techniques to deal with time-varying volatility, and a good portion of the applications of these new techniques have been in finance.[17] Moreover, betas may vary not purely randomly over time, but in response to changing economic conditions. A "conditional" CAPM allows risk and return to change with a set of "conditioning variables."[18]

As importantly, Campbell and Vuolteenaho[19] find that the beta of a security can be decomposed into two components, one of which measures sensitivity to changes in corporate profitability and another which measures sensitivity to changes in the market's discount rates. These are found to be quite different in many cases. Improved econometric techniques such as those proposed in this short survey may help resolve part of the empirical failure of the simple CAPM.

9.5 EXTENSIONS OF THE CAPM

The CAPM uses a number of simplifying assumptions. We can gain greater predictive accuracy at the expense of greater complexity by relaxing some of those assumptions. In this section, we will consider a few of the more important attempts to extend the model. This discussion is not meant to be exhaustive. Rather, it introduces a few extensions of the basic model to provide insight into the various attempts to improve empirical content.

The Zero-Beta Model

Efficient frontier portfolios have a number of interesting characteristics, independently derived by Merton and Roll.[20] Three of these are

1. Any portfolio that is a combination of two frontier portfolios is itself on the efficient frontier.

[17]Engle's work gave rise to the widespread use of so-called ARCH models. ARCH stands for autoregressive conditional heteroskedasticity, which is a fancy way of saying that volatility changes over time, and that recent levels of volatility can be used to form optimal estimates of future volatility.

[18]There is now a large literature on conditional models of security market equilibrium. Much of it derives from Ravi Jagannathan and Zhenyu Wang, "The Conditional CAPM and the Cross-Section of Expected Returns," *Journal of Finance* 51 (March 1996), vol pp. 3–53.

[19]John Campbell and Tuomo Vuolteenaho, "Bad Beta, Good Beta," *American Economic Review* 94 (December 2004), pp. 1249–75.

[20]Robert C. Merton, "An Analytic Derivation of the Efficient Portfolio Frontier," *Journal of Financial and Quantitative Analysis,* 1972. Roll, see footnote 14.

2. The expected return of any asset can be expressed as an exact linear function of the expected return on any two efficient-frontier portfolios P and Q according to the following equation:

$$E(r_i) - E(r_Q) = [E(r_P) - E(r_Q)]\frac{\text{Cov}(r_i, r_P) - \text{Cov}(r_P, r_Q)}{\sigma_P^2 - \text{Cov}(r_P, r_Q)} \tag{9.11}$$

3. Every portfolio on the efficient frontier, except for the global minimum-variance portfolio, has a "companion" portfolio on the bottom (inefficient) half of the frontier with which it is uncorrelated. Because it is uncorrelated, the companion portfolio is referred to as the **zero-beta portfolio** of the efficient portfolio. If we choose the market portfolio M and its zero-beta companion portfolio Z, then Equation 9.11 simplifies to the CAPM-like equation

$$E(r_i) - E(r_Z) = [E(R_M) - E(R_Z)]\frac{\text{Cov}(r_i, r_M)}{\sigma_M^2} = \beta_i[E(r_M) - E(r_Z)] \tag{9.12}$$

   Equation 9.12 resembles the SML of the CAPM, except that the risk-free rate is replaced with the expected return on the zero-beta companion of the market index portfolio.

Fischer Black used these properties to show that Equation 9.12 is the CAPM equation that results when investors face restrictions on borrowing and/or investment in the risk-free asset.[21] In this case, at least some investors will choose portfolios on the efficient frontier that are not necessarily the market index portfolio. Because average returns on the zero-beta portfolio are greater than observed T-bill rates, the zero-beta model can explain why average estimates of alpha values are positive for low-beta securities and negative for high-beta securities, contrary to the prediction of the CAPM. Despite this, the model is not sufficient to rescue the CAPM from empirical rejection.

Labor Income and Nontraded Assets

An important departure from realism is the CAPM assumption that all risky assets are traded. Two important asset classes that are *not* traded are human capital and privately held businesses. The discounted value of future labor income exceeds the total market value of traded assets. The market value of privately held corporations and businesses is of the same order of magnitude. Human capital and private enterprises are different types of assets with possibly different implications for equilibrium returns on traded securities.

Privately held business may be the lesser of the two sources of departures from the CAPM. Nontraded firms can be incorporated or sold at will, save for liquidity considerations that we discuss in the next section. Owners of private business also can borrow against their value, further diminishing the material difference between ownership of private and public business. Suppose that privately held business have similar risk characteristics as those of traded assets. In this case, individuals can partially offset the diversification problems posed by their nontraded entrepreneurial assets by reducing their portfolio demand for securities of similar, traded assets. Thus, the CAPM expected return–beta equation may not be greatly disrupted by the presence of entrepreneurial income.

To the extent that risk characteristics of private enterprises differ from those of traded securities, a portfolio of traded assets that best hedges the risk of typical private business

[21]Fischer Black, "Capital Market Equilibrium with Restricted Borrowing," *Journal of Business*, July 1972.

would enjoy excess demand from the population of private business owners. The price of assets in this portfolio will be bid up relative to the CAPM considerations, and the expected returns on these securities will be lower in relation to their systematic risk. Conversely, securities highly correlated with such risk will have high equilibrium risk premiums and may appear to exhibit positive alphas relative to the conventional SML. In fact, Heaton and Lucas show that adding proprietary income to a standard asset-pricing model improves its predictive performance.[22]

The size of labor income and its special nature is of greater concern for the validity of the CAPM. The possible effect of labor income on equilibrium returns can be appreciated from its important effect on personal portfolio choice. Despite the fact that an individual can borrow against labor income (via a home mortgage) and reduce some of the uncertainty about future labor income via life insurance, human capital is less "portable" across time and may be more difficult to hedge using traded securities than nontraded business. This may induce pressure on security prices and result in departures from the CAPM expected return–beta equation. For one example, surely an individual seeking diversification should avoid investing in his employer's stock and limit investments in the same industry. Thus, the demand for stocks of labor-intensive firms may be reduced, and these stocks may require a higher expected return than predicted by the CAPM.

Mayers[23] derives the equilibrium expected return–beta equation for an economy in which individuals are endowed with labor income of varying size relative to their nonlabor capital. The resultant SML equation is

$$E(R_i) = E(R_M)\frac{\text{Cov}(R_i, R_M) + \frac{P_H}{P_M}\text{Cov}(R_i, R_H)}{\sigma_M^2 + \frac{P_H}{P_M}\text{Cov}(R_M, R_H)} \tag{9.13}$$

where

P_H = value of aggregate human capital,

P_M = market value of traded assets (market portfolio),

R_H = excess rate of return on aggregate human capital.

The CAPM measure of systematic risk, beta, is replaced in the extended model by an adjusted beta that also accounts for covariance with the portfolio of aggregate human capital. Notice that the ratio of human capital to market value of all traded assets, P_H/P_M, may well be greater than 1, and hence the effect of the covariance of a security with labor income, $\text{Cov}(R_i, R_H)$, relative to the average, $\text{Cov}(R_M, R_H)$, is likely to be economically significant. When $\text{Cov}(R_i, R_H)$ is positive, the adjusted beta is greater when the CAPM beta is smaller than 1, and vice versa. Because we expect $\text{Cov}(R_i, R_H)$ to be positive for the average security, the risk premium in this model will be greater, on average, than predicted by the CAPM for securities with beta less than 1, and smaller for securities with beta greater than 1. The model thus predicts a security market line that is less steep than that of the standard CAPM. This may help explain the average negative alpha of high-beta securities and positive alpha of low-beta securities that lead to the statistical failure of the CAPM equation. In Chapter 13 on empirical evidence we present additional results along these lines.

[22]John Heaton and Deborah Lucas, "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk, *Journal of Finance* 55 (June 2000). This paper offers evidence of the effect of entrepreneurial risk on both portfolio choice and the risk–return relationship.

[23]See footnote 8.

A Multiperiod Model and Hedge Portfolios

Robert C. Merton revolutionized financial economics by using continuous-time models to extend many of our models of asset pricing.[24] While his (Nobel Prize–winning) contributions to option-pricing theory and financial engineering (along with those of Fischer Black and Myron Scholes) may have had greater impact on the investment industry, his solo contribution to portfolio theory was equally important for our understanding of the risk–return relationship.

In his basic model, Merton relaxes the "single-period" myopic assumptions about investors. He envisions individuals who optimize a lifetime consumption/investment plan, and who continually adapt consumption/investment decisions to current wealth and planned retirement age. When uncertainty about portfolio returns is the only source of risk and investment opportunities remain unchanged through time, that is, there is no change in the probability distribution of the return on the market portfolio or individual securities, Merton's so-called intertemporal capital asset pricing model (ICAPM) predicts the same expected return–beta relationship as the single-period equation.[25]

But the situation changes when we include additional sources of risk. These extra risks are of two general kinds. One concerns changes in the parameters describing investment opportunities, such as future risk-free rates, expected returns, or the risk of the market portfolio. For example, suppose that the real interest rate may change over time. If it falls in some future period, one's level of wealth will now support a lower stream of real consumption. Future spending plans, for example, for retirement spending, may be put in jeopardy. To the extent that returns on some securities are correlated with changes in the risk-free rate, a portfolio can be formed to hedge such risk, and investors will bid up the price (and bid down the expected return) of those hedge assets. Investors will sacrifice some expected return if they can find assets whose returns will be higher when other parameters (in this case, the risk-free rate) change adversely.

The other additional source of risk concerns the prices of the consumption goods that can be purchased with any amount of wealth. Consider as an example inflation risk. In addition to the expected level and volatility of their nominal wealth, investors must be concerned about the cost of living—what those dollars can buy. Therefore, inflation risk is an important extramarket source of risk, and investors may be willing to sacrifice some expected return to purchase securities whose returns will be higher when the cost of living changes adversely. If so, hedging demands for securities that help to protect against inflation risk would affect portfolio choice and thus expected return. One can push this conclusion even further, arguing that empirically significant hedging demands may arise for important subsectors of consumer expenditures; for example, investors may bid up share prices of energy companies that will hedge energy price uncertainty. These sorts of effects may characterize any assets that hedge important extramarket sources of risk.

More generally, suppose we can identify K sources of extramarket risk and find K associated hedge portfolios. Then, Merton's ICAPM expected return–beta equation would generalize the SML to a multi-index version:

$$E(R_i) = \beta_{iM} E(R_M) + \sum_{k=1}^{K} \beta_{ik} E(R_k) \qquad \textbf{(9.14)}$$

where β_{iM} is the familiar security beta on the market-index portfolio, and β_{ik} is the beta on the kth hedge portfolio.

[24]Merton's classic works are collected in *Continuous-Time Finance* (Oxford, U.K.: Basil Blackwell, 1992).

[25]Eugene F. Fama also made this point in "Multiperiod Consumption-Investment Decisions," *American Economic Review* 60 (1970).

Other multifactor models using additional factors that do not arise from extramarket sources of risk have been developed and lead to SMLs of a form identical to that of the ICAPM. These models also may be considered extensions of the CAPM in the broad sense. We examine these models in the next chapter.

A Consumption-Based CAPM

The logic of the CAPM together with the hedging demands noted in the previous subsection suggests that it might be useful to center the model directly on consumption. Such models were first proposed by Mark Rubinstein, Robert Lucas, and Douglas Breeden.[26]

In a lifetime consumption plan, the investor must in each period balance the allocation of current wealth between today's consumption and the savings and investment that will support future consumption. When optimized, the utility value from an additional dollar of consumption today must be equal to the utility value of the expected future consumption that can be financed by that additional dollar of wealth.[27] Future wealth will grow from labor income, as well as returns on that dollar when invested in the optimal complete portfolio.

Suppose risky assets are available and you wish to increase expected consumption growth by allocating some of your savings to a risky portfolio. How would we measure the risk of these assets? As a general rule, investors will value additional income more highly during difficult economic times (when consumption opportunities are scarce) than in affluent times (when consumption is already abundant). An asset will therefore be viewed as riskier in terms of consumption if it has positive covariance with consumption growth—in other words, if its payoff is higher when consumption is already high and lower when consumption is relatively restricted. Therefore, equilibrium risk premiums will be greater for assets that exhibit higher covariance with consumption growth. Developing this insight, we can write the risk premium on an asset as a function of its "consumption risk" as follows:

$$E(R_i) = \beta_{iC}\mathrm{RP}_C \tag{9.15}$$

where portfolio C may be interpreted as a *consumption-tracking portfolio* (also called a *consumption-mimicking portfolio*), that is, the portfolio with the highest correlation with consumption growth; β_{iC} is the slope coefficient in the regression of asset i's excess returns, R_i, on those of the consumption-tracking portfolio; and, finally, RP_C is the risk premium associated with consumption uncertainty, which is measured by the expected excess return on the consumption-tracking portfolio:

$$\mathrm{RP}_C = E(R_C) = E(r_C) - r_f \tag{9.16}$$

Notice how similar this conclusion is to the conventional CAPM. The consumption-tracking portfolio in the CCAPM plays the role of the market portfolio in the conventional CAPM. This is in accord with its focus on the risk of *consumption* opportunities rather than the risk and return of the *dollar* value of the portfolio. The excess return on the

[26]Mark Rubinstein, "The Valuation of Uncertain Income Streams and the Pricing of Options," *Bell Journal of Economics and Management Science* 7 (1976), pp. 407–25; Robert Lucas, "Asset Prices in an Exchange Economy," *Econometrica* 46 (1978), pp. 1429–45; Douglas Breeden, "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics* 7 (1979), pp. 265–96.

[27]Wealth at each point in time equals the market value of assets in the balance sheet plus the present value of future labor income. These models of consumption and investment decisions are often made tractable by assuming investors exhibit constant relative risk aversion, or CRRA. CRRA implies that an individual invests a constant proportion of wealth in the optimal risky portfolio regardless of the level of wealth. You might recall that our prescription for optimal capital allocation in Chapter 6 also called for an optimal investment proportion in the risky portfolio regardless of the level of wealth. The utility function we employed there also exhibited CRRA.

consumption-tracking portfolio plays the role of the excess return on the market portfolio, *M*. Both approaches result in linear, single-factor models that differ mainly in the identity of the factor they use.

In contrast to the CAPM, the beta of the market portfolio on the market factor of the CCAPM is not necessarily 1. It is perfectly plausible and empirically evident that this beta is substantially greater than 1. This means that in the linear relationship between the market index risk premium and that of the consumption portfolio,

$$E(R_M) = \alpha_M + \beta_{MC}E(R_C) + \varepsilon_M \tag{9.17}$$

where α_M and ε_M allow for empirical deviation from the exact model in Equation 9.15, and β_{MC} is not necessarily equal to 1.

Because the CCAPM is so similar to the CAPM, one might wonder about its usefulness. Indeed, just as the CAPM is empirically flawed because not all assets are traded, so is the CCAPM. The attractiveness of this model is in that it compactly incorporates consumption hedging and possible changes in investment opportunities, that is, in the parameters of the return distributions in a single-factor framework. There is a price to pay for this compactness, however. Consumption growth figures are published infrequently (monthly at the most) compared with financial assets, and are measured with significant error. Nevertheless, recent empirical research[28] indicates that this model is more successful in explaining realized returns than the CAPM, which is a reason why students of investments should be familiar with it. We return to this issue, as well as empirical evidence concerning the CCAPM, in Chapter 13.

9.6 LIQUIDITY AND THE CAPM

Standard models of asset pricing (such as the CAPM) assume frictionless markets, meaning that securities can be traded costlessly. But these models actually have little to say about trading activity. For example, in the equilibrium of the CAPM, all investors share all available information and demand identical portfolios of risky assets. The awkward implication of this result is that there is no reason for trade. If all investors hold identical portfolios of risky assets, then when new (unexpected) information arrives, prices will change commensurately, but each investor will continue to hold a piece of the market portfolio, which requires no exchange of assets. How do we square this implication with the observation that on a typical day, more than 3 billion shares change hands on the New York Stock Exchange alone? One obvious answer is heterogeneous expectations, that is, beliefs not shared by the entire market. Such private information will give rise to trading as investors attempt to profit by rearranging portfolios in accordance with their now-heterogeneous demands. In reality, trading (and trading costs) will be of great importance to investors.

The **liquidity** of an asset is the ease and speed with which it can be sold at fair market value. Part of liquidity is the cost of engaging in a transaction, particularly the bid–ask spread. Another part is price impact—the adverse movement in price one would encounter when attempting to execute a larger trade. Yet another component is immediacy—the ability to sell the asset quickly without reverting to fire-sale prices. Conversely, **illiquidity** can be measured in part by the discount from fair market value a seller must accept if the asset is to be sold quickly. A perfectly liquid asset is one that would entail no illiquidity discount.

[28]Ravi Jagannathan and Yong Wang, "Lazy Investors, Discretionary Consumption, and the Cross-Section of Stock Returns," *Journal of Finance* 62 (August 2007), pp. 1633–61.

STOCK INVESTORS PAY HIGH PRICE FOR LIQUIDITY

Given a choice between liquid and illiquid stocks, most investors, to the extent they think of it at all, opt for issues they know are easy to get in and out of.

But for long-term investors who don't trade often—which includes most individuals—that may be unnecessarily expensive. Recent studies of the performance of listed stocks show that, on average, less-liquid issues generate substantially higher returns—as much as several percentage points a year at the extremes.

ILLIQUIDITY PAYOFF

Among the academic studies that have attempted to quantify this illiquidity payoff is a recent work by two finance professors, Yakov Amihud of New York University and Tel Aviv University, and Haim Mendelson of the University of Rochester. Their study looks at New York Stock Exchange issues over the 1961–1980 period and defines liquidity in terms of bid–asked spreads as a percentage of overall share price.

Market makers use spreads in quoting stocks to define the difference between the price they'll bid to take stock off an investor's hands and the price they'll offer to sell stock to any willing buyer. The bid price is always somewhat lower because of the risk to the broker of tying up precious capital to hold stock in inventory until it can be resold.

If a stock is relatively illiquid, which means there's not a ready flow of orders from customers clamoring to buy it, there's more of a chance the broker will lose money on the trade. To hedge this risk, market makers demand an even bigger discount to service potential sellers, and the spread will widen further.

The study by Profs. Amihud and Mendelson shows that liquidity spreads—measured as a percentage discount from the stock's total price—ranged from less than 0.1%, for widely held International Business Machines Corp., to as much as 4% to 5%. The widest-spread group was dominated by smaller, low-priced stocks.

The study found that, overall, the least-liquid stocks averaged an 8.5 percent-a-year higher return than the most-liquid stocks over the 20-year period. On average, a one percentage point increase in the spread was associated with a 2.5% higher annual return for New York Stock Exchange stocks. The relationship held after results were adjusted for size and other risk factors.

An extension of the study of Big Board stocks done at *The Wall Street Journal*'s request produced similar findings. It shows that for the 1980–85 period, a one percentage-point-wider spread was associated with an extra average annual gain of 2.4%. Meanwhile, the least-liquid stocks outperformed the most-liquid stocks by almost six percentage points a year.

COST OF TRADING

Since the cost of the spread is incurred each time the stock is traded, illiquid stocks can quickly become prohibitively expensive for investors who trade frequently. On the other hand, long-term investors needn't worry so much about spreads, since they can amortize them over a longer period.

In terms of investment strategy, this suggests "that the small investor should tailor the types of stocks he or she buys to his expected holding period," Prof. Mendelson says. If the investor expects to sell within three months, he says, it's better to pay up for the liquidity and get the lowest spread. If the investor plans to hold the stock for a year or more, it makes sense to aim at stocks with spreads of 3% or more to capture the extra return.

Source: Barbara Donnelly, *The Wall Street Journal,* April 28, 1987, p. 37. Reprinted by permission of *The Wall Street Journal.*

Liquidity (or the lack of it) has long been recognized as an important characteristic that affects asset values. For example, in legal cases, courts have routinely applied very steep discounts to the values of businesses that cannot be publicly traded. But liquidity has not always been appreciated as an important factor in security markets, presumably due to the relatively small trading cost per transaction compared with the large costs of trading assets such as real estate. The breakthrough came in the work of Amihud and Mendelson[29] (see the nearby box) and today, liquidity is increasingly viewed as an important determinant of prices and expected returns. We supply only a brief synopsis of this important topic here and provide empirical evidence in Chapter 13.

[29] Yakov Amihud and Haim Mendelson, "Asset Pricing and the Bid–Ask Spread," *Journal of Financial Economics* 17(1986). A summary of the ensuing large body of literature on liquidity can be found in Yakov Amihud, Haim Mendelson, and Lasse Heje Pedersen, "Liquidity and Asset Prices," *Foundations and Trends in Finance* 1, no. 4 (2005).

Early models of liquidity focused on the inventory management problem faced by security dealers. Dealers in over-the-counter markets post prices at which they are willing to buy a security (the bid price) or sell it (the ask price). The willingness of security dealers to add to their inventory or sell shares from their inventory makes them crucial contributors to overall market liquidity. The fee they earn for supplying this liquidity is the bid–ask spread. Part of the bid–ask spread may be viewed as compensation for bearing the price risk involved in holding an inventory of securities and allowing their inventory levels to absorb the fluctuations in overall security demand. Assuming the fair price of the stock is the average of the bid and ask prices, an investor pays half the spread upon purchase and another half upon sale of the stock. A dealer on the other side of the transaction earns these spreads. The spread is one important component of liquidity—it is the cost of transacting in a security.

The advent of electronic trading has steadily diminished the role of dealers, but traders still must contend with a bid–ask spread. For example, in electronic markets, the limit-order book contains the "inside spread," that is, the difference between the highest price at which some investor will purchase any shares and the lowest price at which another investor is willing to sell. The effective bid–ask spread will also depend on the size of the desired transaction. Larger purchases will require a trader to move deeper into the limit-order book and accept less-attractive prices. While inside spreads on electronic markets often appear extremely low, effective spreads can be much larger, because the limit orders are good for only small numbers of shares.

Even without the inventory problems faced by traditional securities dealers, the importance of the spread persists. There is greater emphasis today on the component of the spread that is due to asymmetric information. By asymmetric information, we mean the potential for one trader to have private information about the value of the security that is not known to the trading partner. To see why such an asymmetry can affect the market, think about the problems facing someone buying a used car. The seller knows more about the car than the buyer, so the buyer naturally wonders if the seller is trying to get rid of the car because it is a "lemon." At the least, buyers worried about overpaying will shave the prices they are willing to pay for a car of uncertain quality. In extreme cases of asymmetric information, trading may cease altogether.[30] Similarly, traders who post offers to buy or sell at limit prices need to be worried about being picked off by better-informed traders who hit their limit prices only when they are out of line with the intrinsic value of the firm.

Broadly speaking, we may envision investors trading securities for two reasons. Some trades are driven by "noninformational" motives, for example, selling assets to raise cash for a big purchase, or even just for portfolio rebalancing. These sorts of trades, which are not motivated by private information that bears on the value of the traded security, are called *noise trades.* Security dealers will earn a profit from the bid–ask spread when transacting with noise traders (also called *liquidity traders* because their trades may derive from needs for liquidity, i.e., cash).

Other transactions are motivated by private information known only to the seller or buyer. These transactions are generated when traders believe they have come across information that a security is mispriced, and try to profit from that analysis. If an information trader identifies an advantageous opportunity, it must be disadvantageous to the other party in the transaction. If private information indicates a stock is overpriced, and the trader decides to sell it, a dealer who has posted a bid price or another trader who has posted a

[30]The problem of informational asymmetry in markets was introduced by the 2001 Nobel Laureate George A. Akerlof and has since become known as the *lemons problem.* A good introduction to Akerlof's contributions can be found in George A. Akerlof, *An Economic Theorist's Book of Tales* (Cambridge, U.K.: Cambridge University Press, 1984).

limit-buy order and ends up on the other side of the transaction will purchase the stock at what will later be revealed to have been an inflated price. Conversely, when private information results in a decision to buy, the price at which the security is traded will eventually be recognized as less than fair value.

Information traders impose a cost on both dealers and other investors who post limit orders. Although on average dealers make money from the bid–ask spread when transacting with liquidity traders, they will absorb losses from information traders. Similarly, any trader posting a limit order is at risk from information traders. The response is to increase limit-ask prices and decrease limit-bid orders—in other words, the spread must widen. The greater the relative importance of information traders, the greater the required spread to compensate for the potential losses from trading with them. In the end, therefore, liquidity traders absorb most of the cost of the information trades because the bid–ask spread that they must pay on their "innocent" trades widens when informational asymmetry is more severe.

The discount in a security price that results from illiquidity can be surprisingly large, far larger than the bid–ask spread. Consider a security with a bid–ask spread of 1%. Suppose it will change hands once a year for the next 3 years and then will be held forever by the third buyer. For the last trade, the investor will pay for the security 99.5% or .995 of its fair price; the price is reduced by half the spread that will be incurred when the stock is sold. The second buyer, knowing the security will be sold a year later for .995 of fair value, and having to absorb half the spread upon purchase, will be willing to pay $.995 - .005/1.05 = .9902$ (i.e., 99.02% of fair value), if the cost of trading is discounted at a rate of 5%. Finally, the current buyer, knowing the loss next year, when the stock will be sold for .9902 of fair value (a discount of .0098), will pay for the security only $.995 - .0098/1.05 = .9857$. Thus the discount has ballooned from .5% to 1.43%. In other words, the present values of all three future trading costs (spreads) are discounted into the current price.[31] To extend this logic, if the security will be traded once a year forever, its current illiquidity cost will equal immediate cost plus the present value of a perpetuity of .5%. At an annual discount rate of 5%, this sum equals $.005 + .005/.05 = .105$, or 10.5%! Obviously, liquidity is of potentially large value and should not be ignored in deriving the equilibrium value of securities.

Consider three stocks with equal bid–ask spreads of 1%. The first trades once a year, the second once every 2 years, and the third every 3 years. We have already calculated the price discount due to illiquidity as the present value of illiquidity costs for the first as 10.5%. The discount for the second security is .5% plus the present value of a biannual perpetuity of .5%, which at a discount rate of 5% amounts to $.5 + .5/(1.05^2 - 1) = 5.38\%$. Similarly, the cost for the security that trades only every 3 years is 3.67%. From this pattern of discounts—10.5%, 5.38%, and 3.67%—it seems that for any *given* spread, the price discount will increase almost in proportion to the frequency of trading. It also would appear that the discount should be proportional to the bid–ask spread. However, trading frequency may well vary inversely with the spread, and this will impede the response of the price discount to the spread.

An investor who plans to hold a security for a given period will calculate the impact of illiquidity costs on expected rate of return; liquidity costs will be amortized over the anticipated holding period. Investors who trade less frequently therefore will be less affected by high trading costs. The reduction in the rate of return due to trading costs is lower the longer the security is held. Hence in equilibrium, investors with long holding periods will,

[31] We will see another instance of such capitalization of trading costs in Chapter 13, where one explanation for large discounts on closed-end funds is the substantial present value of a *stream* of apparently small per-period expenses.

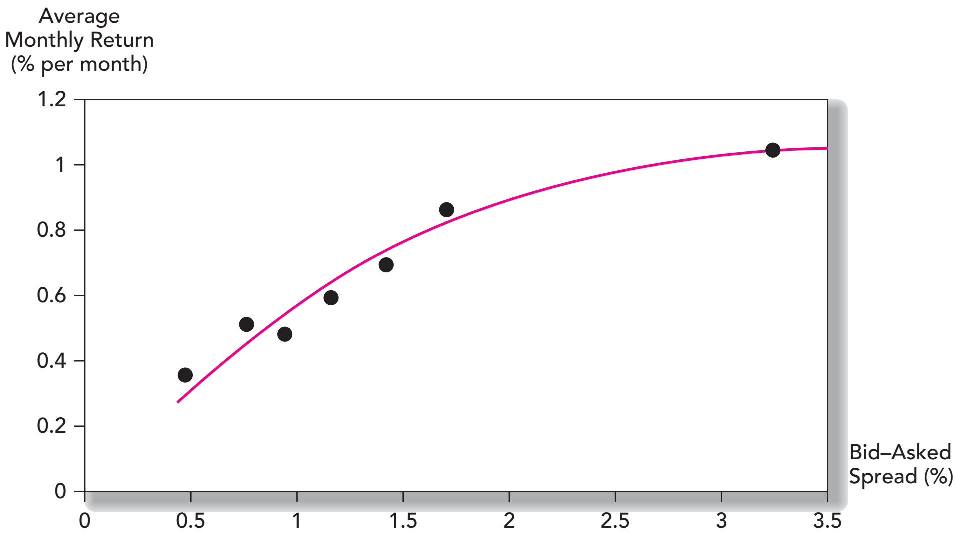


FIGURE 9.5 The relationship between illiquidity and average returns

Source: Derived from Yakov Amihud and Haim Mendelson, "Asset Pricing and the Bid–Ask Spread," *Journal of Financial Economics* 17 (1986), pp. 223–49.

on average, hold more of the illiquid securities, while short-horizon investors will more strongly prefer liquid securities. This "clientele effect" mitigates the effect of the bid–ask spread for illiquid securities. The end result is that the liquidity premium should increase with the bid–ask spread at a decreasing rate. Figure 9.5 confirms this prediction.

So far, we have shown that the expected level of liquidity can affect prices, and therefore expected rates of return. What about unanticipated changes in liquidity? Investors may also demand compensation for *liquidity risk.* The bid–ask spread of a security is not constant through time, nor is the ability to sell a security at a fair price with little notice. Both depend on overall conditions in security markets. If asset liquidity fails at times when it is most desired, then investors will require an additional price discount beyond that required for the expected cost of illiquidity.[32] In other words, there may be a *systematic* component to liquidity risk that affects the equilibrium rate of return and hence the expected return–beta relationship.

As a concrete example of such a model, Acharya and Pedersen[33] consider the impacts of both the level and the risk of liquidity on security pricing. They include three components to liquidity risk—each captures the extent to which liquidity varies systematically

[32] A good example of systematic effects in liquidity risk surrounds the demise of Long-Term Capital Management in the summer of 1998. Despite extensive analysis that indicated its portfolio was highly diversified, many of its assets went bad at the same time when Russia defaulted on its debt. The problem was that despite the fact that short and long positions were expected to balance price changes based on normal market fluctuations, a massive decline in the market liquidity and prices of some assets was not offset by increased prices of more liquid assets. As a supplier of liquidity to others, LTCM was a large holder of less-liquid securities and a liquidity shock of this magnitude was at that time an unimaginable event. While its portfolio may have been diversified in terms of exposure to traditional business condition shocks, it was undiversified in terms of exposure to liquidity shocks.

[33] V. V. Acharya and L. H. Pedersen, "Asset Pricing with Liquidity Risk," *Journal of Financial Economics* 77 (2005).

with other market conditions. They identify three relevant "liquidity betas," which measure in turn: (i) the extent to which the stock's illiquidity varies with market illiquidity; (ii) the extent to which the stock's return varies with market illiquidity; and (iii) the extent to which the stock illiquidity varies with the market return. Therefore, expected return depends on expected liquidity, as well as the conventional "CAPM beta" and three additional liquidity-related betas:

$$E(R_i) = kE(C_i) + \lambda(\beta + \beta_{L1} - \beta_{L2} - \beta_{L3}) \qquad \textbf{(9.18)}$$

where

$E(C_i)$ = expected cost of illiquidity,

k = adjustment for average holding period over all securities,

λ = market risk premium net of average market illiquidity cost, $E(R_M - C_M)$,

β = measure of systematic market risk,

$\beta_{L1}, \beta_{L2}, \beta_{L3}$ = liquidity betas.

Compared to the conventional CAPM, the expected return–beta equation now has a predicted firm-specific component that accounts for the effect of security liquidity. Such an effect would appear to be an alpha in the conventional index model.

The market risk premium itself is measured net of the average cost of illiquidity, that is, $\lambda = E(R_M - C_M)$, where C_M is the market-average cost of illiquidity.

The overall risk of each security now must account for the three elements of liquidity risk, which are defined as follows:[34]

$$\beta_{L1} = \frac{\mathrm{Cov}(C_i, C_M)}{\mathrm{Var}(R_M - C_M)}$$

Measures the sensitivity of the security's illiquidity to market illiquidity. Investors want additional compensation for holding a security that becomes illiquid when general liquidity is low.[34]

$$\beta_{L2} = \frac{\mathrm{Cov}(R_i, C_M)}{\mathrm{Var}(R_M - C_M)}$$

Measures the sensitivity of the stock's return to market illiquidity. This coefficient appears with a negative sign in Equation 9.18 because investors are willing to accept a lower average return on stocks that will provide higher returns when market illiquidity is greater.

$$\beta_{L3} = \frac{\mathrm{Cov}(C_i, R_M)}{\mathrm{Var}(R_M - C_M)}$$

Measures the sensitivity of security illiquidity to the market rate of return. This sensitivity also appears with a negative sign, because investors will be willing to accept a lower average return on securities that can be sold more easily (have low illiquidity costs) when the market declines.

A good number of variations on this model can be found in the current (and rapidly growing) literature on liquidity.[35] What is common to all liquidity variants is that they improve on the explanatory power of the CAPM equation and hence there is no doubt that, sooner or later, practitioner optimization models and, more important, security analysis will incorporate the empirical content of these models.

[34]Several papers have shown that there is important covariance across asset illiquidity. See for example, T. Chordia, R. Roll, and A. Subramanyam, "Commonality in Liquidity," *Journal of Financial Economics* 56 (2000), pp. 3–28 or J. Hasbrouck and D. H. Seppi "Common Factors in Prices, Order Flows and Liquidity," *Journal of Financial Economics* 59 (2001), pp. 383–411.

[35]Another influential study of liquidity risk and asset pricing is L. Pastor and R. Stambaugh, "Liquidity Risk and Expected Stock Returns," *Journal of Political Economy* 111 (2003), pp. 642–85.

SUMMARY

1. The CAPM assumes that investors are single-period planners who agree on a common input list from security analysis and seek mean-variance optimal portfolios.
2. The CAPM assumes that security markets are ideal in the sense that:
   a. They are large, and investors are price-takers.
   b. There are no taxes or transaction costs.
   c. All risky assets are publicly traded.
   d. Investors can borrow and lend any amount at a fixed risk-free rate.
3. With these assumptions, all investors hold identical risky portfolios. The CAPM holds that in equilibrium the market portfolio is the unique mean-variance efficient tangency portfolio. Thus a passive strategy is efficient.
4. The CAPM market portfolio is a value-weighted portfolio. Each security is held in a proportion equal to its market value divided by the total market value of all securities.
5. If the market portfolio is efficient and the average investor neither borrows nor lends, then the risk premium on the market portfolio is proportional to its variance, σ_M^2, and to the average coefficient of risk aversion across investors, *A:*

$$E(r_M) - r_f = \overline{A}\sigma_M^2$$

6. The CAPM implies that the risk premium on any individual asset or portfolio is the product of the risk premium on the market portfolio and the beta coefficient:

$$E(r_i) - r_f = \beta_i[E(r_M) - r_f]$$

   where the beta coefficient is the covariance of the asset with the market portfolio as a fraction of the variance of the market portfolio

$$\beta_i = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2}$$

7. When risk-free investments are restricted but all other CAPM assumptions hold, then the simple version of the CAPM is replaced by its zero-beta version. Accordingly, the risk-free rate in the expected return–beta relationship is replaced by the zero-beta portfolio's expected rate of return:

$$E(r_i) = E[r_{Z(M)}] + \beta_i E[r_M - r_{Z(M)}]$$

8. The simple version of the CAPM assumes that investors are myopic. When investors are assumed to be concerned with lifetime consumption and bequest plans, but investors' tastes and security return distributions are stable over time, the market portfolio remains efficient and the simple version of the expected return–beta relationship holds. But if those distributions change unpredictably, or if investors seek to hedge nonmarket sources of risk to their consumption, the simple CAPM will give way to a multifactor version in which the security's exposure to these nonmarket sources of risk command risk premiums.
9. The consumption-based capital asset pricing model (CCAPM) is a single-factor model in which the market portfolio excess return is replaced by that of a consumption-tracking portfolio. By appealing directly to consumption, the model naturally incorporates consumption-hedging considerations and changing investment opportunities within a single-factor framework.
10. The Security Market Line of the CAPM must be modified to account for labor income and other significant nontraded assets.
11. Liquidity costs and liquidity risk can be incorporated into the CAPM relationship. Investors demand compensation for both expected costs of illiquidity as well as the risk surrounding those costs.

Related Web sites for this chapter are available at **www.mhhe.com/bkm**

KEY TERMS

homogeneous expectations
market portfolio
mutual fund theorem
market price of risk
beta
expected return–beta relationship
security market line (SML)
alpha
market model
zero-beta portfolio
liquidity
illiquidity

PROBLEM SETS

Quiz

1. What must be the beta of a portfolio with $E(r_P) = 18\%$, if $r_f = 6\%$ and $E(r_M) = 14\%$?
2. The market price of a security is \$50. Its expected rate of return is 14%. The risk-free rate is 6% and the market risk premium is 8.5%. What will be the market price of the security if its correlation coefficient with the market portfolio doubles (and all other variables remain unchanged)? Assume that the stock is expected to pay a constant dividend in perpetuity.
3. Are the following true or false? Explain.
   a. Stocks with a beta of zero offer an expected rate of return of zero.
   b. The CAPM implies that investors require a higher return to hold highly volatile securities.
   c. You can construct a portfolio with beta of .75 by investing .75 of the investment budget in T-bills and the remainder in the market portfolio.

Problems

4. You are a consultant to a large manufacturing corporation that is considering a project with the following net after-tax cash flows (in millions of dollars):

Years from Now	After-Tax Cash Flow
0	−40
1–10	15

The project's beta is 1.8. Assuming that $r_f = 8\%$ and $E(r_M) = 16\%$, what is the net present value of the project? What is the highest possible beta estimate for the project before its NPV becomes negative?

5. Consider the following table, which gives a security analyst's expected return on two stocks for two particular market returns:

Market Return	Aggressive Stock	Defensive Stock
5%	−2%	6%
25	38	12

   a. What are the betas of the two stocks?
   b. What is the expected rate of return on each stock if the market return is equally likely to be 5% or 25%?
   c. If the T-bill rate is 6% and the market return is equally likely to be 5% or 25%, draw the SML for this economy.
   d. Plot the two securities on the SML graph. What are the alphas of each?
   e. What hurdle rate should be used by the management of the aggressive firm for a project with the risk characteristics of the defensive firm's stock?

For Problems 6 to 12: If the simple CAPM is valid, which of the following situations are possible? Explain. Consider each situation independently.

6.

Portfolio	Expected Return	Beta
A	20	1.4
B	25	1.2

7.

Portfolio	Expected Return	Standard Deviation
A	30	35
B	40	25

8.

Portfolio	Expected Return	Standard Deviation
Risk-free	10	0
Market	18	24
A	16	12

9.

Portfolio	Expected Return	Standard Deviation
Risk-free	10	0
Market	18	24
A	20	22

10.

Portfolio	Expected Return	Beta
Risk-free	10	0
Market	18	1.0
A	16	1.5

11.

Portfolio	Expected Return	Beta
Risk-free	10	0
Market	18	1.0
A	16	0.9

12.

Portfolio	Expected Return	Standard Deviation
Risk-free	10	0
Market	18	24
A	16	22

For Problems 13 to 15 assume that the risk-free rate of interest is 6% and the expected rate of return on the market is 16%.

13. A share of stock sells for $50 today. It will pay a dividend of $6 per share at the end of the year. Its beta is 1.2. What do investors expect the stock to sell for at the end of the year?
14. I am buying a firm with an expected perpetual cash flow of $1,000 but am unsure of its risk. If I think the beta of the firm is .5, when in fact the beta is really 1, how much *more* will I offer for the firm than it is truly worth?
15. A stock has an expected rate of return of 4%. What is its beta?
16. Two investment advisers are comparing performance. One averaged a 19% rate of return and the other a 16% rate of return. However, the beta of the first investor was 1.5, whereas that of the second was 1.
    a. Can you tell which investor was a better selector of individual stocks (aside from the issue of general movements in the market)?
    b. If the T-bill rate were 6% and the market return during the period were 14%, which investor would be the superior stock selector?
    c. What if the T-bill rate were 3% and the market return were 15%?

17. Suppose the rate of return on short-term government securities (perceived to be risk-free) is about 5%. Suppose also that the expected rate of return required by the market for a portfolio with a beta of 1 is 12%. According to the capital asset pricing model:
    a. What is the expected rate of return on the market portfolio?
    b. What would be the expected rate of return on a stock with $\beta = 0$?
    c. Suppose you consider buying a share of stock at $40. The stock is expected to pay $3 dividends next year and you expect it to sell then for $41. The stock risk has been evaluated at $\beta = -.5$. Is the stock overpriced or underpriced?
18. Suppose that borrowing is restricted so that the zero-beta version of the CAPM holds. The expected return on the market portfolio is 17%, and on the zero-beta portfolio it is 8%. What is the expected return on a portfolio with a beta of .6?
19. *a.* A mutual fund with beta of .8 has an expected rate of return of 14%. If $r_f = 5\%$, and you expect the rate of return on the market portfolio to be 15%, should you invest in this fund? What is the fund's alpha?
    b. What passive portfolio comprised of a market-index portfolio and a money market account would have the same beta as the fund? Show that the difference between the expected rate of return on this passive portfolio and that of the fund equals the alpha from part (*a*).

Challenge Problem

20. Outline how you would incorporate the following into the CCAPM:
    a. Liquidity
    b. Nontraded assets (Do you have to worry about labor income?)

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CFA® PROBLEMS

1. *a.* John Wilson is a portfolio manager at Austin & Associates. For all of his clients, Wilson manages portfolios that lie on the Markowitz efficient frontier. Wilson asks Mary Regan, CFA, a managing director at Austin, to review the portfolios of two of his clients, the Eagle Manufacturing Company and the Rainbow Life Insurance Co. The expected returns of the two portfolios are substantially different. Regan determines that the Rainbow portfolio is virtually identical to the market portfolio and concludes that the Rainbow portfolio must be superior to the Eagle portfolio. Do you agree or disagree with Regan's conclusion that the Rainbow portfolio is superior to the Eagle portfolio? Justify your response with reference to the capital market line.
    b. Wilson remarks that the Rainbow portfolio has a higher expected return because it has greater nonsystematic risk than Eagle's portfolio. Define nonsystematic risk and explain why you agree or disagree with Wilson's remark.
2. Wilson is now evaluating the expected performance of two common stocks, Furhman Labs Inc. and Garten Testing Inc. He has gathered the following information:
    - The risk-free rate is 5%.
    - The expected return on the market portfolio is 11.5%.
    - The beta of Furhman stock is 1.5.
    - The beta of Garten stock is .8.

    Based on his own analysis, Wilson's forecasts of the returns on the two stocks are 13.25% for Furhman stock and 11.25% for Garten stock. Calculate the required rate of return for Furhman Labs stock and for Garten Testing stock. Indicate whether each stock is undervalued, fairly valued, or overvalued.
3. The security market line depicts:
    a. A security's expected return as a function of its systematic risk.
    b. The market portfolio as the optimal portfolio of risky securities.

c. The relationship between a security's return and the return on an index.
d. The complete portfolio as a combination of the market portfolio and the risk-free asset.

4. Within the context of the capital asset pricing model (CAPM), assume:
   - Expected return on the market = 15%.
   - Risk-free rate = 8%.
   - Expected rate of return on XYZ security = 17%.
   - Beta of XYZ security = 1.25.

   Which one of the following is correct?

   a. XYZ is overpriced.
   b. XYZ is fairly priced.
   c. XYZ's alpha is −.25%.
   d. XYZ's alpha is .25%.
5. What is the expected return of a zero-beta security?

   a. Market rate of return.
   b. Zero rate of return.
   c. Negative rate of return.
   d. Risk-free rate of return.
6. Capital asset pricing theory asserts that portfolio returns are best explained by:

   a. Economic factors.
   b. Specific risk.
   c. Systematic risk.
   d. Diversification.
7. According to CAPM, the expected rate of return of a portfolio with a beta of 1.0 and an alpha of 0 is:

   a. Between r_M and r_f.
   b. The risk-free rate, r_f.
   c. $\beta\,(r_M - r_f)$.
   d. The expected return on the market, r_M.

The following table shows risk and return measures for two portfolios.

Portfolio	Average Annual Rate of Return	Standard Deviation	Beta
R	11%	10%	0.5
S&P 500	14%	12%	1.0

8. When plotting portfolio *R* on the preceding table relative to the SML, portfolio *R* lies:

   a. On the SML.
   b. Below the SML.
   c. Above the SML.
   d. Insufficient data given.
9. When plotting portfolio *R* relative to the capital market line, portfolio *R* lies:

   a. On the CML.
   b. Below the CML.
   c. Above the CML.
   d. Insufficient data given.

10. Briefly explain whether investors should expect a higher return from holding portfolio *A* versus portfolio *B* under capital asset pricing theory (CAPM). Assume that both portfolios are fully diversified.

	Portfolio A	Portfolio B
Systematic risk (beta)	1.0	1.0
Specific risk for each individual security	High	Low

11. Joan McKay is a portfolio manager for a bank trust department. McKay meets with two clients, Kevin Murray and Lisa York, to review their investment objectives. Each client expresses an interest in changing his or her individual investment objectives. Both clients currently hold well-diversified portfolios of risky assets.

    a. Murray wants to increase the expected return of his portfolio. State what action McKay should take to achieve Murray's objective. Justify your response in the context of the CML.

    b. York wants to reduce the risk exposure of her portfolio but does not want to engage in borrowing or lending activities to do so. State what action McKay should take to achieve York's objective. Justify your response in the context of the SML.

12. Karen Kay, a portfolio manager at Collins Asset Management, is using the capital asset pricing model for making recommendations to her clients. Her research department has developed the information shown in the following exhibit.

Forecast Returns, Standard Deviations, and Betas

	Forecast Return	Standard Deviation	Beta
Stock *X*	14.0%	36%	0.8
Stock *Y*	17.0	25	1.5
Market index	14.0	15	1.0
Risk-free rate	5.0		

a. Calculate expected return and alpha for each stock.

b. Identify and justify which stock would be more appropriate for an investor who wants to

i. add this stock to a well-diversified equity portfolio.
ii. hold this stock as a single-stock portfolio.

STANDARD & POOR'S

Go to **www.mhhe.com/edumarketinsight** and link to *Company*, then *Population*. Select a company of interest to you and link to the Company Research page. Look for the *Excel Analytics section*, and choose *Valuation Data*, then review the *Profitability* report. Find the row that shows the historical betas for your firm. Is beta stable from year to year? Go back to the *Company Research* page and look at the latest available *S&P Stock Report* for your firm. What beta does the report indicate for your firm? Why might this be different from the one in the *Profitability Report*? Based on current risk-free rates (available at **finance.yahoo.com**), and the historical risk premiums discussed in Chapter 5, estimate the expected rate of return on your company's stock by using the CAPM.

E-Investments

Beta and Security Returns

Fidelity provides data on the risk and return of its funds at **www.fidelity.com.** Click on the *Research* link, then choose *Mutual Funds* from the submenu. In the *Fund Evaluator* section, choose *Advanced Search*. Scroll down until you find the *Risk/ Volatility Measures* section and indicate that you want to screen for funds with betas less than or equal to .50. Click *Search Funds* to see the results. Click on the link that says *View All Matching Fidelity Funds.* Select five funds from the resulting list and click *Compare*. Rank the five funds according to their betas and then according to their standard deviations. Do both lists rank the funds in the same order? How would you explain any difference in the rankings? Note the 1-Year return for one of the funds (use the load-adjusted return if it is available). Repeat the exercise to compare five funds that have betas greater than or equal to 1.50.

SOLUTIONS TO CONCEPT CHECKS

1. We can characterize the entire population by two representative investors. One is the "uninformed" investor, who does not engage in security analysis and holds the market portfolio, whereas the other optimizes using the Markowitz algorithm with input from security analysis. The uninformed investor does not know what input the informed investor uses to make portfolio purchases. The uninformed investor knows, however, that if the other investor is informed, the market portfolio proportions will be optimal. Therefore, to depart from these proportions would constitute an uninformed bet, which will, on average, reduce the efficiency of diversification with no compensating improvement in expected returns.

2. *a.* Substituting the historical mean and standard deviation in Equation 9.2 yields a coefficient of risk aversion of

$$\overline{A} = \frac{E(r_M) - r_f}{\sigma_M^2} = \frac{.084}{.203^2} = 2.04$$

b. This relationship also tells us that for the historical standard deviation and a coefficient of risk aversion of 3.5 the risk premium would be

$$E(r_M) - r_f = \overline{A}\sigma_M^2 = 3.5 \times .203^2 = .144 = 14.4\%$$

3. For these investment proportions, w_{Ford}, w_{GM}, the portfolio β is

$$\begin{aligned}\beta_P &= w_{\text{Ford}}\beta_{\text{Ford}} + w_{\text{GM}}\beta_{\text{GM}} \\ &= (.75 \times 1.25) + (.25 \times 1.10) = 1.2125\end{aligned}$$

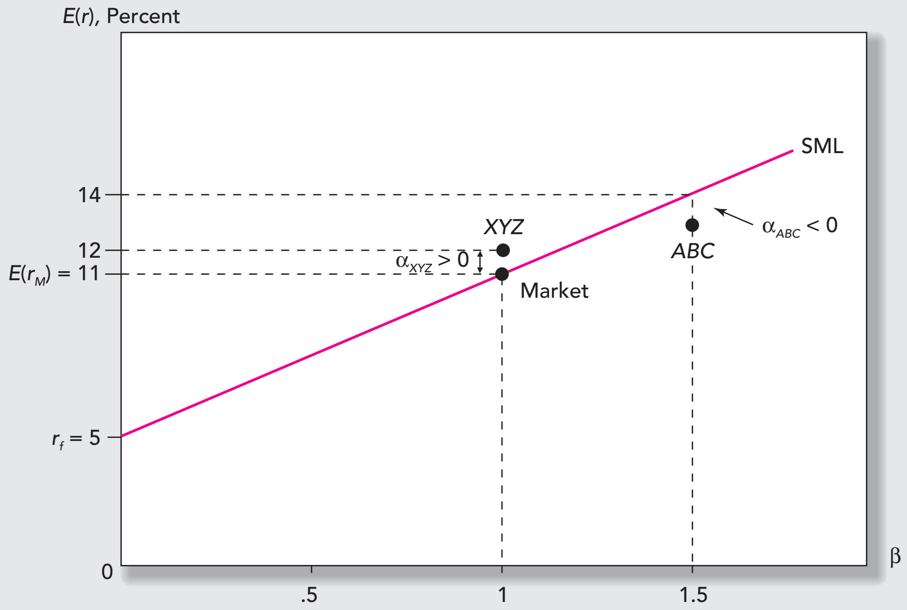
As the market risk premium, $E(r_M) - r_f$, is 8%, the portfolio risk premium will be

$$\begin{aligned}E(r_P) - r_f &= \beta_P[E(r_M) - r_f] \\ &= 1.2125 \times 8 = 9.7\%\end{aligned}$$

4. The alpha of a stock is its expected return in excess of that required by the CAPM.

$$\alpha = E(r) - \{r_f + \beta[E(r_M) - r_f]\}$$
$$\alpha_{XYZ} = 12 - [5 + 1.0(11 - 5)] = 1\%$$
$$\alpha_{ABC} = 13 - [5 + 1.5(11 - 5)] = -1\%$$

ABC plots below the SML, while *XYZ* plots above.



5. The project-specific required return is determined by the project beta coupled with the market risk premium and the risk-free rate. The CAPM tells us that an acceptable expected rate of return for the project is

$$r_f + \beta[E(r_M) - r_f] = 8 + 1.3(16 - 8) = 18.4\%$$

which becomes the project's hurdle rate. If the IRR of the project is 19%, then it is desirable. Any project with an IRR equal to or less than 18.4% should be rejected.

6. The CAPM is a model that relates expected rates of return to risk. It results in the expected return–beta relationship, where the expected risk premium on any asset is proportional to the expected risk premium on the market portfolio with beta as the proportionality constant. As such the model is impractical for two reasons: (i) expectations are unobservable, and (ii) the theoretical market portfolio includes every risky asset and is in practice unobservable. The next three models incorporate additional assumptions to overcome these problems.

The single-factor model assumes that one economic factor, denoted *F*, exerts the only common influence on security returns. Beyond it, security returns are driven by independent, firm-specific factors. Thus for any security, *i*,

$$r_i = E(r_i) + \beta_i F + e_i$$

The single-index model assumes that in the single-factor model, the factor *F* can be replaced by a broad-based index of securities that can proxy for the CAPM's theoretical market portfolio. The index model can be stated as $R_i = \alpha_i + \beta_i R_M + e_i$.

At this point it should be said that many interchange the meaning of the index and market models. The concept of the market model is that rate of return *surprises* on a stock are proportional to corresponding surprises on the market index portfolio, again with proportionality constant β.