Diagnosing Market Power in Chile's Electricity Industry[1]

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Abstract

Chile's electricity market is modeled as a Cournot duopoly with a competitive fringe. Due to the importance of hydro-storage resources (62% of total generation in 2000) particular care was given to the hydro scheduling issue. The model was estimated over a 1-month planning horizon using real cost and load data for April 2000. I found that if an unregulated spot market were implemented in Chile's electricity industry, large generators, especially the largest (Endesa) would have the incentive and ability to exercise market power unilaterally. Endesa would exercise market power by keeping its thermal portfolio outside of the market and by allocating its hydro production in order to take advantage of differences in price elasticity of demand. In particular, it would allocate too little supply to high demand periods and too much to low demand periods (relative to the competitive equilibrium). Endesa has so much market power, especially when demand is high, that the second largest producer (Gener)'s optimal strategy is to produce at capacity. It only has the incentive to exercise market power by constraining its production when demand is low. The source of Endesa's market power is its hydro capacity. Indeed, when hydro flows are reduced, so is its market power. Under these conditions, Gener has the incentive and ability to constrain its production in order to hold prices above competitive levels. Still, prices are lower than when Endesa had all of its hydro capacity. Endesa's incentive to exercise market power by shifting hydro production from one month to another depends on how large are the inter-month differences in price elasticity. As expected, final equilibrium is also very sensitive to the value of the price elasticity of demand.

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I. Introduction

Chile was the first country that reformed and restructured its power industry. The power sector reform, implemented in the early 80's, was an element of a more general economic reform that introduced competition rules to different industries. Basically, the government tried to limit its market interventions only to those industries where the private sector was not able to make appropriate investment, production and/or consumption decisions. In particular, the regulatory authority decided that, since the generation segment of the electricity industry was not a natural monopoly, the appropriate approach was to promote competition among generators. Entry into the generation business was opened up to competitors and generators were encouraged to enter into supply contracts with large industrial customers and distribution companies. The reforms also created a spot market, but the prices in the spot market were determined based on the marginal costs of generators in the system and the associated least cost dispatch. That is, generators effectively were required to bid their available capacity and associated audited marginal costs into the spot market. The marginal cost of the last generator required to balance supply and demand, taking into account transmission constraints and losses, then determined a simulated spot market clearing price at each node on the system. The prices for supplies made under long-term contracts to distribution companies were based on 4-year projections of these nodal prices as determined by the regulator. Thus, generation prices were not "deregulated" in the usual sense of the term, except for the very largest industrial customers who chose to enter into contracts directly with generators[4]. Rather, the system defined a "simulated" perfectly competitive set of spot and forward contract prices. The latter, have been constrained indirectly by a requirement that they be no higher than 110% and no lower than 90% of the prices charged to large industrial customers who negotiate prices directly with generators. The transmission and distribution segments continued to be regulated based on traditional cost-of-service regulatory principles because of their natural monopoly features. This economic policy was implemented in conjunction with a huge privatization effort, where most of the electricity companies were re-organized and then sold to the private sector.[5]

Two decades have passed since the restructuring of the industry took place. It is fair to say that in general terms, the reforms have had a positive effect: installed capacity increased and both efficiency and quality of service and quality of supply indices have exhibited a positive trend. However, in the past years there have been some warning signs that call for new changes; private companies for instance, have been reluctant to increase their installed capacity in spite of the signs of future shortages. Currently, both Chilean regulatory authorities and private companies are analyzing the question of moving to a system in which spot prices for electric energy would be truly deregulated by creating a uniform market clearing price "power exchange" through which generators would be free to bid whatever prices they choose with competition between generators determining the

[4] Distribution companies were not allowed to purchase energy from the simulated spot market. Instead, they were required to sign long term contracts with generators. The contract price was the regulated price.

[5] For more information on the privatization process, see Luders and Hachette (1991).

bids and market clearing prices.[6] One major concern that has been raised regarding this deregulation proposal is that the incumbent generators would be in a position to exercise market power, leading to prices far above competitive levels. In particular the high degree of concentration in the generation segment has raised concerns regarding the desirability of implementing a system like this one.

The electricity industry has been subject in the past years to restructuring and reforms in different countries (UK, many states in the US, Argentina, Colombia, Australia, New Zealand and so on). There is not a unique approach to follow as different countries have implemented different reforms with different results. However, some lessons have been learned from them. First of all, it is clear that in order to get the benefits of a competitive wholesale electricity market, reforms must be designed and implemented correctly. It is important to look at the industry as a whole and do not treat its different segments (generation, transmission, system operation, distribution and retailing) independently as they continually interact, and the effectiveness (and success) of one segment's regulation certainly depends on the other segments'. This interaction partially explains some of the problems experienced by California: utilities had to buy electricity at a volatile (and at sometimes very high) spot price, sell to retail consumers at a fixed price, and were not able to hedge against the risk that spot market prices could rise above the fixed retail price at which the power purchased in the spot market had to be resold at retail. In addition, the market design must realize that the electricity industry is prone to the exercise of market power, as electricity cannot be stored, producers interact very frequently, capacity constraints may be binding when demand is high and thus even a small supplier may be able profitably and unilaterally increase the market price by withholding supplies from the market. The importance of an adequate number of generating companies that compete in the wholesale market has also been emphasized. For instance, the experience in the UK, although successful in many aspects, has shed light of the problems that high concentration may yield to, especially with respect to the exercise of market power[7].

In this paper I analyze whether the high concentration in the generation sector will result in anti-competitive behavior if all electricity supplies were traded in an hourly unregulated spot market.[8] Following Borenstein and Bushnell (1999) and Bushnell (1998) I model Chile's electricity market as a Cournot duopoly with a competitive fringe. Particular care is given to the modeling of hydro resources, which are not only important because of its large share in total installed capacity and in total generation (61% and 62% respectively in 2000), but because of its impact on the incentives firms face when competing. As it will be analyzed in the paper, having hydro resources as a source of electric generation, means that firms do not take static production decisions at each

[6] There is no consensus regarding how the price paid by distribution companies for the supply of their franchise's small consumers would be regulated. Different alternatives have been proposed. Among them: to allow distribution companies to freely negotiate the price with generators and to use the regulated price (as it is currently calculated) as a price cap.
[7] For more details on the lessons learned from the UK and US experience, see Joskow (2002).
[8] See Borenstein et al (2000) for an analysis of why concentration indices are not a sufficient indicator of market power.

moment in time, but that firms have to take account that more water used today, means less water is available for tomorrow: the model becomes dynamic rather than static.

Quantitative simulation results obtained from this model depend on assumptions that, unfortunately, cannot be supported (or rejected) by strong empirical evidence, as the latter simply doesn't exist in many cases. For instance, prices, the production level, markups, and many other variables are a function of price-elasticity of demand, a variable for which there is no precise estimation. Even if there were, it is likely that after the power exchange and other reforms are implemented, demand for electricity (and price elasticity) will change. Accordingly the main contribution of this paper is given by the qualitative results. I do not expect the reader to remember exactly how high prices or markups would be. Instead, I expect him/her to remember what conditions are more favorable to the exercise of market power, who is really able to exercise it and how. In addition, this model will be a useful tool to evaluate the impact of different policies that could be implemented in this industry to mitigate market power.

I found that if an unregulated spot market were implemented in Chile prices could rise far above competitive levels as suppliers, in particular Endesa, the largest supplier, would exercise unilateral market power. The large thermal portfolio owned by Gener, the second largest generator, is not enough for the exercise of market power, as the relevant plants are mostly base load plants. Indeed, Endesa has so much market power and can move prices up so much, that Gener's optimal strategy is to produce at full capacity, profiting from the high prices set by Endesa. Even though Endesa keeps most of its thermal plants outside of the market, the real source of its market power is its large hydro capacity. In particular, it schedules its hydro production in order to exploit differences in price elasticity, allocating too little supply to high demand periods and too much to low demand periods, relative to the competitive equilibrium. This hydro scheduling strategy may be observed no matter what planning horizon is assumed in the model (a month, a year); the only "requirement" is that there is enough "inter-period" differences in demand elasticity. The smaller the fluctuation, the closer is the hydro scheduling strategy to the traditional competitive supply-demand or value-maximizing optimization analysis' conclusions (i.e. water is stored when it is relatively abundant and released when it is relatively scarce). The importance of hydro resources for Endesa is such that when hydro flows are reduced (as it occurs if the hydrological year is "dry"), Endesa loses its market power. Under these circumstances, Gener has incentives to act strategically to increase prices, but the resulting prices are still lower than when Endesa had all of its capacity.

Not surprisingly, alternative assumptions about the elasticity of demand for electricity turned out to be very important, as the more elastic is demand, the less market power can be exercised.

The paper is organized as follows: in Chapter II, I review the main findings of the literature that are related to this topic. In Chapter III, I briefly describe the Chilean power industry. In the fourth Chapter I analyze the model that will be used to diagnose market power. Data used to estimate the model is described in Chapter V while results are reported in Chapter VI. Two extensions to the main model are analyzed and estimated in Chapter VII. The final chapter concludes and gives directions for further research.

II. Empirical literature Review

This paper is related to two areas of research: the modeling of electricity markets in order to (ex-ante) simulate strategic behavior after the industry has been deregulated and to the analysis of the strategic use of hydro resources to exercise market power.

Three types of model have been used to simulate the strategic behavior of electricity firms. In the supply function equilibrium (SFE) approach, used by Green and Newbery (1992) and Halseth (1998) based on the work by Klemperer and Meyer (1989), the producers bid a supply function that relates quantity supplied to the market price. In the general case, the duopoly supply lies between the competitive and Cournot equilibrium; the range of feasible equilibria is reduced when uncertainty is added to the model.

Green and Newbery (1992) modeled the England and Wales electricity market using the Supply Function Equilibrium (SFE) framework developed by Klemperer and Meyer (1989) as applied to an empirical characterization of supply and demand, designed to match the attributes of the electricity system in England and Wales (making alternative assumptions about the elasticity of demand)[9]. They first present simulated values for prices, output and welfare for the duopoly case. They found that generators were able to drive prices far above competitive levels, depending on the assumed elasticity of demand, while creating a significant deadweight loss and producing supra-competitive profits for the generators..[10] They then examined the impact of restructuring the industry so that there were five equal-sized firms. In this case, the equilibrium price was significantly lower and close to competitive levels. In order to take account of the effect of potential entry, they examined the effects of entry in the duopoly case under alternative assumptions about the price responses of the incumbents to the entry of generators who acted as price takers. If the incumbents adopted a strategy of not responding to entry by lowering prices, substantial entry was attracted by the excess profits in the system. Eventually entry eroded the incumbents' profits completely, yielding an equilibrium with inefficient expenditures on new generating capacity and high prices. The welfare losses in these cases were very large.

Halseth (1998) used the SFE approach to analyze the potential for market power in the Nordic market. In his model, the supply function is restricted to be linear, with a constant markup over marginal cost. This markup is independent of the particular technology used by the producer but it varies between the different time periods. Asymmetry in production technologies is incorporated through the marginal cost function (each production level is associated to a specific marginal technology (hydro, nuclear or thermal). Due to the importance of hydro production in the Nordic market (it accounts for

[9] Von der Fehr and Harbord (1993) and Halseth (1998) criticize on theoretical grounds the Green and Newbery (1992) use of the SFE approach.

[10] Wolfram (1999) found that prices in the British market had been much lower than what Green and Newbery (1992) predicted.

50% of annual production), the hydro scheduling issue is explicitly modeled. In particular, annual hydro production is restricted to be less than the annual inflow and the water inflow that is stored between periods has to be within the reservoir capacity. He found that the potential for market power was less than expected due to the fringe's excess capacity. Only two of the six largest producers had incentives to reduce production.[11] Remaining producers did not have incentive to do so. In particular, he found that hydro producers were not interested in reducing its market supply. He argued that since all of its income came from hydro production (with a very low marginal cost), the price increase had to be very large in order to induce it not to use its generating capacity to the full.[12] It should be noted that all the results of this model are reported in annual terms. In particular, he found that hydro generating capacity was used to the full in the year. However, nothing is said regarding how it is allocated throughout the year. This is an important omission because it may be the case that hydro producers do not exercise market power by using less than its hydro capacity but through a strategy that distinguishes between periods of high demand from periods of low demand.

Auction theory has also been used to analyze strategic behavior in the electricity market. Von der Fehr and Harbord (1993) model the UK electricity spot market as a first price, sealed-bid, multiple unit private-value auction with a random number of units. In their model, generators simultaneously bid supply schedules (reflecting different prices for each individual plant), then demand is realized and the market price is given by the offer price of the marginal plant. They argue that producers face two opposing forces when bidding: by bidding a high price, the producer gets higher revenue but a lower probability of being dispatched. Equilibrium has different properties depending on the demand level. In particular, when demand is low, producers bid a price equal to the marginal cost of the least efficient generator and equilibrium is unique. When demand is high, there are multiple equilibria and the price is equal to the highest admissible price.[13] They remark that some of these equilibria may result in inefficient dispatching: the high cost generator will be dispatched with its total capacity if it submitted the lowest bid, while the low cost generator will be dispatched for only a fraction of it. Finally they argue that their model is supported by the bidding behavior observed in the UK electricity industry from May 1990 to April 1991. In particular, they report that while bids were close to generation cost at the beginning of the period, they diverged thereafter, Even though contracts were in place in the first part of the analyzed period, they argue that contracting practice is not a plausible explanation to the observed bidding behavior because contracts started to expire after the change of pattern took place. The coincidence of the first period with the low demand season (warm weather) and the second with the high demand season (cold weather) makes their model a more appropriate explanation. It should be noticed however that they analyzed a very short period. In order to be really able to separate the contract

[11] This two producers are Vattenfall and IVO. The portfolio of the first one is split between hydro (42%), nuclear (48%) and conventional thermal plants (10%). IVO is mostly a thermal producer.

[12] Johnsen et al (1999) concluded from this result that market power cannot be exercised in a market dominated by hydroelectric producers, to what they add, unless transmission constraint binds.

[13] Multiplicity of equilibria is given by the fact that both producers want to be the "low bidder" because the received price is the same but the producer is ranked first, and thus output is greater.

effect from the high/low demand effect, and in this way to get more conclusive support to their theory, the following seasons should be analyzed.[14]

Finally a third approach that has been used in the literature is to model the electricity industry as a Cournot oligopoly where producers are assumed to bid fix quantities. Andersson and Bergman (1995) simulated market behavior of the Swedish electricity industry after deregulation took place. They assumed a constant elasticity demand function (with an elasticity of demand equal to 0.3 in the main case), constant marginal costs for hydro and nuclear power plants and a non-linear marginal cost function for conventional thermal units. They found that prices would increase and production would be constrained. In particular, they found that the Cournot price equilibrium was 36% higher than the current (base) case and 62% than the Bertrand equilibrium. Markups were not analyzed. They also analyzed the impact of alternative market structures like splitting the largest company in 2 firms of the same size and a merge between the six smallest companies. In both cases equilibrium price was reduced below the base case. Finally they analyzed the impact of increased price responsiveness solving the model for a higher elasticity value (0.6). Since hydro production is modeled on an average basis, nothing is said regarding how resources are allocated within the year (for instance there is no differentiation between peak and off peak periods). In addition, nothing is said regarding how the portfolio of resources is used and how it compares to the base and Bertrand equilibrium cases. This is an important omission given the importance of hydro resources in the Swedish electricity market.

Borenstein and Bushnell (1999) and Bushnell (1998) modeled the California power industry as a Cournot triopoly with a competitive fringe.[15] Cournot producers face a residual demand where must run generation, the fringe's supply and hydro generation in the case of Borenstein and Bushnell (1999) are subtracted from total demand. Marginal cost functions were estimated using cost data at the plant level. A big difference between those articles is given by the treatment of hydro resources: Bushnell (1998) assumes that Cournot producers use them strategically while Borenstein and Bushnell (1999) assume that they are allocated competitively.[16] In other words, in Bushnell (1998)'s model, hydro producers are "allowed" to store water inflows from one period and use them in another one in order to manipulate prices. As a result, in his model the different periods are not independent and thus the maximization has to be solved simultaneously over the entire planning horizon, as opposed to Borenstein and Bushnell (1999)'s model where each period can be treated independently. Borenstein and Bushnell (1999) use a constant elasticity demand and estimate the model for a range of demand elasticity values (-0.1, -0.4 and -1.0) and six different demand levels. They found that the potential for market power was greater when demand was high and the fringe's capacity was exhausted, making it impossible for the small producers to increase production. In lower demand hours, Cournot producers had less incentive to withhold production because the fringe had excess capacity. In addition they found that the more elastic was demand, the less

[14] Wolfram (1998) analyzes the bidding behavior in the UK and tests the theoretical predictions of the multi unit auction theory.
[15] Their market definitions are slightly different.
[16] In particular, hydro production is allocated over the period using a peak shaving technique.

was the incentive to exercise market power. Finally they analyzed the hydro scheduling issue by allocating hydro production across periods so as to equalize marginal revenue. They found that even though the resulting hydro allocation was very different from the one implied by the peak shaving approach, prices did not change much because as hydro production was moved out from one period, the resulting price increase induces the other large producers and the fringe to increase production. This result is different from Bushnell (1998)'s findings.

Overall, the literature seems to agree on the following conclusions: more market power can be exercised when the fringe's capacity is exhausted (which usually occurs when demand is high) because this makes the residual demand curve faced by the firms with market power less elastic. The exercise of market power results in high prices, reduced output and in an inefficient allocation (production costs are not minimized). Results are very sensitive to the elasticity of demand as well as the elasticity of fringe supply. In order to say something regarding the role of hydro resources in the exercise of market power, a formal study of the hydro scheduling issue is needed.

The analysis of the hydro scheduling issue is always done following a similar approach: producers maximize their inter-temporal profits subject to certain constraints such as hydro generation being within a range determined by min and max flow constraints and by the availability of water. Then, an assumption is made regarding what sort of strategy producers may choose. Scott and Read (1996), Scott (1998) and Bushnell (1998) used a quantity strategy and the industry was modeled as a Cournot oligopoly. The main difference between their approaches is given by the method they chose to solve the optimization problem. While Scott and Read used a dual dynamic programming methodology (DDP), Bushnell solved the model by searching for the dual variables that satisfied the equilibrium conditions of the model. In particular, Scott and Read used DDP to optimize reservoir management for the New Zealand electricity market over a medium term planning horizon (1 year). They estimate a "water value surface" (WVS) that relates the optimal storage level at each period to the marginal value of water (MVW). The latter is interpreted as the marginal cost of generating at the hydro stations. The schedule of the system is determined by running each period a Cournot model in which the hydro plant is treated as a thermal plant using the WVS to determine the marginal cost of water (i.e. MVW), given the period and the storage level at that period. [17] The Scott and Read approach is rich in details as hydro allocation for the whole planning horizon is derived as a function of the MVW. However it is computationally intensive, especially when there is more than one producer who owns hydro-storage plants. It is also data demanding as information on water inflows is required on a very frequent basis. Bushnell modeled the Western US electricity market, where the three largest producers had hydro-storage

[17] The water value surface consists on a set of curves, one for each period, which relates the storage level at a certain period to the marginal value of water. It is derived recursively. The storage level at the beginning of a certain period is calculated by adding to the end of period storage level a demand curve for release of water (DCR), which is a function of MVW, and subtracting the (expected) water inflows of the period. This is done recursively starting from the end of the planning horizon, resulting on a water value surface. The demand curve for release of water is calculated by running a one stage Cournot model for a representative range of MWVs holding all other inputs constant. The DCR is given by plotting hydro generation versus MWV.

plants. He adopted a dual method to solve the model, treating the marginal value of water multiplier and the shadow prices on the flow constraints as the decision variables. He derived an analytic solution by searching for values of the dual variables that satisfy the equilibrium conditions at every stage of the multi-period problem. In order to solve his model, he simplified it by assuming that demand and the marginal cost functions were linear[18]. The planning horizon was assumed to be one month and the model was estimated for March, June and September. Bushnell (1998) found that firms could profit from shifting production from peak to off peak hours, i.e. from hours when the fringe was capacity constrained to when it was not. In particular, he estimated that hydro production was reduced by 10% (relative to perfect competition) during the peak hours, resulting in more than 100% price increase. Based on the estimated marginal water values for different months, he found that against what it was expected producers did not shift production from months of high demand to months of low demand. He argued “since the market is relatively competitive at least some of the time in each month, strategic firms do not need to reallocate across months in order to find hours in which extra output will have little impact on prices” (p.30)

III. Chile’s Electricity industry

Electricity supply in Chile is provided through four non-interconnected electric systems: Interconnected System of Norte Grande (SING) in the north, Central Interconnected System (SIC) in the center and Aysen and Magallanes in the south of the country. Total installed capacity in 2000 amounted to 9713 MW. The SIC is largest system in the country in terms of installed capacity and it concentrates more than 90% of the country’s population. Due to differences in resource availability, each system generates energy from different sources. While the north relies almost completely in thermal sources, the rest of the country also generates energy from hydroelectric sources and recently from natural gas. The most important source of energy in Chile is hydrological resources. They are concentrated in the central and southern part of the country, which explains why the SIC relies heavily on hydro generation. Fuel resources are not abundant: natural gas and a large fraction of the oil used are imported and Chilean coal is not of good quality. In what follows, all the analysis and estimations will refer to the SIC, the biggest electric system.

Gross generation in 2000 amounted to 29.577 GWh, 37% of which was hydro-reservoir generation, 38% thermal generation and 26% hydro-Run-of-River (ROR) generation. Maximum demand in the year 2000 amounted to 4576 MW (April). The generating sector is highly concentrated: 93% of total installed capacity and 90% of total generation are in hands of three economic groups (Endesa, Gener and Colbun) Endesa being the largest of them. (See Table 1). The Hirschmann-Herfindahl index is 3716. In order to simplify the reading of the paper, I will refer to these companies as “Firm 1” (Endesa), “Firm 2” (Gener), and “Firm 3”(Colbun). These three firms differ in terms of size, their generating plants portfolio and the associated marginal cost functions (See Figure 1).

[18] The slope of the demand function was assumed to be constant across periods and set at a level such that the elasticity of demand at the peak forecasted quantity was -0.1.

While Endesa relies mostly on hydro sources, Gener owns the majority of the thermal plants of the system.

Firm 3 has the lowest marginal cost plant, but is also the smallest firm in terms of capacity. Firms 1 and 2 both own low and high marginal cost plants, being this feature more accentuated in the case of Firm 1.[19]

Generating companies are allowed to sell to three different customers: large consumers, distribution companies and other generating companies[20]. Transfers of energy between generators take place in a simulated "uniform price" spot market. The "market" clearing price, called the Short Run Marginal Cost (SRMC), is given by the marginal cost of the last generator required to balance supply and demand, taking into account transmission constraints and losses. It is calculated by an independent entity, called the "Load and Economic Dispatch Center" (CDEC), according to marginal cost information reported by the generators themselves. Neither distribution companies nor large consumers have access to the simulated spot market. Large consumers are entitled to enter into contracts directly with generators and to freely negotiate the price for electricity. Distribution companies are required to enter into long-term contracts with the generators, at a regulated price, in order to purchase electricity for the supply of their regulated consumers. This regulated price is set every 6 months by the regulatory agency called the National Energy Commission (CNE). A particular feature of the Chilean pricing system is that the nodal price is required to lie within a band of +/- 10% of the non-regulated price; this means that every time the regulated price is set, the regulator has to get information on the free price and make sure that that constraint is satisfied.

The CDEC is in charge of the dispatch, of the operation of the system, the estimation of the SRMC, of guaranteeing open access to transmission lines and of other administrative tasks. For a detailed analysis of the Chilean regulation, see Arellano (2001a, 2001b)

IV. Theoretical Model

I will estimate an ex-ante model much in the spirit of Green and Newbery (1992), Borenstein and Bushnell (1999) and Bushnell (1998) using real demand and cost data for the year 2000. Following Borenstein and Bushnell (1999) and Bushnell (1998) the industry is modeled as a Cournot duopoly (Firms 1 and 2) with a competitive fringe[21]. The model that is analyzed in this Chapter will be referred to as the "base model".

The portfolio of generation sources is very important; in fact, it defines the *way* market power can be exercised. The whole idea behind the exercise of market power is to reduce

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[19] In addition, there is an important degree of vertical integration in the SIC. In particular, Enersis, the owner of Endesa, is also the owner of two large distribution companies, Chilectra and Rio Maipo whose customers amount to 43% of the SIC.

[20] Large consumers are those whose maximum demand > 2 MW. They amount to 50% of total consumption. 70% of them are really free to choose whom to contract with.

[21] I also estimated the model assuming that the third largest firm (Colbun, "Firm 3") had market power but it turned out that it always ended up behaving as a price taker. In other words, it wasn't big enough to be able to use its resources strategically.

output in order to increase market price. However, the decisions that producers can make are different depending on whether they are in a purely thermal / purely hydro or in a mixed electric system. In a purely thermal system, the only decision that can be taken is when to switch on or off a plant and how much to produce at every moment in time; in this context, market power is exercised by reducing output when rival generators are capacity constrained, which usually corresponds to periods of high demand. A system with hydro-reservoirs, on the other hand, allows producers to store water during some periods and release it in some others; in other words, they are able to "store" power and release it to the market at their convenience. Therefore, hydro producers are entitled to decide not only when to switch on or off their plants and how much to produce, but also to decide *when* they want to use their hydro resources over a certain period of time. This (dynamic) scheduling decision is not available to thermal producers[22]. In a purely hydro system producers exercise market power by exploiting differences in demand elasticities in different hours. In particular, they shift production from periods where demand elasticity is high to periods when it is low.[23]

Only water from hydro reservoirs can be used strategically. Since water from run of the river (ROR) sources can't be stored, it can't be used by producers to manipulate the price. ROR plants will be treated in the model as "must-run" (MR) units except for those ROR plants that are associated to a reservoir system upstream, which will be included as part of the reservoir complex. In the Chilean system, Firm 1 and the Fringe own hydro-reservoir plants. Their hydro capacity amounts to 78% and 22% of total hydro-reservoir capacity respectively. Firm 2 is a purely thermal plant, concentrating the largest fraction of thermal resources in the SIC (46%). See Table 1 for more detailed information. In order to simplify the model as much as possible, I will assume that Firm 1 and the fringe only have one reservoir complex. They will be made up by the aggregate of individual reservoirs.

The model will determine hydro scheduling by Firm 1. However, since the Fringe also owns a medium size reservoir, it will be necessary to allocate its hydro production in a certain manner. In particular, I will use the Peak Shaving approach. The basic idea is the following: when there are no flow constraints, producers schedule hydro generation so as to equalize the marginal profit that they earn from one more unit of production over the whole period in which the hydro plant is being used. If the market were perfectly competitive, prices would be equalized. If there were market power, then generators would equalize marginal revenues over time. As long as demand level is a good indicator of the firm's marginal revenue, a peak shaving strategy would consist in allocating hydro production to the periods of higher demand.[24] In addition, producers also have to take account of minimum flow constraints, given by technical requirements and irrigation needs, and maximum flow constraints, given by capacity. As a result, hydro production by the fringe was distributed across periods allocating as much as possible (given

[22] Notice that even in a perfectly competitive market producers are able to hydro schedule. The difference is that when the market is competitive, difference between on peak and off-peak hours is *reduced* as opposed to when producers exercise market power in which case difference is *enlarged*.
[23] See Johnsen et al (1999), Bushnell (1998) and Halseth (1998).
[24] This is true when using either a linear or a constant price-elasticity demand.

min/max flow constraints) to every period in order to eliminate or reduce demand peaks[25].

Cournot producers face a residual demand given by:

$D^R (P_t) = D(P_t) – S^f (P_t) – q^{MR}_t - q^{PS}_{h\,t}$

where D(P) is market demand, D^R (P) is residual demand, S^f (P) is the Fringe's thermal supply function , q^{MR} is must-run units' generation and q^{PS}_h is the Fringe's hydro production from reservoirs distributed across periods according to a Peak shaving strategy.

Each firm's maximization problem is given by:

Firm 1's Optimization problem

$$\text{Max} \sum_t \{ P_t (q_t)*(q_{1ht} + q_{1Tht}) – CT_1 (q_{1Th\,t}) \} \tag{1}$$

s.t.

(2) $q_{1Th\ MIN} \leq q_{1Th\ t} \leq q_{1Th\ MAX}$ $\forall$ t (thermal production min/max constraints)

(3) $q_{1h\ MIN} \leq q_{1h\,t} \leq q_{1h\ MAX}$ $\forall$ t (hydro production min/max constraints)

(4) $\Sigma_t\, q_{1h\,t} \leq q_{1h\ tot}$ (hydro resources availability)

Firm 2's optimization problem

$$\text{Max} \sum_t \{P_t (q_t) * (q_{2Tht}) – CT_2 (q_{2Tht}) \} \tag{5}$$

s.t.

(6) $q_{2Th\ MIN} \leq q_{2\,Th\,t} \leq q_{2Th\ MAX}$ $\forall$ t (thermal production min/max constraints)

where:

$P_t (q_t)$ = is the inverse function of the residual demand in period t

Q_t = is total production by firms 1 and 2 in period t, ($Q_t = q_{1t} + q_{2t}$),

$q_{it} = q_{iTh\,t} + q_{ih\,t}$ is total production by Firm i in period t,

$q_{iTh\,t}$ = total energy produced by Firm i out of thermal plants, period t

$q_{1h\,t}$= total energy produced by Firm 1 out of hydro-storage plants, period t

$CT_i(q_{iTh\,t})$ = Total Cost function, thermal plants, firm i

$q_{iTh\ MIN}$ = Minimum thermal production, Firm i, period t

[25] For more detail on the peak shaving approach see Borenstein and Bushnell (1999).

$q_{iTh\ MAX}$ = Maximum thermal production, Firm i, period t

$q_{1h\ MIN}$ = Minimum hydro production, Firm 1, period t

$q_{1h\ MAX}$ = Maximum hydro production, Firm 1, period t

$q_{1h\ tot}$ = available hydro production for the whole period

t =time period within the planning horizon. The planning horizon of the model will be assumed to be a month and will be divided in 6 sub-periods (t=1,2,..6) of equal length.

Firm 1's Lagrangean is given by:

$$L= \sum_t \{ P_t (q_t)*(q_{1ht} + q_{1Tht}) - CT_1 (q_{1Tht}) - \lambda_{1t}*(q_{1Tht} - q_{1Th\ MAX}) - \alpha_{1t} *(q_{1Th\ MIN} - q_{1Tht})$$

$$- \gamma_{1t} (q_{1ht} - q_{1h\ MAX}) - \delta_{1t} (q_{1hMIN} - q_{1h\,t}) \} - \sigma_1 (\Sigma_t\, q_{1h\,t} - q_{1h\,tot}) \qquad (7)$$

Firm 2's optimization problem is simpler because it only owns thermal plants. Its Lagrangean is given by

$$L = \sum_t \{P_t (q_t)*(q_{2Tht}) - CT_2 (q_{2Tht}) - \lambda_{2t}*(q_{2Tht} - q_{2Th\ MAX}) - \alpha_{2\,t}*(q_{1Th\ MIN} - q_{2Tht})\} \quad (8)$$

Where λ_{it}, α_{it} , γ_{1t} , δ_{1t} and σ_1 are the Lagrange multipliers for maximum thermal capacity, minimum thermal capacity, maximum hydro capacity, minimum hydro capacity and available hydro flows constraint respectively. They all must be positive. It is important to keep in mind that σ_1 is the only multiplier that is constant over time; it indicates the marginal value of water, i.e. the additional profit Firm 1 would get if an additional unit of water became available.

FOC for Firms 1 and 2 are[26]:

$$\frac{\partial L}{\partial q_{1Tht}} = Pt (q_t) + q_{1t} * \frac{\partial Pt (q_t)}{\partial q_t} - \frac{\partial CT_1 (q_{1Tht})}{\partial q_{1t}} - \lambda_{1t} + \alpha_{1t} = 0 \qquad (9)$$

$$\frac{\partial L}{\partial q_{1ht}} = Pt (q_t) + q_{1t} * \frac{\partial Pt (q_t)}{\partial q_t} - \gamma_{1t} + \delta_{1t} - \sigma_1 = 0 \qquad (10)$$

$$\frac{\partial L}{\partial q_{2Tht}} = Pt (q_t) + q_{2Tht} * \frac{\partial Pt (q_t)}{\partial q_{2\,t}} - \frac{\partial CT_2 (q_{2Tht})}{\partial q_{2Tht}} - \lambda_{2t} + \alpha_{2t} = 0 \qquad (11)$$

These conditions can be reformulated as follows:

[26] Slackness conditions are not reported.

$$(9')\ MR_{1t} = c_1 + \lambda_{1t} - \alpha_{1t}$$

$$(10')\ MR_{1t} = \sigma_1 + \gamma_{1t} - \delta_{1t} = \Omega_{1t}$$

$$(11')\ MR_{2t} = c_2 + \lambda_{2t} - \alpha_{2t}$$

where MR_i is Firm i's marginal revenue and c_i is Firm i's thermal production marginal cost.

Each firm schedules its production in order to equalize marginal revenue to thermal marginal cost each period (adjusted for shadow prices), as expected (constraints 9' and 11'). In addition, Firm 1 allocates water across time so as to equalize the marginal cost of water (Ω_{1t}) with the cost of producing an additional unit of power from the marginal thermal plant (constraints 9' and 10')[27]. This means that an extra unit of water will be generated until its cost is equal to the cost of the most expensive thermal plant in use. The intuition of this is the following: an additional unit of water would replace production from the least efficient thermal plant that is in use and profits would increase by the cost of production that has been saved. If minimum and maximum hydro production constraints were not binding, then marginal cost and marginal revenue would be constant as the marginal value of water (σ) is constant over time. Firm 1 would allocate hydro storage resources in order to equalize marginal cost across periods. Firm 1 peak shaves marginal revenues rather than prices.

If thermal and/or hydro min/max capacity constraints are binding, these conclusions still hold but applied to a broader definition of marginal cost / marginal value of water that includes the shadow price of increasing/decreasing installed capacity.

The Fringe solves exactly the same optimization problem solved by Firm 1; the only difference is that $\partial Pt(q_t) / \partial q_{Ft} = 0$ as it does not have any market power, and thus behaves as a price taker. As a consequence, the fringe uses its plants (thermal and hydro) until the marginal cost (thermal or hydro plants) is equal to the market price:

$$(12)\ P = c_F + \lambda_{Ft} - \alpha_{Ft}$$

$$(13)\ P = \sigma_F + \gamma_{Ft} - \delta_{Ft} = \Omega_{Ft}$$

Some final remarks regarding the model that will be used to analyze the exercise of market power are in order. First of all, and as the reader has probably noticed, this is a completely deterministic model. In particular, hydrological resources, marginal costs and load levels are assumed to be known in advance by the agents. Certainty with respect to thermal marginal cost functions and demand fluctuations should not be a real concern, as the former are well known in the electricity industry and the shape of the load curve has been relatively stable in the past years. Certainty with respect to hydrological inflows is clearly a more arbitrary assumption. In the context of my model, this should not be too

[27] Notice that Firm 1 allocates its plants (thermal and hydro) efficiently *given* the total level of production (which is inefficient as the firm produces until marginal cost = marginal revenue < price).

problematic either because I assumed that producers maximize over a short time horizon (one month). The longer the planning horizon, the more uncertain are the hydro inflows, and the more important it is to incorporate uncertainty into the model. Secondly, the model lacks of dynamic competition elements. This omission is clearly important for this particular industry. In the context of a power exchange system, the producers interact on a very frequent basis providing optimal conditions to engage in (tacit) collusive practices. For instance, producers can easily learn their competitors' strategies, monitor their behavior and credibly threat in case of deviating from the "collusive" strategy. In this sense, the results of the model should be seen as a *lower* bound of market power. On the other hand, the model does not incorporate the effect of high prices on potential entry or in consumption patterns; accordingly market power might be *over*estimated. Finally, transmission constraints and contracts were not taken into account yet. [28]

V. Model estimation

Supply side

Each firm's marginal cost function was calculated aggregating their thermal plants' marginal cost functions. I assumed that each plant had a constant marginal cost up to its expected capacity level.[29] The constant marginal cost at the plant level (and at the plant "mouth") was calculated as the monthly average of the weekly marginal cost reported by the CDEC. This reported value does not incorporate transmission losses. Since market behavior will be modeled as if all transactions took place at the same geographic node, it is necessary to incorporate the fact that the MC of delivering energy at one node of the system is different from the MC of "producing" energy because a fraction of the energy that is generated in the plant is lost while it is being transmitted to the consumption node. In other words, the marginal cost of a KW produced by a plant located in node A and consumed at node B is "production MC^A" + "transmission charge". In order to incorporate this, I calculated for each plant a "system-equivalent marginal cost" as Production MC x Penalty factor (calculated by the CNE).

Each plant's capacity was adjusted for transmission losses, auto-consumption and average availability[30]. Unfortunately it was not possible to get separate data for scheduled and non-scheduled (non-expected) maintenance periods[31]. Related papers do not adjust for transmission losses that occur within the market but only for those that take place when energy is imported. I think this assumption is not appropriate for the Chilean case. The distance from North to South in the SIC is approximately 2300 kms. (about 1430 miles) and so transmission losses are likely to be important. In order to take them into account I adjusted capacity by the transmission loss factor. In other words, if maximum

[28] These issues are analyzed in two companion papers. See Arellano (2002a and 2002b).

[29] Start-up costs were not taken into account.

[30] As it was discussed in Borenstein et al (2000) the use of average availability may underestimate true expected capacity.

[31] Availability figures are high for international standards. This may be due to the way they are calculated: a plant is considered to be available if it doesn't go down when it is dispatched. However plants that are not dispatched but are available are also considered being available. The issue here is that there is no certainty that those apparently available, non-dispatched plants would be effectively available if dispatched. In addition, availability data seems to include maintenance periods, which is a strategic variable.

capacity is q, then the maximum delivered capacity is q*(1-LF) where LF is the loss factor. Finally since demand will be calculated as the *sales* of the system, auto-consumption must also be subtracted from total production. I used the last 5 years average for both the transmission loss and auto-consumption factors (4.6% and 2% respectively).

Resulting marginal cost functions are plotted in Figure 2. Notice that both Firms own low and high marginal cost plants, being this feature more accentuated in the case of Firm 1.

Demand

As it was said before, Cournot producers face a residual demand given by:

$$D^R (P_t) = D(P_t) - S^f (P_t) - q^{MR}_t - q^{PS}_{ht}$$

Where D(P) is market demand, D^R (P) is residual market demand, S^f (P) is the fringe supply's function (adjusted by transmission losses), q^{MR} is must-run units' generation and q^{PS}_h is the hydro production from reservoirs owned by the fringe that is distributed across periods according to a Peak shaving strategy.

a) Market demand: I constructed a step function representation of April-2000's load curve with 6 discrete load levels (t=1 for the highest load level)[32]. The load level of each step was set equal to the average of the loads covered by those hours in the full load profile[33]. See Figure 3. Each load level has an associated price given by the regulated price, which is the price paid by final consumers. This price-quantity point will be referred to as the "anchor point" for each period (Figure 4). Given that there is only one price-quantity observation for each period, it is not possible to directly estimate the market demand function; all that can be done is to assume a functional form and parameterize it using each period's anchor point and an assumption for the price-elasticity of demand for electricity.

Demand is assumed to be linear $D(p_t) = A_t - BP_t$.[34] As a consequence, price elasticity increases as the level of production is reduced and the elasticity of demand at the price where the market clears is always higher when there is market power.

The empirical literature has emphasized the importance of price elasticity of demand in the results. In my model, demand elasticity will also turn out to be very important, as Cournot equilibrium will be closer to the competitive equilibrium the more elastic is demand. In addition hydro scheduling will be determined in part by demand elasticity. Estimates of the price elasticity of demand for electricity vary widely in the literature. As Dahl (1993) pointed out, the estimation of price elasticity is sensitive to the type of model used, to the estimation technique and to the data set used. In addition, studies differ on

[32] I chose April because historically it has been the month where the maximum demand of the year takes place.

[33] The observed load per hour was increased by 13% to take account of spinning reserves.

[34] A linear functional form is consistent with the peak shaving criteria that will be used later to allocate hydro generation: periods of high demand are also periods of high marginal revenue.

their definition of short run and long run price elasticity[35]. In lagged adjustment models short run is defined as the 1-year response to a permanent increase in prices. Garcia-Cerruti (2000) using panel aggregate data for selected California counties (1983-1997) estimated that short run price elasticity went from –0.132 to –0.172, while the range for the long run was from –0.17 to –0.19. In the particular case of Chile, Galetovic et al (2001) used a partial adjustment model to estimate the demand for electricity by commercial and residential users. Their estimates of short run (long run) price elasticity were –0.33 (-0.41) and -0.19 (-0.21) for residential and commercial users respectively.[36] Short run estimates of price elasticity are lower when the period in which the consumption pattern may be adjusted is shorter. Wolak and Patrick (2001) looked for changes in electricity consumption due to half hourly price changes in the England and Wales market. They focused on 5 large and medium sized industrial and commercial customers. Not surprisingly, they got much lower estimates of price elasticity. In the water supply industry, which was the most price responsive industry analyzed, price elasticity estimates went from nearly zero (at peak) to -0.27. The steel tube industry was the least price responsive industry, with price elasticity estimates going from nearly zero to –0.007 (there is no indication of the demand level at which the upper estimate was observed). Finally, Dahl (1992) found no clear evidence that the developing world's energy demand were less price elastic than for the industrial world.

Because of the large variation in the price-elasticity estimates, I follow the traditional approach of estimating and reporting the results of the model for different values of elasticity. In particular, the market demand will be estimated for 5 different assumptions of price elasticity of demand E = {-0.1, -1/3, -1/2, -2/3, -1.0}, measured at the anchor point at peak hours. In the main text of the paper I only report results for –1/3 and –2/3.[37] These values may appear to be high compared to some of the estimates reported. However and under the assumption that consumers are sensitive to price changes at least until a certain degree, it is not reasonable to assume that consumers will not react to the exercise of market power. In particular, we should expect them to learn, after a while, that the price is higher in certain periods than in others and to adjust their consumption behavior accordingly.[38] This change should mitigate the potential for market power. It is very difficult to explicitly incorporate this demand side reaction to market power into the model. An indirect way of doing it is to assume that the market is more price responsive than short run estimates of price elasticity indicate. Results for the E=-2/3 assumption are

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[35] Nesbakken (1999) suggested that since there is a lot of individual variation in energy used, estimates based on micro data were more reliable.

[36] As I mentioned before, the regulated price in Chile is fixed for a period of 6 months. During that period, it changes mostly according to he evolution of inflation. This means that the authors did not have much price variation over time. However, since the price that was used to estimate price elasticity was the final price, and since that price includes transmission and distribution charges that vary across consumers according to different parameters, they did have cross-section price variation.

[37] For comparison purposes, I report price elasticity values ("E") assumed by other authors. A constant elasticity of demand was assumed by Borenstein and Bushnell (1999), estimating the model for E=-0.1, -0.4 and -1.0 and by Andersson and Bergman (1995) who used E=-0.3. A linear demand was assumed by Wolfram (1999) with E=-0.17 at the mean price and quantity and by Bushnell (1998) who assumed E=-0.1 at peak forecasted price/quantity point.

[38] See Wolak and Patrick (2001) and Herriges et al (1993) for estimations of elasticity of substitution within the day.

reported as a way to illustrate the effect of increasing price-sensitivity of demand. The results for the case of E = -0.1, E=-1/2 and E=-1.0 are reported in the Appendix.

The price elasticity assumption was incorporated in the model through the slope parameter B, which was calculated such that the elasticity at the *peak* demand level was equal to "E". This implies that I will work with parallel demands ("same slope"). The intercept was calculated so as to fit anchor quantity and anchor price at each demand level (given the calculated slope B).[39] See Table 2 for demand parameters used assuming E = -1/3.

By assuming that market demand is linear and the slope is constant across load levels, I am implicitly assuming that market demand at peak hours is less elastic than demand at off peak hours (at a constant price). [40] Neither the linear demand assumption nor the anchor point chosen had any influence on the results. The main conclusions (even order of magnitudes) were the same when running the simulation assuming that the slope was not constant[41].

a) *Fringe's supply:* In order to minimize the number of steps that the residual demand faced by Cournot producers have, I decided to use a linear approximation of the Fringe's supply function. This linear function is given by the following expression (see Figure 5):

$$
(14) \qquad MC^F \begin{cases} = 3.66 & \text{for } 0 \leq Q^F \leq 54.9 \text{ MW} \\ = -114.60441 + 2.156038Q^F & \text{for } 54.9 \leq Q^F \leq 58.5 \text{ MW} \\ = 11.51217 & \text{for } 58.5 \leq Q^F \leq 399.9 \text{ MW} \\ = -333.526 + 0.8628848Q^F & \text{for } 399.9 \leq Q^F \leq 433.7 \text{ MW} \end{cases}
$$

b) *Must run quantity:* The plants that have to be dispatched all the time (no matter the price) and thus cannot be used strategically by their owners were designated as "must run" plants. This includes two small co-generator thermal plants that produce electricity and steam and all the hydro-ROR plants that are not associated to any reservoir system. q^{MR} was calculated as April 2000's average generation per hour in the case of thermal plants, and in the case of hydro-ROR plants, as the average

[39] A similar approach was used by Bushnell (1998).

[40] Empirical evidence supports the assumption of price elasticity being a function of the output level as the linear functional form implies. However, evidence is not conclusive regarding whether demand at peak hours is more or less elastic than at off peak hours. Aigner et al (1994) estimated that demand for electricity in the winter was more elastic during peak periods while in the spring/autumn season it was the off peak demand the one that was more price responsive.

[41] In the "different slope approach", the slope parameter B was such that the elasticity at every anchor point was equal to "E". Results are reported in Appendix #1. I decided to report in the main text the results for the same slope approach because when using the different slope approach, residual demands intersect on a certain (and relevant) price range making it more difficult to interpret results. Results are almost the same under both approaches.

generation in a normal hydro year calculated according to the Energy Matrix provided by the CDEC. [42]

c) *Hydro-reservoir generation by the Fringe* ($q^{PS}{}_h$). In order to allocate the hydro-storage generation by the Fringe, I calculated, for each plant, the average generation per month (in this case April) in a normal hydrological year based on the Energy Matrix estimated by the CDEC. This monthly hydro generation was assumed to be total hydro production available for the period. It was allocated over the month according to the peak shaving strategy described before. Minimum and maximum flow constraints were also taken into account. $q^{PS}{}_h$ used to estimate the model is the average hydro generation per hour allocated to each sub-period according to this approach.

   Since the Fringe owns relatively small hydro-storage plants, the amount of hydro production that can be allocated through a peak shaving approach is also small. Peaks are only slightly reduced and the shape of the "shaved load" curve remains mostly the same. (See Figure 6).

d) *Residual demand*: See Table 3 for a summary of what was subtracted from market demand (April 2000) to get the residual demand faced by the Cournot producers. The shape and position of residual demands faced by Cournot producers is explained by a combination of three elements: the anchor point, the fringe's supply for thermal production and the load curve shape that results after allocating fringe's hydro production through a peak shaving strategy and**.** (See Figure 7).

Hydro data

Minimum hydro production per hour is given by technical requirements and by irrigation contracts. Maximum hydro production per hour is determined by technical requirements. Total April's available hydro production is 1118.1 GWh according to the Energy Matrix provided by the CDEC (See Table 4)[43]. Fringe's hydro production was allocated according to the Peak shaving strategy, as was explained before. Hydro scheduling by Firm 1 will be a result of the model.

VI. Results

Competitive equilibrium

As a benchmark case, I calculated the competitive equilibrium. System's hydro-storage production (from the fringe *and* Firm 1) was allocated according to the peak shaving strategy. The competitive equilibrium was calculated as the quantity - price point where

[42] Since must run plants' production was subtracted from total demand, must run plants were also removed from the set of available units (in other words, they are not included in the aggregated marginal cost function).

[43] Unfortunately, the CDEC does not have and estimation for the Laja system (the largest in the country). Because of that I used the observed average generation of that hydro system in April of a normal year.

demand (net of hydro production) and the aggregate marginal cost function intersect. Resulting competitive equilibrium is reported in Tables 5 and 6.

Observe that the equilibrium is exactly the same for the first four periods (t=1 to 4) and almost the same for the fifth one. This is a consequence of net demand being the same in those periods, or, in other words, of hydro production being so large that its allocation across the month completely flattens demand in those periods, eliminating (reducing) the peaks[44]. See Figure 8.

Cournot equilibrium

The Cournot-fringe was solved with GAMS/CONOPT using an iterative process. I started assuming that each firm produced at the average level observed in April 2000, and then solved for the Cournot equilibrium for Firm 2. Given the resulting production schedule, I solved for Firm 1 and used the resulting Cournot equilibrium as an input for Firm 2's maximization problem. I continued this iteration process until the model converged to a solution for each of the firms[45]**.** Results are reported in Tables 7 and 8.

Total quantity is smaller than in the competitive model and prices are considerably higher, especially when demand is high (see figures 9 and 10). Notice that as demand falls, the Cournot equilibrium (price and production) converges monotonically to the competitive equilibrium. The exception is given by the last period when demand is at its lowest level which may be explained by Firm 2 increasingly constraining production as demand falls. The more elastic is demand, the larger is total production and the closer is hydro scheduling to the competitive equilibrium

When demand is at high and medium levels Firm 1 is the one that really enjoys market power. Indeed, Firm 1 has so much market power and is able to drive prices up by so much that Fim 2's optimal strategy is to produce at capacity. Firm 2 exercises market power only in the last 2 periods, when demand is low. (See Figure 12) [46].

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[44] A similar result holds for the "different slope approach". The difference is that since the slope is different, net demand functions are not exactly the same in the first four periods but they are very close.

[45] Uniqueness of equilibrium was not investigated theoretically but empirically. In particular, the simulation was solved for 400 randomly chosen starting points. The model always converged to the same aggregated equilibrium: prices, each firm's total production, marginal cost, marginal value of water and profits. The only exception is given by the Firm 1's production strategy. Even though it is true that the equilibrium for Firm 1's total production is unique, this is not true for its production strategy, i.e. the decision of how much is produced from its thermal and hydro-storage plants. Multiplicity of equilibrium is explained by Firm 1 being able to allocate hydro production over time and by marginal cost being constant over relevant intervals of output. Indeed, observe that the FOCs are in terms of MR and MC and that the MR is a function of total sales and independent of what plants were used. This problem only affects Firm 1 as it is the only one who is able to allocate hydro production over time and that is able to combine thermal and hydro plants to produce a certain output level. I want to remark that in spite of this multiplicity of equilibrium, all the qualitative conclusions hold and magnitudes are very similar. Values reported in the tables for q1h and q1th are averages calculated over 400 diferent estimations of the model.

[46] Strictly speaking, Firm 2 is not producing at capacity as it still has some thermal plants that are not being run. However, the big difference observed between the marginal cost of Firm 2's next available plant and the marginal plant at that demand level (almost $30) prevents Firm 2 from increasing production. By contrast, Firm 1 has a large capacity at a relatively low marginal cost. See Figure 2

Firm 1 chooses to satisfy demand mainly through hydro production. In particular, it uses *all* the hydro production that is available but allocates it differently than in the competitive model. Firm 1 allocates relatively *less* water to high demand periods and relatively *more* water to the low demand periods (See Figure 12) [47]. This hydro allocation enlarges the difference between peak and off-peak periods, as opposed to what is observed under competition. This effect is *smaller* the more elastic is demand. [48]. Firm 1's hydro scheduling strategy is consistent with what has been found in the literature. In their study of the Norwegian electricity market, Johnsen et al (1999) argued that "market power can not be exercised in markets dominated by hydroelectric producers unless there are transmission constraints". The hydro producer exploits differences in demand elasticity, reducing production when the constraint is binding (and demand is less elastic). Notice that whether or not transmission constraints bind has an impact on the elasticity of the residual demand faced by the Cournot producers as imports/exports of energy take place (or not). My model results show that transmission constraints are not a necessary condition for the exercise of market power by hydro producers. Capacity constraints (supply constraints) will have the same effect. In other words if a hydro producer competes with a thermal producer, the first one may choose to restrict production when the thermal producer is capacity constrained, and to increase production the rest of the time. In this way, one could say that Firm 1 faced a less elastic demand when Firm 2 is capacity constrained. As a consequence, the shifting of hydro production is also the result of Firm 1 exploiting differences in price elasticity.

Firm 1's large hydro capacity is the source of its market power. Accordingly, Firm 1 explicitly schedules its hydro production in order to exploit differences in price elasticity and exercises as much market power as it can. However Firm 1 also exercises market power in a less observable way, namely the use (or more strictly speaking the "no use") of its thermal capacity. Indeed Firm 1 uses, on average, only 15% of its thermal capacity. If Firm 1's thermal portfolio were in a third generator's portfolio, Firm 1 would be more constrained in the exercise of its market power[49]. On the other side, Firm 2's large thermal capacity was not enough to enable it to exercise market power. Behind this result is the fact that a large fraction of its capacity are baseload plants, which are usually not marginal and thus do not set the market price.

Firm 1's markups are decreasing as the demand it faces falls and they go from 66% to 76%, with a weighted average of 72% when demand elasticity is –1/3 . The more elastic is demand, the smaller are the Lerner Indices, as expected. Firm 2's markups do not

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[47] This finding is consistent with Bushnell (1998)'s findings.

[48] It cannot be argued that the demand assumptions are driving the results. Even though I am implicitly imposing that peak demand is less elastic than off-peak demand (at a constant price) I am not imposing in any way how the water should be allocated across periods. This is just a result of the model.

[49] Arellano (2002a) estimates the market equilibrium assuming that Firm 1's thermal portfolio is divested. In particular, she analyzes the impact of selling Firm 1's thermal portfolio to two different set of agents: i) a unique producer and ii) many small producers with no market power.

exhibit the same monotone pattern. In particular, the Lerner index is larger during the middle hours (Table 9). [50]

The Cournot equilibrium is not only inefficient because production falls short the competitive equilibrium production level but also because costs of production are not minimized. In particular, the fringe is running plants that are less efficient (higher marginal cost) than the ones that are being withheld by Firm 1 and hydro production is used to increase the difference between peak and off peak periods.[51]

A final question regarding the exercise of market power in this industry is who are the winners and the losers. In order to analyze this issue, I calculated each firm's producer surplus, the consumer surplus and welfare (as the sum of producer and consumer surplus). Results are reported in Table 10.

All of the producers are better off when market power can be exercised. As expected the less elastic is demand, the better off producers are and the worse off consumers are as more market power can be exercised. Observe that even though it is Firm 1 the one who is really able to constrain production and drive prices up, the real winner, in relative terms, is Firm 2. The reason behind this result is clear: since Firm 2 is capacity constrained when demand is high, its production level is very close to the competitive level but the price is considerable higher.[52] Finally notice that the increase in the producer surplus almost compensates the consumer surplus' reduction, keeping welfare loss at a relatively low level.

VII. Extensions to the model

In order to evaluate how sensitive these results are to the assumptions used, I re-estimate the model under two different conditions. First, I analyze the effect of a smaller amount of water inflows being available to allocate over the planning horizon. This exercise is motivated by the role hydro resources play in Firm 1's market power. Secondly, it is more realistic to think that Firm 1 may prefer to schedule its hydro production over a longer planning horizon, saving a fraction of the hydropower available in a particular month to be used in a later period. Therefore, the effect of producers maximizing over an entire year rather than a month will be studied.

Cournot equilibrium with reduced hydro inflows.

The model was estimated assuming that the hydrology was "dry" rather than normal. In particular, I assumed that hydro flows were 60% of the flows registered in a "normal"

[50] The Lerner index shows market power that is exercised. Since Firm 2 is "capacity constrained", it is probably not exercising market power. Consequently, Firm 2's Lerner index doesn't have an economic meaning during those periods.
[51] The possibility of inefficient dispatching was pointed out by Von der Fehr and Harbord (1993), Borenstein et al (2000) and Wolfram (1998).
[52] Firm 1's profits are considerably larger than Firm 2's and so in absolute terms, the winner is Firm 1.

year[53]. Since all hydro flows are assumed to be smaller, the fringe is also smaller and residual demand is relatively less elastic than in the base model. Data used to calculate residual demand are reported in Table 11 while Cournot and competitive equilibrium for the E=-1/3 case is in Tables 12 and 13. The results for the E=-0.1, E=-1/2, E=-2/3 and E=-1.0 are reported in Appendix #1.

Hydro flows are just enough to satisfy the minimum hydro production constraint; as a consequence, Firm 1 and the Fringe have little hydro scheduling to do. In particular, the fringe is not able to schedule its plants differently than running them at minimum capacity. Therefore the shape of the load curve is completely preserved and peaks are not eliminated, just reduced by a very small amount.

Since hydro resources are not abundant as before, thermal capacity is more intensely used, driving prices considerably up - no matter whether the market is competitive or not. The marginal value of water is also considerably higher as less efficient thermal plants must be dispatched in order to meet demand.

Results reported in Table 13 show that even under these extreme circumstances significant unilateral market power could be exercised. However, while Firm 1's market power has been diminished, Firm 2 has the incentive and the ability to act strategically by withholding supplies from the market to keep them from falling further. Prices are lower than when Firm 1's hydro capacity was larger but at the same time, they are still above the competitive equilibrium. Unlike what happened before, prices are increasingly different the lower is demand. How can this be explained? Observe that Firm 1 is capacity constrained when demand is at its maximum level. Therefore, Firm 1 constrains more its thermal production (relative to the competitive equilibrium) the lower is demand. In addition, and contrarily to what was found before, relatively more water is allocated to periods of high demand and relatively less to periods of low demand[54]. Firm 1 behaves like a price taker when demand is high and exercises some market power when demand is low. On the other hand, Firm 2 constrains more its (thermal) production when demand is high.

Firm 2 is still the producer who is benefited the most by the exercise of market power, but similarly to what happens to the remaining producers, its net benefit is smaller than when hydro flows are "normal". (See Table 14). Observe that in relative terms, consumers' loss is smaller (relative to the competitive equilibrium) when hydro flows are reduced as less market power is exercised. Since this more than compensates the smaller increase in the producer surplus, welfare loss is also smaller.[55] The finding that the

[53] Actual values for the 1998/99 year (the driest in the past 40 years) are 55%, 78% and 60% for hydro reservoirs, ROR and total hydro flows.

[54] The shadow price of the minimum hydro production constraint is, for the first time, different than zero. In particular, Firm 1is willing to pay to reduce the lower bound limit when demand is at medium and low levels. Indeed it is willing to pay more the lower is the level of demand. This result is consistent with a producer who is able to exercise market power only when demand is at medium-low levels.

[55] Welfare loss from the exercise of market power being smaller when hydro flows are reduced (as opposed to normal) does not mean that in absolute terms society is also better off. Indeed society is better off when hydro flows are normal than dry, even is market power is exercised.

welfare loss from the exercise of market power is smaller when hydro flows are reduced (as opposed to normal) does not mean that in absolute terms society is also better off. Indeed society is better off when hydro flows are normal than dry, even if market power is exercised.

Hydro scheduling over a longer time horizon

The model that was used to analyze the exercise of market power in Chile's electricity industry assumed that the hydro producer was able to allocate its hydro production over a 1 month planning horizon. Indeed, the optimization problem solved by Firm 1 in that model, assumed that the water left in the reservoir at the end of the period had no value; accordingly, the incentive to use less water than what is available is reduced. In real life, however, producers usually maximize over a longer time horizon, and thus it is reasonable to expect them to save a fraction of their hydro flows available in a particular month to be used in a later period. An interesting question is whether Firm 1 would follow the same hydro scheduling strategy that was found optimal in the 1-month model if the planning horizon was longer. In particular, would Firm 1 exploit inter-month differences in demand elasticity by moving water around? If so, then water should be stored in those months in which demand is less elastic and released when demand is more elastic. According to Table 15, this means that hydro production should be shifted from October, November and December to March, January and the winter months (June-August).[56]

There are at least two different approaches to investigate Firm 1's hydro scheduling strategy over the year: analysis of the marginal value of water (MVW) and a direct estimation of the model assuming that the planning horizon is longer. Since the data set required to estimate a comprehensive 1-year version of the model is not available, and therefore many arbitrary assumptions would be needed, I will concentrate on the MVW analysis. A very simple version of a 1-year model is analyzed and estimated in Appendix #2.

The basic idea behind the MVW analysis is the following: given that generators allocate their hydro production in order to equalize the marginal revenue across periods (subject to capacity and/or production constraints) and that in equilibrium marginal revenue is equal to the MVW, it is possible to use the latter to determine the hydro scheduling strategy (when is water stored/ when it is released) that the producer would follow over the year. In particular, if min/max flow constraints and min/max reservoir capacity constraints are not binding, the MVW analysis would conclude that water inflows are stored in months in which water is cheap (low MVW) and released in months in which water is more expensive (high MVW). In other words, hydro production would be shifted from low MR months to high MR months. If the hydro scheduling strategy that I found that was optimal in the 1-month model is also followed by producers over the year, I

[56] Market demand was parameterized as before, and thus it was assumed that price elasticity was –1/3 at the peak anchor point. Residual demand is not constant throughout the year because must run production and the fringe's production fluctuates over the different months.

should find that: water is cheap (low MVW) in October, November and December and more costly in March, January and the winter months (June-August) [57].

In order to check for this hypothesis, I run the model and calculated the MVW for each month of the year (Table 16). Observe that the MVW fluctuates over the year. Water is more valuable in the first months of the year and less valuable in October, June, July and November. Accordingly, if Firm 1 could reallocate hydro flows throughout the year, it would store a fraction of them in the second half of the year and would use them in the first half of the next year.[58] The MVW is positively correlated to the inter-month differences in demand elasticity suggesting that the hydro scheduling strategy is consistent with the conclusions of the 1-month model, as hydro production would be shifted from low demand elasticity months (low MVW) to high demand elasticity months (High MVW). MVW is also positively correlated to the intra-month variation of demand elasticity (fluctuations of demand elasticity *within* each month). According to this evidence, the hydro producer would choose to "flood" the market not only when average residual demand is relatively more elastic (and hence the market more competitive) but also when the difference between min/max demand and price elasticity across periods is larger, reinforcing the first effect.

Even though the hydro producer has incentive to exploit inter-month differences in demand elasticity, and intra-month demand fluctuations, *the incentive is not strong enough* to completely explain the hydro scheduling strategy over a longer planning horizon. Indeed, the correlation coefficient between average elasticity of demand and the MVW and between intra-month demand fluctuation and the MVW are relatively small (0.24 and 0.26 respectively). Unlike that, the correlation between MVW and the ratio inflows/net demand is very strong (correlation coefficient = -0.93) suggesting that producers store hydro flows and release them across months according to how scarce these flows are relative to demand. This relationship being stronger implies that the value of water and its allocation over the year is mainly explained by a traditional supply-demand analysis (i.e. store water when it is relatively abundant) than by a market power argument (i.e. store water when demand is less elastic).

Why is it that Firm 1 would not schedule its hydro plants in order to exploit inter-month differences in demand elasticity? There are, at least, two possible explanations. First, producers may not need to allocate water from less elastic months to more elastic months in order to exercise market power if they are already able to exercise it within a month. In the 1-month version of the model, I found that Firm 1 exercised market power by constraining its hydro production in periods of high demand, as its rivals were capacity constrained and by using the "stored" water when the market was more competitive (low demand). If there is enough intra-month demand variability, it is more likely that there will be periods in which the producer will be able to release the stored water without a

[57] Winter in the Southern hemisphere goes from June 21st to September 21st. January, February and September coincide with vacation periods. In addition, the first two months are in the middle of the summer. AC use is not spread in Chile.
[58] This simple analysis does not take into account any technical constraint regarding maximum and minimum capacity of reservoirs.

big impact on the market price. The Cournot equilibrium converging each month to the competitive equilibrium as the demand level falls, indicates that the market is competitive a fraction of the time, providing qualitative support to this argument[59]. In order to check for this hypothesis more formally, I estimated a calibrated version of the model assuming that there were only inter-month differences in demand elasticity but *no* intra-month demand fluctuation. If it is true that intra-month fluctuations are the reason why Firm 1 does not exploit differences in demand elasticity across months, then the correlation coefficient should be higher. Results are reported in Table 17, column (2). As expected, the correlation coefficient between the elasticity of demand and the MVW is higher but only marginally. At the same time, the relationship between the value of water and its availability (measured by the ratio inflows/net demand) is still very strong. These results suggest that the intra-month demand fluctuation is only a second order explanation to the hydro scheduling strategy followed over the year.

Alternatively, Firm 1 may not have incentive to exercise market power by shifting water from one month to another *because inter-month differences in price elasticity are not big enough.* Remember that the 1-month model, and associated marginal value of water was estimated assuming that the price elasticity of market demand for electricity was –1/3 at the peak anchor point (see Chapter V). Even though elasticity of residual demand fluctuates both intra and inter-month as the size of the fringe and must run production varies, it may be possible that this fluctuation is not sufficiently large. In order to check for this hypothesis, I estimated the model assuming that the elasticity of *market* demand was different over the year (but constant within each month).[60] As a result of larger differences in price elasticity (almost ten times), the correlation between the marginal value of water and the average elasticity of demand also increased; the correlation coefficient amounted to 0.88 (Table 17, column (3)). In other words, the larger are the differences in inter-month price elasticity of demand, the greater the incentive to exercise market power by shifting water from one month to another and the smaller the relationship between the availability of water relative to demand and the value of water. The larger the inter-month difference, the closer is the hydro scheduling strategy to the market power explanation and further from the traditional supply/demand analysis. [61]

Summing up, producers exercise market power by exploiting differences in price elasticity, allocating too little supply to periods in which demand is relatively less elastic, and relatively too much to periods with more elastic demand. This hydro scheduling strategy may be observed no matter what the planning horizon of the model is; the only "requirement" is that there is enough "inter-period" demand variability. The smaller the fluctuation, the closer is the hydro scheduling strategy to the traditional supply-demand analysis' conclusions (i.e. more water to periods in which water is relatively scarce).

[59] The correlation coefficient between the price ratio Cournot/Competitive equilibrium and the MVW is –0.72.

[60] It could take one of the following values: -0.1, -0.33, -0.5, -0.67, -1.0. These were randomly allocated to each month.

[61] In order to completely discard that this effect is explained by greater demand fluctuation within each month, I re-calibrated the model, assuming the same inter-month difference in price elasticity but no intra-month fluctuation (Table 17, column (4)). The correlation coefficient between the marginal value of water and the average elasticity is almost unchanged.

VIII. Conclusions

Currently Chilean authorities are evaluating the desirability of moving one step forward in the de-regulation process; in particular they are analyzing whether to stop regulating the nodal price. Concerns have been raised due to the high concentrations indices that the industry exhibits. In this paper I analyzed the question of whether these high concentration indices would translate into prices significantly higher than the competitive level as a result of suppliers exercising market power.

A model with 2 Cournot producers (Firms 1 and 2) and a competitive fringe was built. Both producers own thermal plants but only one of them (Firm 1) also owns hydro-storage plants. These plants turned out to be very important because they allowed the producer to store water (and power) and to allocate it over a certain planning horizon at its convenience. This alternative was not available to Firm 2.

I found that if an unregulated spot market were implemented in Chile's electricity industry, generators would have the incentive and ability to exercise market power, raising prices significantly above competitive levels. In particular, Firm 1 would keep its thermal plants outside of the market and would allocate its hydro resources in order to take advantage of differences in price elasticity of demand; this means allocating too little supply to high demand periods and too much to low demand periods (relative to the competitive equilibrium). As a consequence, differences between peak and off peak periods would be enlarged rather than reduced (as it would be if the market were competitive). On the other side, Firm 2 would not be able to use its large thermal portfolio to exercise market power most of the time. In fact, when demand is high, Firm 1 has so much market power and can drive prices up by so much that Firm 2's best strategy is to produce at capacity. Firm 2 is able to constrain its production only when demand is low. Results slightly change when hydro flows are reduced, as it is Firm 2 the one who really enjoys and exercises market power.

Conditions for the exercise of market power are more favorable the more inelastic is demand, an expected result. However, since in this case a generator is able to "shift power" from one period to another, there is an additional element that must be looked at when diagnosing market power: the inter-period differences in price elasticity. In particular, the larger the difference of price elasticity of demand across periods, the greater the incentive Firm 1 has to exercise market power by shifting hydro production from one period to another. This result applies no matter the length of the planning horizon assumed in the model. These conclusions call for more accurate estimations of elasticity of demand for electricity.

There are some elements of the model that should be kept in mind when analyzing its results. First of all, results in general, and the magnitude of distortions in particular, depend on the demand elasticity. The approach taken in this paper was to estimate the model assuming different parameters for the demand elasticity. Because of the importance of this parameter in the final result, more rigorous research in this topic is required for the Chilean case. Secondly, the model has no dynamic elements although a power exchange system is clearly a perfect scenario for repeated competition. It is reasonable to think that this omission results in my model underestimating market power.

In light of these results, Chilean authorities should be aware of the risks entailed in de-regulating prices. A careful analysis of complementary measures that could be implemented to mitigate market power is recommended. The model used in this paper would certainly be a useful tool in that task, as it will allow estimating the impact of alternative policies and doing comparative analysis. Chapter 2 examines policies (a) that mitigate market power by breaking up the largest generating company through divestiture of generating capacity and (b) that impose fixed price forward supply contracts on varying amounts of the capacity owned by firms with market power.

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Tables

Table 1: Installed capacity in the SIC (December 2000), MW

Economic Group	Thermal	Hydro-ROR	Hydro-reservoir	Hydro	Total	% thermal	% hydro	% total
Endesa (Firm 1)	939	238	2454	2693	3632	25.9%	74.1%	55%
Gener (Firm 2)	1212	245	0	245	1457	83.2%	16.8%	22%
Colbun (Firm 3)	370	0	697	697	1067	34.7%	65.3%	16%
Other	102	403	0	403	504	20.1%	79.9%	8%
Total	2622	886	3151	4037	6660	39.0%	61.0%	100%

Source: CDEC-SIC

Table 2: Demand Estimation, April 2000, Elasticity = -1/3

Demand level	# hours	Anchor quantity MW	Anchor price US$/MW	A	B	Elasticity at anchor point
1	120	4749.7	31.1	6332.9	50.9	0.333
2	120	4329.6	31.1	5912.8	50.9	0.366
3	120	4091.1	31.1	5674.3	50.9	0.387
4	120	3643.3	31.1	5226.5	50.9	0.435
5	120	3270.8	31.1	4854.0	50.9	0.484
6	120	2988.5	31.1	4571.7	50.9	0.530

Table 3: Data to calculate residual demand (MW), Base Model

Demand level	Hydro peak shaving	Thermal must run generation	Hydro ROR Must run generation	Total
1	274.6	14.2	380.3	669.1
2	183.8	14.2	380.3	578.2
3	183.8	14.2	380.3	578.2
4	183.8	14.2	380.3	578.2
5	183.8	14.2	380.3	578.2
6	183.8	14.2	380.3	578.2

Table 4: Hydro Data used to estimate the Base Model

Firm	q^h_{min} (MW)	q^h_{max} (MW)	$q^{\sim h}$ (GWh month)
Firm 1	743.7	2436.1	974.9
Fringe	183.8	489.2	143.2

Table 5: Base Model, Competitive results (E= -1/3)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	Qtot	Price
1	673.1	944.4	1617.5	2133.1	0.0	2133.1	3750.6	420.5	4839.0	29.4
2	673.1	944.4	1617.5	1802.6	0.0	1802.6	3420.1	420.5	4418.9	29.4
3	673.1	944.4	1617.5	1564.1	0.0	1564.1	3181.6	420.5	4180.4	29.4
4	673.1	944.4	1617.5	1116.3	0.0	1116.3	2733.8	420.5	3732.6	29.4
5	673.1	944.4	1617.5	764.7	0.0	764.7	2382.2	420.1	3380.5	28.9
6	566.2	944.4	1510.6	743.7	0.0	743.7	2254.4	416.6	3249.3	26.0

Table 6: Base Model, Competitive results (E= -2/3)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	Qtot	Price
1	673.1	944.4	1617.5	2133.1	0.0	2133.1	3750.6	421.5	4840.0	30.2
2	673.1	944.4	1617.5	1802.6	0.0	1802.6	3420.1	421.5	4419.9	30.2
3	673.1	944.4	1617.5	1564.1	0.0	1564.1	3181.6	421.5	4181.4	30.2
4	673.1	944.4	1617.5	1116.3	0.0	1116.3	2733.8	421.5	3733.6	30.2
5	673.1	944.4	1617.5	764.7	0.0	764.7	2382.2	421.3	3381.7	30.0
6	673.1	944.4	1617.5	743.7	0.0	743.7	2361.2	418.4	3357.8	27.5

Table 7: Base Model, Cournot Equilibrium (E=-1/3)

t	qth1 #	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	Qtot	Price
1	133.7	944.4	1078.1	1743.0	0.0	1743.0	2821.3	441.2	3931.3	47.2
2	140.1	944.4	1084.5	1572.0	0.0	1572.0	2656.5	437.5	3672.3	44.0
3	120.3	944.4	1064.7	1472.6	0.0	1472.6	2537.3	434.9	3550.4	41.7
4	122.0	944.4	1066.4	1247.0	0.0	1247.0	2313.4	429.9	3321.5	37.4
5	127.0	867.7	994.7	1094.1	0.0	1094.1	2088.7	426.6	3093.6	34.6
6	131.9	773.6	905.4	995.1	0.0	995.1	1900.6	424.5	2903.3	32.8

# denotes Multiple equilibrium. Values reported are averages over 400 different simulations

Table 8: Base Model, Cournot Equilibrium (E=-2/3)

t	qth1#	qth2	QRth	qh1#	qh2	QRh	QR	Qthfringe	Qtot	Price
1	377.7	944.4	1322.1	1768.1	0.0	1768.1	3090.2	429.0	4188.3	36.6
2	376.0	944.4	1320.4	1605.2	0.0	1605.2	2925.6	427.1	3930.9	35.0
3	378.3	944.4	1322.7	1483.6	0.0	1483.6	2806.3	425.8	3810.3	33.9
4	375.8	944.4	1320.2	1262.2	0.0	1262.2	2582.4	423.2	3583.9	31.7
5	379.6	944.4	1324.0	1072.2	0.0	1072.2	2396.2	421.1	3395.6	29.9
6	378.2	944.4	1322.5	932.5	0.0	932.5	2255.0	419.6	3252.8	28.5

# denotes Multiple equilibrium. Values reported are averages over 400 different simulations.

Table 9: Lerner Indices, Base Model

	E = -1/3		E= -2/3	
T	Firm 1	Firm 2	Firm 1	Firm 2
1	76%	38%	57%	25%
2	75%	41%	55%	26%
3	73%	43%	53%	27%
4	70%	48%	50%	29%
5	68%	48%	47%	31%
6	66%	45%	45%	32%

Table 10: Welfare Analysis
Index (Perfect Competition = 100)

Elasticity	Firm 1's Producer surplus	Firm 2's Producer surplus	Fringe's Producer surplus	Producer surplus	Consumer surplus	Welfare
E = -0.1	242.7	586.0	352.6	319.1	64.0	94.6
E= -1/3	124.0	178.9	145.8	138.3	73.3	95.9
E= -1/2	112.4	131.8	118.5	117.0	82.7	98.1
E= -2/3	107.3	120.6	111.7	110.6	84.9	98.4
E= -1.0	104.0	109.8	105.7	105.4	88.4	99.1

Table 11: Data to calculate residual demand (MW). Model with reduced hydro flows

Demand level	Hydro peak shaving	Thermal must run generation	Hydro ROR Must run generation	Total
1	119.3	14.2	228.2	361.7
2	119.3	14.2	228.2	361.7
3	119.3	14.2	228.2	361.7
4	119.3	14.2	228.2	361.7
5	119.3	14.2	228.2	361.7
6	119.3	14.2	228.2	361.7

Table 12: Competitive results with reduced hydro flows (E= -1/3)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	Qtot	Price
1	779.2	992.9	1772.1	833.4	0.0	833.4	2605.5	572.6	3420.4	57.6
2	673.1	992.9	1666.0	808.2	0.0	808.2	2474.2	566.0	3282.6	51.9
3	673.1	992.9	1666.0	808.2	0.0	808.2	2474.2	560.6	3277.2	47.2
4	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	551.5	3219.6	39.4
5	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	543.3	3211.3	32.3
6	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	537.0	3205.0	26.9

Table 13: Cournot Equilibrium with reduced hydro flows (E=-1/3)

t	qth1#	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	Qtot	Price
1	614.3	944.4	1558.7	1029.6	0.0	1029.6	2588.3	453.2	3403.2	57.6
2	607.0	944.4	1551.4	826.8	0.0	826.8	2378.2	448.6	3188.4	53.5
3	527.4	944.4	1471.8	787.1	0.0	787.1	2258.9	445.9	3066.5	51.2
4	454.6	944.4	1399.0	743.6	0.0	743.6	2142.6	438.5	2942.8	44.9
5	426.4	944.4	1370.8	743.6	0.0	743.6	2114.4	430.9	2906.9	38.3
6	341.7	902.7	1244.4	743.6	0.0	743.6	1988.0	427.4	2777.1	35.3

# denotes multiple equilibrium. Values reported are averages over 400 different simulations.

Table 14: Welfare Analysis, Model with reduced hydro flows Index (Perfect Competition = 100)

Elasticity	Firm 1's Producer surplus	Firm 2's Producer surplus	Fringe's Producer surplus	Producer surplus	Consumer surplus	Welfare
E = -0.1	123.4	147.9	144.6	135.3	77.7	96.7
E= -1/3	107.6	115.4	109.7	110.0	87.1	98.7
E= -1/2	103.7	107.4	103.1	104.4	92.2	99.3
E= -2/3	102.5	105.3	101.4	102.8	93.3	99.4
E= -1.0	101.4	103.2	99.9	101.4	94.5	99.4

Table15: Demand Indicators

	Total demand (GWh)	Demand - q^{MR} (GWh) *	Inflows (GWh)	Inflows / (demand – q^{MR})	Average Net demand per day (GWh)	Max demand / Min demand	Average Elasticity / Residual Ddand
January	2733.43	2293.80	1058.98	46%	74.0	2.20	0.49
February	2520.23	2139.77	996.86	47%	76.4	1.90	0.47
March	2911.54	2532.46	1333.09	53%	81.7	1.85	0.51
April	2768.74	2484.73	1118.05	45%	82.8	2.00	0.48
May	2885.11	2587.79	1294.28	50%	83.5	1.69	0.45
June	2794.75	2466.61	1561.06	63%	82.2	1.82	0.50
July	2820.53	2487.82	1475.91	59%	80.3	1.79	0.49
August	2799.88	2478.86	1384.18	56%	80.0	1.75	0.49
September	2649.14	2307.95	1262.55	55%	76.9	1.94	0.48
October	2770.69	2399.68	1478.88	62%	77.4	1.93	0.40
November	2751.61	2328.93	1385.07	59%	77.6	1.90	0.40
December	2859.66	2410.97	1279.78	53%	77.8	1.98	0.40

* q^{MR} includes production from must run thermal plants and hydro production from ROR plants.

Table 16: Marginal Value of Water (MVW)

	E = -0.1	E = -1/3	E= - 0.5	E = -2/3	E = -1.0
January	0.00	11.13	11.13	13.91	17.90
February	0.00	11.13	11.13	13.54	17.75
March	0.00	10.62	11.13	13.36	17.75
April	0.00	11.13	11.89	15.78	19.85
May	0.00	9.53	11.13	12.78	17.29
June	0.00	0.00	7.78	11.13	14.21
July	0.00	1.92	10.08	11.13	15.41
August	0.00	6.45	11.13	11.13	15.78
September	0.00	8.47	11.13	11.98	17.03
October	0.00	0.00	9.82	11.13	15.78
November	0.00	2.21	11.13	11.13	17.01
December	0.00	9.88	11.13	14.61	20.28

Table 17: Marginal Value of Water under different price elasticity assumptions.

	MVW				Average Elasticity of residual demand				St dev intra-month elasticity				Inflows/ net demand		
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)
Jan	11.13	10.08	0.00	0.00	0.49	0.49	0.15	0.15	0.10	0.0	0.03	0.0	46%	110%	46%
Feb	11.13	9.33	17.75	17.25	0.47	0.47	1.36	1.36	0.08	0.0	0.23	0.0	47%	25%	47%
Mar	10.62	8.88	0.00	0.00	0.51	0.51	0.15	0.15	0.08	0.0	0.03	0.0	53%	131%	53%
Apr	11.13	11.13	15.78	15.45	0.48	0.48	0.94	0.94	0.10	0.0	0.19	0.0	45%	35%	45%
May	9.53	7.85	9.53	7.85	0.45	0.45	0.45	0.45	0.06	0.0	0.06	0.0	50%	63%	50%
Jun	0.00	0.00	0.00	0.00	0.50	0.50	0.50	0.50	0.09	0.0	0.09	0.0	63%	84%	63%
Jul	1.92	1.14	1.92	1.14	0.49	0.49	0.49	0.49	0.08	0.0	0.08	0.0	59%	81%	59%
Aug	6.45	5.32	11.13	11.13	0.49	0.49	0.73	0.73	0.09	0.0	0.13	0.0	56%	60%	56%
Sep	8.47	7.02	11.98	11.13	0.48	0.48	0.92	0.92	0.09	0.0	0.16	0.0	55%	45%	55%
Oct	0.00	0.00	11.13	11.13	0.40	0.40	0.72	0.72	0.06	0.0	0.11	0.0	62%	50%	62%
Nov	2.21	1.08	17.01	16.62	0.40	0.40	1.00	1.00	0.06	0.0	0.12	0.0	59%	35%	59%
Dec	9.88	7.90	14.61	14.02	0.40	0.40	0.72	0.72	0.06	0.0	0.09	0.0	53%	44%	53%
Average	6.87	5.81	9.24	8.81	0.46	0.46	0.68	0.68	0.08	0.0	0.11	0.0			
stdev	4.55	4.16	6.93	6.83	0.04	0.04	0.35	0.35	0.02		0.06				
Corr MVW					0.24	0.27	0.88	0.89	0.26		0.77		-0.93		-0.22

Figures

Figure 1

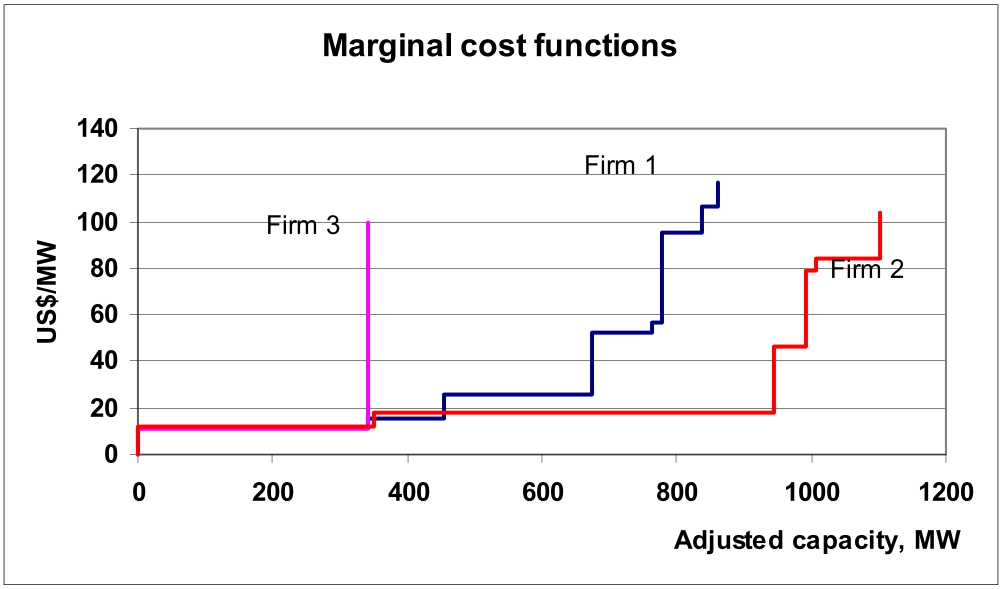


Figure 2

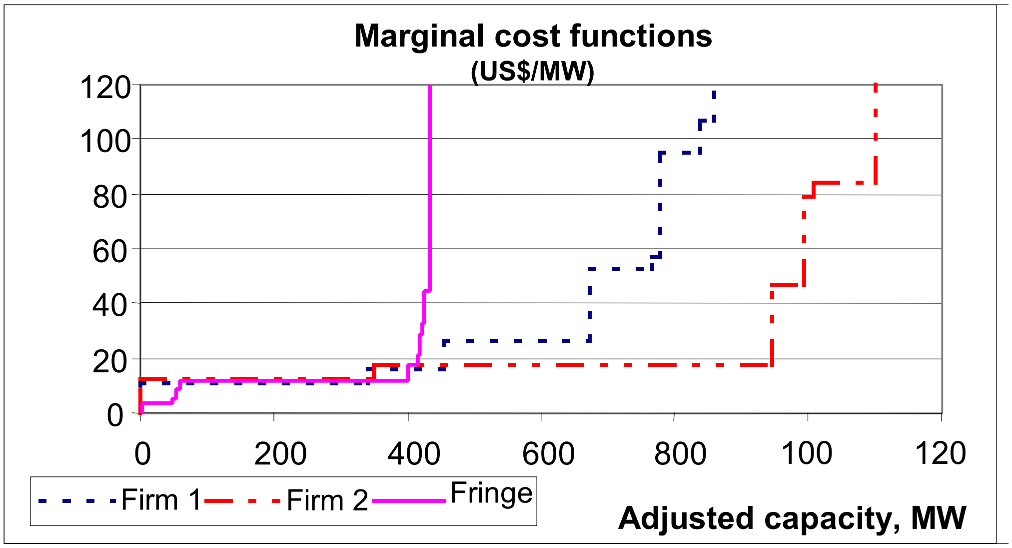


Figure 3

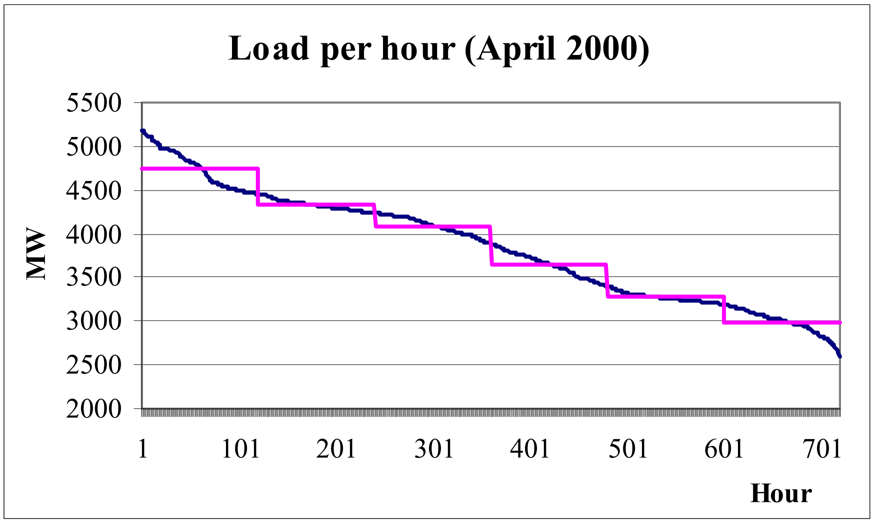


Figure 4

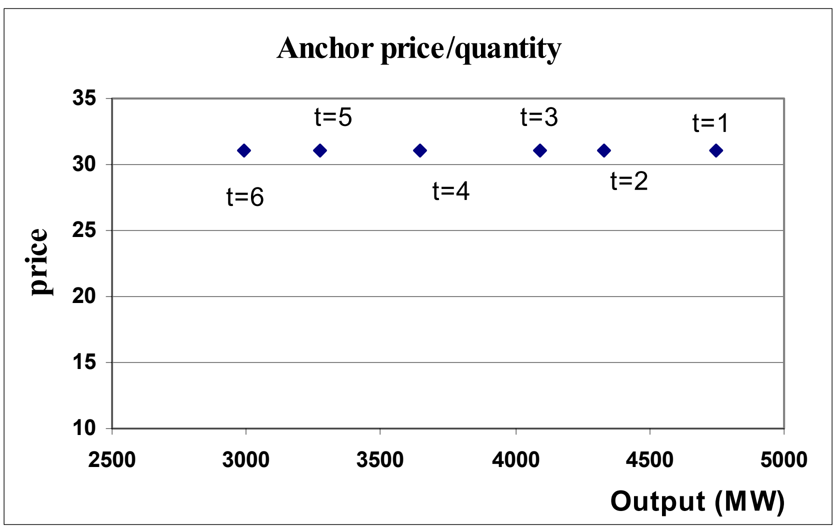


Figure 5

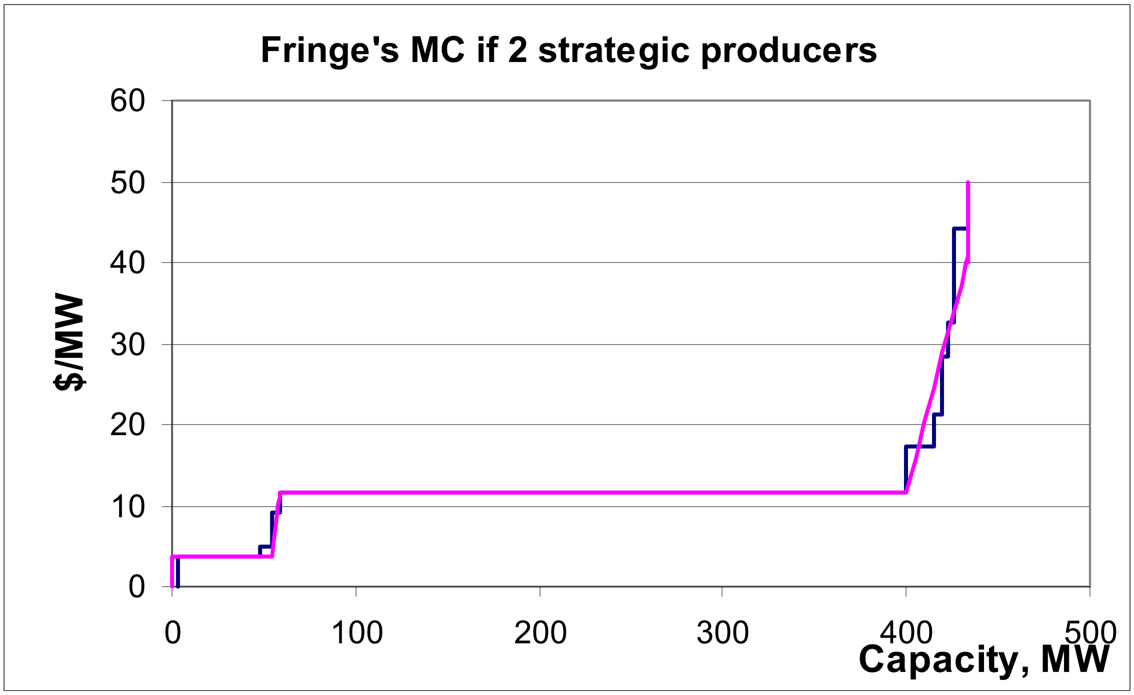


Figure 6

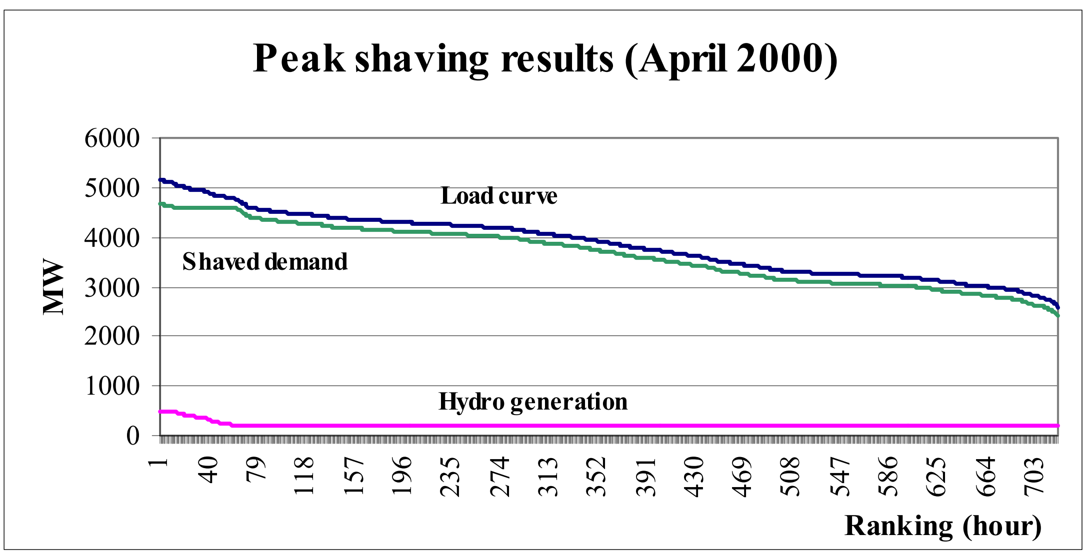


Figure 7

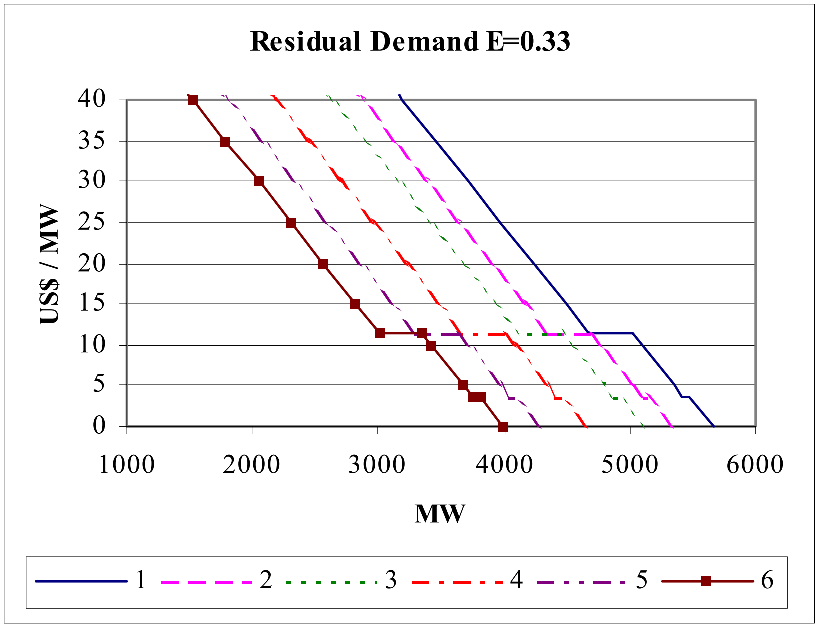


Figure 8

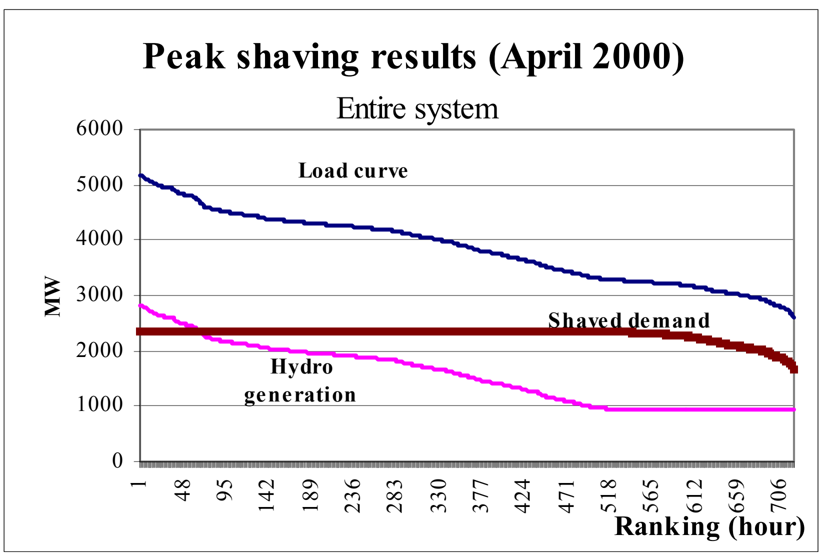


Figure 9

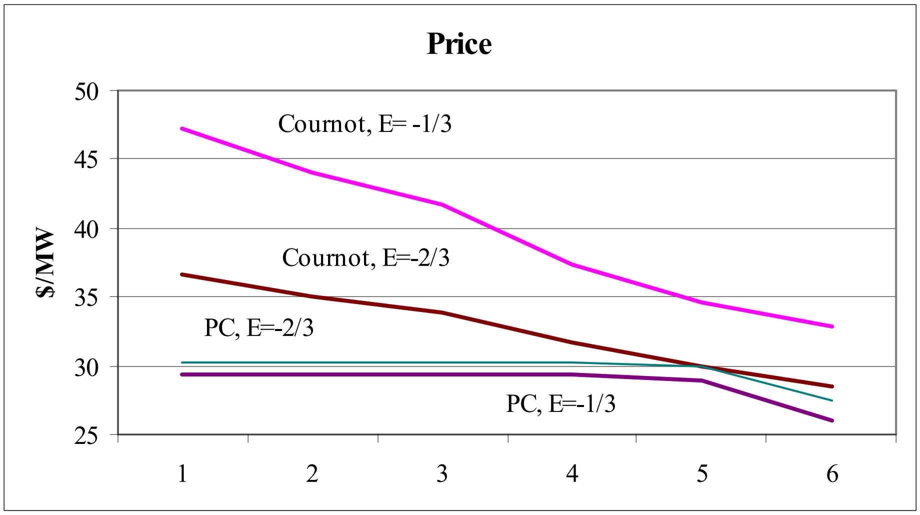


Figure 10

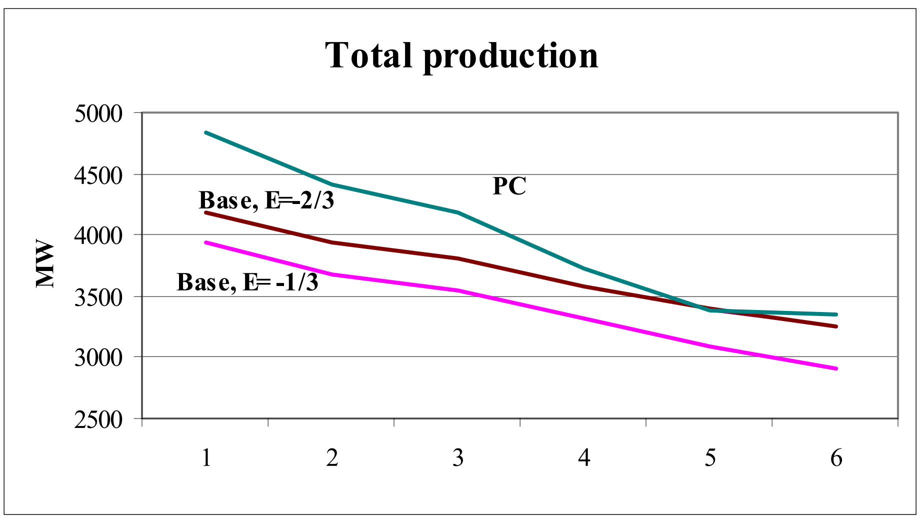


Figure 11

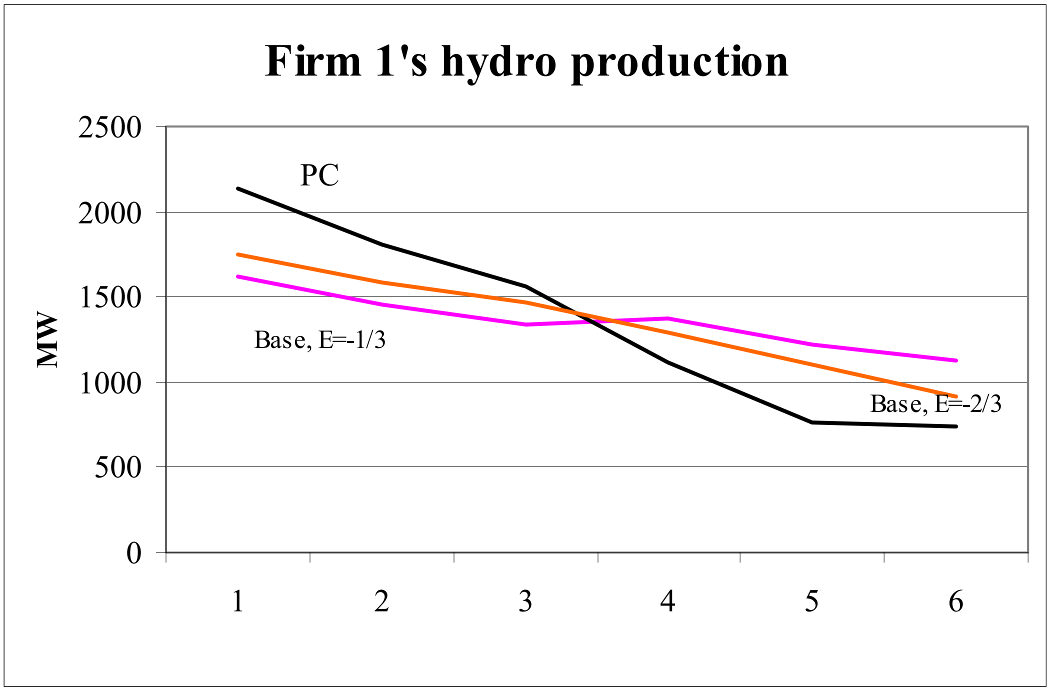
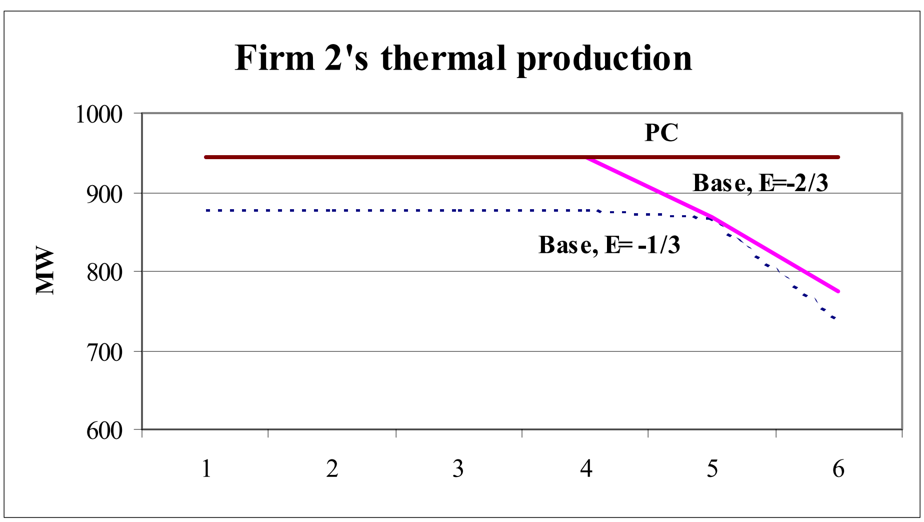


Figure 12



APPENDIX # 1 - TABLES

1. Base Model, Same slope approach

A. Competitive Equilibrium (Same slope approach)

Table A1.1: Competitive equilibrium (E=-0.1)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	665.94	944.40	1610.34	2133.1	0	2133.1	3743.4	690.03	394.46	2406.49	4827.9	26.0
2	665.94	944.40	1610.34	1802.6	0	1802.6	3412.9	600.43	394.46	1986.36	4407.8	26.0
3	665.94	944.40	1610.34	1564.1	0	1564.1	3174.4	600.43	394.46	1747.87	4169.3	26.0
4	665.94	944.40	1610.34	1116.3	0	1116.3	2726.6	600.43	394.46	1300.08	3721.5	26.0
5	645.03	944.40	1589.43	764.7	0	764.7	2354.1	600.43	394.46	948.48	3349.0	26.0
6	454.60	944.40	1399.00	743.7	0	743.7	2142.7	595.43	394.46	927.52	3132.6	21.7

Table A1.2: Competitive equilibrium (E=-1/3)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.1	944.4	1617.5	2133.1	0.0	2133.1	3750.6	693.9	394.46	2406.49	4839.0	29.4
2	673.1	944.4	1617.5	1802.6	0.0	1802.6	3420.1	604.3	394.46	1986.36	4418.9	29.4
3	673.1	944.4	1617.5	1564.1	0.0	1564.1	3181.6	604.3	394.46	1747.87	4180.4	29.4
4	673.1	944.4	1617.5	1116.3	0.0	1116.3	2733.8	604.3	394.46	1300.08	3732.6	29.4
5	673.1	944.4	1617.5	764.7	0.0	764.7	2382.2	603.9	394.46	948.48	3380.5	28.9
6	566.2	944.4	1510.6	743.7	0.0	743.7	2254.4	600.4	394.46	927.52	3249.3	26.0

Table A1.3: Competitive equilibrium (E=-0.5)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.10	944.40	1617.50	2133.1	0	2133.1	3750.6	694.61	394.46	2406.49	4839.7	29.9
2	673.10	944.40	1617.50	1802.6	1	1803.6	3421.1	605.01	394.46	1986.36	4420.5	29.9
3	673.10	944.40	1617.50	1564.1	2	1566.1	3183.6	605.01	394.46	1747.87	4183.0	29.9
4	673.10	944.40	1617.50	1116.3	3	1119.3	2736.8	605.01	394.46	1300.08	3736.2	29.9
5	673.10	944.40	1617.50	764.7	4	768.7	2386.2	604.69	394.46	948.48	3385.3	29.7
6	673.10	944.40	1617.50	743.7	5	748.7	2366.2	600.79	394.46	927.52	3361.5	26.3

Table A1.4: Competitive equilibrium (E=-2/3)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.1	944.4	1617.5	2133.1	0.0	2133.1	3750.6	694.9	394.46	2406.49	4840.0	30.2
2	673.1	944.4	1617.5	1802.6	0.0	1802.6	3420.1	605.3	394.46	1986.36	4419.9	30.2
3	673.1	944.4	1617.5	1564.1	0.0	1564.1	3181.6	605.3	394.46	1747.87	4181.4	30.2
4	673.1	944.4	1617.5	1116.3	0.0	1116.3	2733.8	605.3	394.46	1300.08	3733.6	30.2
5	673.1	944.4	1617.5	764.7	0.0	764.7	2382.2	605.1	394.46	948.48	3381.7	30.0
6	673.1	944.4	1617.5	743.7	0.0	743.7	2361.2	602.2	394.46	927.52	3357.8	27.5

Table A1.5: Competitive equilibrium (E=-1.0)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.10	944.40	1617.50	2133.1	0	2133.1	3750.6	695.28	394.46	2406.49	4840.3	30.5
2	673.10	944.40	1617.50	1802.6	0	1802.6	3420.1	605.68	394.46	1986.36	4420.2	30.5
3	673.10	944.40	1617.50	1564.1	0	1564.1	3181.6	605.68	394.46	1747.87	4181.7	30.5
4	673.10	944.40	1617.50	1116.3	0	1116.3	2733.8	605.68	394.46	1300.08	3733.9	30.5
5	673.10	944.40	1617.50	764.7	0	764.7	2382.2	605.53	394.46	948.48	3382.2	30.4
6	673.10	944.40	1617.50	743.7	0	743.7	2361.2	603.56	394.46	927.52	3359.2	28.7

B. Cournot equilibrium (Same slope approach)

Table A1.6: Cournot equilibrium **(E=-0.1)**

t	qth1 #	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	0.0	944.4	944.4	1612.3	0.0	1612.3	2556.7	500.3	394.46	274.6	3726.1	98.1
2	0.0	944.4	944.4	1447.7	0.0	1447.7	2392.1	488.6	394.46	183.8	3459.0	88.1
3	0.0	944.4	944.4	1328.4	0.0	1328.4	2272.8	480.2	394.46	183.8	3331.3	80.9
4	0.0	854.9	854.9	1149.3	0.0	1149.3	2004.2	467.6	394.46	183.8	3050.0	70.0
5	0.0	730.7	730.7	1025.1	0.0	1025.1	1755.9	458.8	394.46	183.8	2792.9	62.4
6	0.0	636.6	636.6	931.0	0.0	931.0	1567.7	452.2	394.46	183.8	2598.1	56.7

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.7: Cournot equilibrium **(E=-1/3)**

t	qth1 #	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	133.7	944.4	1078.1	1743.0	0.0	1743.0	2821.1	441.2	394.46	274.6	3931.4	47.2
2	140.1	944.4	1084.5	1572.0	0.0	1572.0	2656.5	437.5	394.46	183.8	3672.3	44.0
3	120.3	944.4	1064.7	1472.6	0.0	1472.6	2537.3	434.9	394.46	183.8	3550.4	41.7
4	122.0	944.4	1066.4	1247.0	0.0	1247.0	2313.4	429.9	394.46	183.8	3321.5	37.4
5	127.0	867.7	994.7	1094.1	0.0	1094.1	2088.8	426.6	394.46	183.8	3093.6	34.6
6	131.9	773.6	905.4	995.1	0.0	995.1	1900.6	424.5	394.46	183.8	2903.3	32.8

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.8: Cournot equilibrium **(E=-0.5)**

t	qth1 #	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	341.7	944.4	1286.1	1759.9	0.0	1759.9	3046.0	431.7	394.46	274.6	4146.8	39.0
2	341.7	944.4	1286.1	1595.3	0.0	1595.3	2881.4	429.3	394.46	183.8	3888.9	36.9
3	341.7	944.4	1286.1	1476.0	0.0	1476.0	2762.1	427.5	394.46	183.8	3767.8	35.3
4	341.7	944.4	1286.1	1252.1	0.0	1252.1	2538.2	424.1	394.46	183.8	3540.6	32.4
5	341.7	941.5	1283.2	1067.3	0.0	1067.3	2350.5	421.4	394.46	183.8	3350.1	30.1
6	341.7	847.4	1189.1	973.2	0.0	973.2	2162.3	420.0	394.46	183.8	3160.5	28.9

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.9: Cournot equilibrium **(E=-2/3)**

t	qth1 #	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	377.7	944.4	1322.1	1768.1	0.0	1768.1	3090.2	429.0	394.46	274.6	4188.3	36.6
2	376.0	944.4	1320.4	1605.2	0.0	1605.2	2925.6	427.1	394.46	183.8	3930.9	35.0
3	378.3	944.4	1322.7	1483.6	0.0	1483.6	2806.3	425.8	394.46	183.8	3810.3	33.9
4	375.8	944.4	1320.2	1262.2	0.0	1262.2	2582.4	423.2	394.46	183.8	3583.9	31.7
5	379.6	944.4	1324.0	1072.2	0.0	1072.2	2396.2	421.1	394.46	183.8	3395.6	29.9
6	378.2	944.4	1322.5	932.5	0.0	932.5	2255.0	419.6	394.46	183.8	3252.8	28.5

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.10: Cournot equilibrium **(E=-1.0)**

t	qth1#	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	454.6	944.4	1399.0	1768.2	0.0	1768.2	3167.2	426.3	394.46	274.6	4262.5	34.3
2	454.6	944.4	1399.0	1603.6	0.0	1603.6	3002.6	425.0	394.46	183.8	4005.8	33.2
3	454.6	944.4	1399.0	1484.3	0.0	1484.3	2883.3	424.1	394.46	183.8	3885.7	32.4
4	454.6	944.4	1399.0	1260.4	0.0	1260.4	2659.4	422.4	394.46	183.8	3660.1	31.0
5	454.6	944.4	1399.0	1074.2	0.0	1074.2	2473.2	421.0	394.46	183.8	3472.4	29.8
6	454.6	944.4	1399.0	933.0	0.0	933.0	2332.0	420.0	394.46	183.8	3330.2	28.9

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table **A1.11**: Lerner Indices

	E=-0.1		***E=-1/3***		***E=0.5***		***E= -2/3***		***E=-1.0***	
t	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2
1	100%	59%	76%	38%	70%	31%	57%	25%	42%	18%
2	100%	65%	75%	41%	68%	33%	55%	26%	40%	18%
3	100%	71%	73%	43%	66%	34%	53%	27%	39%	19%
4	100%	74%	70%	48%	63%	38%	50%	29%	36%	20%
5	100%	71%	68%	48%	60%	40%	47%	31%	33%	21%
6	100%	68%	66%	45%	59%	38%	45%	32%	31%	21%

2. Base Model, Different slope approach

A. Competitive Equilibrium (Different slope approach)

Table A1.12: Competitive equilibrium (E=-0.1)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	665.94	944.40	1610.34	2133.1	0	2133.1	3743.4	690.0	394.46	2406.49	4827.9	25.98
2	659.02	944.40	1603.42	1802.6	0	1802.6	3406.0	600.4	394.46	1986.36	4400.9	25.98
3	655.09	944.40	1599.49	1564.1	0	1564.1	3163.6	600.4	394.46	1747.87	4158.5	25.98
4	647.72	944.40	1592.12	1116.3	0	1116.3	2708.4	600.4	394.46	1300.08	3703.3	25.98
5	620.67	944.40	1565.07	764.7	0	764.7	2329.7	600.4	394.46	948.48	3324.6	25.98
6	454.60	931.31	1385.91	743.7	0	743.7	2129.6	591.1	394.46	927.52	3115.2	17.92

Table A1.13: Competitive equilibrium (E=-1/3)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.1	944.4	1617.5	2133.1	0	2133.1	3750.6	693.9	394.46	2406.49	4839.0	29.4
2	673.1	944.4	1617.5	1802.6	0	1802.6	3420.1	604.1	394.46	1986.36	4418.7	29.2
3	673.1	944.4	1617.5	1564.1	0	1564.1	3181.6	604.0	394.46	1747.87	4180.0	29.1
4	673.1	944.4	1617.5	1116.3	0	1116.3	2733.8	603.7	394.46	1300.08	3732.0	28.8
5	673.1	944.4	1617.5	764.7	0	764.7	2382.2	602.8	394.46	948.48	3379.4	28.0
6	469.5	944.4	1413.9	743.7	0	743.7	2157.7	600.4	394.46	927.52	3152.6	26.0

Table A1.14: Competitive equilibrium (E=-0.5)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.10	944.40	1617.50	2133.1	0	2133.1	3750.6	694.6	394.46	2406.49	4839.7	29.93
2	673.10	944.40	1617.50	1802.6	0	1802.6	3420.1	604.9	394.46	1986.36	4419.4	29.81
3	673.10	944.40	1617.50	1564.1	0	1564.1	3181.6	604.8	394.46	1747.87	4180.8	29.74
4	673.10	944.40	1617.50	1116.3	0	1116.3	2733.8	604.6	394.46	1300.08	3732.8	29.58
5	673.10	944.40	1617.50	764.7	0	764.7	2382.2	604.0	394.46	948.48	3380.6	29.02
6	551.59	944.40	1495.99	743.7	0	743.7	2239.7	600.4	394.46	927.52	3234.6	25.98

Table A1.15: Competitive equilibrium (E=-2/3)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.1	944.4	1617.5	2133.1	0.0	2133.1	3750.6	694.9	394.46	2406.49	4840.0	30.2
2	673.1	944.4	1617.5	1802.6	0.0	1802.6	3420.1	605.2	394.46	1986.36	4419.8	30.1
3	673.1	944.4	1617.5	1564.1	0.0	1564.1	3181.6	605.2	394.46	1747.87	4181.2	30.1
4	673.1	944.4	1617.5	1116.3	0.0	1116.3	2733.8	605.0	394.46	1300.08	3733.3	30.0
5	673.1	944.4	1617.5	764.7	0.0	764.7	2382.2	604.5	394.46	948.48	3381.2	29.5
6	633.6	944.4	1578.0	743.7	0.0	743.7	2321.7	600.4	394.46	927.52	3316.6	26.0

Table A1.16: Competitive equilibrium (E=-1.0)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.10	944.40	1617.50	2133.1	0	2133.1	3750.6	695.3	394.46	2406.49	4840.3	30.51
2	673.10	944.40	1617.50	1802.6	0	1802.6	3420.1	605.6	394.46	1986.36	4420.1	30.45
3	673.10	944.40	1617.50	1564.1	0	1564.1	3181.6	605.6	394.46	1747.87	4181.6	30.42
4	673.10	944.40	1617.50	1116.3	0	1116.3	2733.8	605.5	394.46	1300.08	3733.7	30.33
5	673.10	944.40	1617.50	764.7	0	764.7	2382.2	605.1	394.46	948.48	3381.8	30.05
6	673.10	944.40	1617.50	743.7	0	743.7	2361.2	601.9	394.46	927.52	3357.6	27.26

B. Cournot equilibrium (Different slope approach)

Table A1.17: Cournot equilibrium **(E=-0.1)**

t	qth1 #	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	0.0	944.4	944.4	1612.3	0.0	1612.3	2556.7	500.1	394.46	274.6	3725.9	98.0
2	0.0	944.4	944.4	1426.7	0.0	1426.7	2371.1	496.3	394.46	183.8	3445.6	94.7
3	0.0	944.4	944.4	1295.5	0.0	1295.5	2239.9	491.1	394.46	183.8	3309.2	90.2
4	0.0	860.7	860.7	1091.1	0.0	1091.1	1951.8	484.9	394.46	183.8	3014.9	84.9
5	0.0	738.4	738.4	947.3	0.0	947.3	1685.8	480.7	394.46	183.8	2744.7	81.3
6	0.0	645.7	645.7	838.4	0.0	838.4	1484.1	476.8	394.46	183.8	2539.2	77.9

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.18: Cournot equilibrium **(E=-1/3)**

t	qth1 #	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	61.2	944.4	1005.6	1815.6	0.0	1815.6	2821.2	441.2	394.46	274.6	3931.5	47.2
2	59.0	944.4	1003.4	1608.1	0.0	1608.1	2611.5	440.0	394.46	183.8	3629.8	46.2
3	49.7	944.4	994.0	1472.7	0.0	1472.7	2466.8	438.6	394.46	183.8	3483.6	45.0
4	31.4	944.4	975.7	1219.2	0.0	1219.2	2194.9	435.5	394.46	183.8	3208.6	42.2
5	21.1	833.9	855.0	1058.6	0.0	1058.6	1913.6	434.0	394.46	183.8	2925.8	40.9
6	9.0	733.3	742.3	949.6	0.0	949.6	1691.9	432.9	394.46	183.8	2703.1	40.0

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.19: Cournot equilibrium **(E=-0.5)**

t	qth1 #	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	248.6	944.4	1193.0	1882.4	0.0	1882.4	3075.4	431.3	394.46	274.6	4175.7	38.6
2	241.2	944.4	1185.6	1657.7	0.0	1657.7	2843.2	430.5	394.46	183.8	3852.0	38.0
3	242.8	944.4	1187.2	1498.5	0.0	1498.5	2685.7	429.6	394.46	183.8	3693.5	37.1
4	232.2	944.4	1176.6	1213.3	0.0	1213.3	2389.9	427.5	394.46	183.8	3395.6	35.3
5	259.0	871.2	1130.2	977.1	0.0	977.1	2107.3	426.1	394.46	183.8	3111.6	34.1
6	206.6	767.5	974.1	894.8	0.0	894.8	1868.9	425.4	394.46	183.8	2872.5	33.5

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.20: Cournot equilibrium **(E=-2/3)**

t	qth1 #	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	341.7	944.4	1286.1	1933.0	0.0	1933.0	3219.1	427.5	394.46	274.6	4315.7	35.4
2	341.7	944.4	1286.1	1688.1	0.0	1688.1	2974.2	426.9	394.46	183.8	3979.3	34.9
3	341.7	944.4	1286.1	1523.2	0.0	1523.2	2809.3	426.2	394.46	183.8	3813.8	34.3
4	341.7	944.4	1286.1	1213.8	0.0	1213.8	2499.9	424.7	394.46	183.8	3502.8	32.9
5	341.7	944.4	1286.1	956.4	0.0	956.4	2242.5	423.0	394.46	183.8	3243.7	31.5
6	341.7	848.2	1190.0	809.3	0.0	809.3	1999.3	422.4	394.46	183.8	2999.9	30.9

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.21: Cournot equilibrium **(E=-1.0)**

t	qth1 #	qth2	QRth	qh1 #	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	454.6	944.4	1399.0	1969.6	0.0	1969.6	3368.5	424.8	394.46	274.6	4462.4	33.0
2	454.6	944.4	1399.0	1711.2	0.0	1711.2	3110.2	424.4	394.46	183.8	4112.8	32.7
3	454.6	944.4	1399.0	1538.8	0.0	1538.8	2937.7	423.9	394.46	183.8	3939.9	32.3
4	454.6	944.4	1399.0	1215.0	0.0	1215.0	2614.0	422.8	394.46	183.8	3615.1	31.3
5	454.6	944.4	1399.0	945.7	0.0	945.7	2344.6	421.8	394.46	183.8	3344.6	30.4
6	454.6	944.4	1399.0	743.6	0.0	743.6	2142.6	420.7	394.46	183.8	3141.5	29.5

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table **A1.22**: Lerner Indices

	E=-0.1		***E=-1/3***		***E=0.5***		***E= -2/3***		***E=-1.0***	
t	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2
1	100%	59%	76%	38%	71%	32%	62%	26%	48%	19%
2	100%	66%	76%	43%	71%	35%	62%	29%	47%	21%
3	100%	73%	75%	47%	70%	38%	61%	31%	47%	22%
4	100%	79%	74%	56%	69%	45%	60%	36%	45%	25%
5	100%	78%	73%	56%	67%	48%	58%	42%	43%	29%
6	100%	77%	72%	55%	67%	47%	57%	42%	42%	33%

3. Base Model, Same slope approach, Reduced Hydro Flows

A. Competitive Equilibrium (Same slope approach)

Table A1.23: Competitive equilibrium (E=-0.1)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	860.0	1101.3	1961.3	833.4	0.0	833.4	2794.7	629.9	242.4	952.7	3666.9	107.0
2	779.2	1101.3	1880.5	808.2	0.0	808.2	2688.7	606.1	242.4	927.5	3537.1	86.5
3	779.2	992.9	1772.1	808.2	0.0	808.2	2580.3	596.2	242.4	927.5	3418.9	78.0
4	705.2	992.9	1698.1	808.2	0.0	808.2	2506.3	566.8	242.4	927.5	3315.4	52.6
5	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	546.2	242.4	927.5	3214.3	34.8
6	535.8	944.4	1480.2	808.2	0.0	808.2	2288.4	536.0	242.4	927.5	3066.7	26.0

Table A1.24: Competitive equilibrium (E=-1/3)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	779.2	992.9	1772.1	833.4	0.0	833.4	2605.5	572.6	242.4	952.7	3420.4	57.6
2	673.1	992.9	1666.0	808.2	0.0	808.2	2474.2	566.0	242.4	927.5	3282.6	51.9
3	673.1	992.9	1666.0	808.2	0.0	808.2	2474.2	560.6	242.4	927.5	3277.2	47.2
4	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	551.5	242.4	927.5	3219.6	39.4
5	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	543.3	242.4	927.5	3211.3	32.3
6	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	537.0	242.4	927.5	3205.0	26.9

Table A1.25: Competitive equilibrium (E=-0.5)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.1	992.9	1666.0	833.4	0.0	833.4	2499.4	564.0	242.4	952.7	3305.7	50.2
2	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	558.7	242.4	927.5	3226.8	45.6
3	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	555.1	242.4	927.5	3223.2	42.5
4	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	548.4	242.4	927.5	3216.4	36.7
5	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	542.8	242.4	927.5	3210.9	31.9
6	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	538.6	242.4	927.5	3206.6	28.2

Table A1.26: Competitive equilibrium (E=-2/3)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.1	944.4	1617.5	833.4	0.0	833.4	2450.9	559.0	242.4	952.7	3252.3	45.9
2	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	554.5	242.4	927.5	3222.6	42.0
3	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	551.8	242.4	927.5	3219.9	39.7
4	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	546.8	242.4	927.5	3214.8	35.3
5	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	542.6	242.4	927.5	3210.6	31.7
6	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	539.4	242.4	927.5	3207.5	29.0

Table A1.27: Competitive equilibrium (E=-1.0)

t	qth1	qth2	QRth	qh1	qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.1	944.4	1617.5	833.4	0.0	833.4	2450.9	553.3	242.4	952.7	3246.5	40.9
2	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	550.3	242.4	927.5	3218.4	38.4
3	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	548.5	242.4	927.5	3216.6	36.8
4	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	545.2	242.4	927.5	3213.2	33.9
5	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	542.4	242.4	927.5	3210.4	31.5
6	673.1	944.4	1617.5	808.2	0.0	808.2	2425.7	540.2	242.4	927.5	3208.3	29.7

B. Cournot equilibrium, Same slope approach, Reduced Hydro Flows

Table A1.28: Cournot equilibrium (E=-0.1)

t	qth1 #	qth2	QRth	qh1 #	Qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	486.8	992.9	1479.8	1041.5	0.0	1041.5	2521.2	524.4	242.4	119.3	3407.4	119.0
2	488.9	983.0	1472.0	834.2	0.0	834.2	2306.2	510.0	242.4	119.3	3177.9	106.5
3	455.6	944.4	1399.9	767.8	0.0	767.8	2167.7	502.9	242.4	119.3	3032.3	100.4
4	341.7	944.4	1286.1	743.6	0.0	743.6	2029.7	481.1	242.4	119.3	2872.5	81.6
5	231.8	863.9	1095.7	743.6	0.0	743.6	1839.3	468.2	242.4	119.3	2669.2	70.5
6	137.7	769.8	907.5	743.6	0.0	743.6	1651.1	461.6	242.4	119.3	2474.4	64.8

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.29: Cournot equilibrium **(E=-1/3)**

t	qth1 #	qth2	QRth	qh1 #	Qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	614.3	944.4	1558.7	1029.6	0.0	1029.6	2588.3	453.2	242.4	119.3	3403.2	57.6
2	607.0	944.4	1551.4	826.8	0.0	826.8	2378.2	448.6	242.4	119.3	3188.4	53.5
3	527.4	944.4	1471.8	787.1	0.0	787.1	2258.9	445.9	242.4	119.3	3066.5	51.2
4	454.6	944.4	1399.0	743.6	0.0	743.6	2142.6	438.5	242.4	119.3	2942.8	44.9
5	426.4	944.4	1370.8	743.6	0.0	743.6	2114.4	430.9	242.4	119.3	2906.9	38.3
6	341.7	902.7	1244.4	743.6	0.0	743.6	1988.0	427.4	242.4	119.3	2777.1	35.3

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.30: Cournot equilibrium **(E=-0.5)**

t	qth1 #	qth2	QRth	qh1 #	Qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	662.7	944.4	1607.1	1046.3	0.0	1046.3	2653.4	442.2	242.4	119.3	3457.3	48.0
2	662.7	944.4	1607.1	836.3	0.0	836.3	2443.4	439.0	242.4	119.3	3244.1	45.3
3	618.9	944.4	1563.3	760.9	0.0	760.9	2324.1	437.3	242.4	119.3	3123.1	43.8
4	454.6	944.4	1399.0	743.6	0.0	743.6	2142.6	433.3	242.4	119.3	2937.6	40.3
5	454.6	944.4	1399.0	743.6	0.0	743.6	2142.6	427.7	242.4	119.3	2932.0	35.5
6	454.6	944.4	1399.0	743.6	0.0	743.6	2142.6	423.5	242.4	119.3	2927.8	31.9

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.31: Cournot equilibrium **(E=-2/3)**

t	qth1 #	qth2	QRth	qh1 #	Qh2	QRh	QR	Qthfringe	qMR	qhPS	Qtot	Price
1	673.1	944.4	1617.5	1055.0	0.0	1055.0	2672.4	437.1	242.4	119.3	3471.3	43.7
2	673.1	944.4	1617.5	844.9	0.0	844.9	2462.4	434.8	242.4	119.3	3258.8	41.6
3	673.1	944.4	1617.5	743.6	0.0	743.6	2361.1	433.2	242.4	119.3	3156.0	40.3
4	477.5	944.4	1421.8	743.6	0.0	743.6	2165.4	430.4	242.4	119.3	2957.5	37.8
5	454.6	944.4	1399.0	743.6	0.0	743.6	2142.6	426.4	242.4	119.3	2930.7	34.4
6	454.6	944.4	1399.0	743.6	0.0	743.6	2142.6	423.3	242.4	119.3	2927.5	31.7

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table A1.32: Cournot equilibrium **(E=-1.0)**

t	**qth1 #**	**qth2**	**QRth**	**qh1 #**	**Qh2**	**QRh**	**QR**	**Qthfringe**	**qMR**	**qhPS**	**Qtot**	**Price**
1	673.1	944.4	1617.5	1055.0	0.0	1055.0	2672.4	432.3	242.4	119.3	3466.4	39.5
2	673.1	944.4	1617.5	844.9	0.0	844.9	2462.4	430.7	242.4	119.3	3254.8	38.1
3	673.1	944.4	1617.5	743.6	0.0	743.6	2361.1	429.7	242.4	119.3	3152.4	37.2
4	607.8	944.4	1552.2	743.6	0.0	743.6	2295.8	426.8	242.4	119.3	3084.3	34.8
5	454.6	944.4	1399.0	743.6	0.0	743.6	2142.6	425.2	242.4	119.3	2929.4	33.3
6	454.6	944.4	1399.0	743.6	0.0	743.6	2142.6	423.0	242.4	119.3	2927.3	31.5

# denotes multiple equilibrium. Values reported are averages over 400 simulations.

Table **A1.33**: Lerner Indices

	E=-0.1		***E=-1/3***		***E=0.5***		***E= -2/3***		***E=-1.0***	
t	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2	Firm 1	Firm 2
1	78%	51%	55%	32%	46%	25%	38%	21%	28%	16%
2	76%	56%	51%	34%	43%	27%	35%	22%	26%	16%
3	74%	57%	49%	35%	41%	28%	34%	23%	25%	16%
4	81%	70%	51%	40%	38%	30%	31%	24%	25%	18%
5	84%	75%	59%	47%	43%	34%	34%	27%	23%	18%
6	83%	72%	59%	49%	48%	38%	37%	29%	25%	19%

Appendix #2

A simple 1-year Cournot model for Chile's Electricity Industry

The model that was used to analyze the exercise of market power in Chile's electricity industry assumed that the hydro producer was able to allocate its hydro production over a 1 month planning horizon. In addition, in that model no value was given to the water left in the reservoir at the end of the month. Accordingly, even though Firm 1, the hydro producer, was "allowed" by the model to use only a fraction of the water available, incentives pushed more in the direction of using it all.

In real life, however, producers usually maximize over a longer time horizon, and thus it is reasonable to expect them to store a fraction of their hydro flows available in a particular month in order to use them in a later period. An interesting question is whether the hydro scheduling pattern that resulted in the 1-month period model would also be observed when a longer planning horizon is assumed. If so, generators would exploit differences in demand elasticity across months by shifting water from low demand elasticity months to high demand elasticity months. According to the data reported in Table 15, this means that water should be stored in October, November and December and released in April, March, January and the winter months[62]. In the paper I report evidence based on MVW analysis suggesting that Firm 1's incentive to exercise market power by shifting hydro production from one month to another depends on how large are the inter-month differences in demand elasticity/ The smaller the fluctuations, the closer is the hydro scheduling strategy to the traditional competitive supply-demand or value-maximizing optimization analysis' conclusions (i.e. water is stored when it is relatively abundant and released when it is relatively scarce).

The MVW analysis is helpful in that one can draw conclusions regarding how hydro flows would be allocated across months, but it does not provide information regarding the size of the flows that are stored and later released. In addition, it is not possible to compare the market equilibrium when market power is and is not exercised. In order to address these issues, a very simple 1-year model was estimated. This model will still suffer the omission of re-allocation across years but will probably shed lights of the re-allocations that happen within a year.

Given that the estimation of this model was computationally intensive, I had to use many simplifying assumptions. In addition, information on many variables was required but not available; as a consequence, additional assumptions were required.

[62] Market demand was parameterized as before, and thus it was assumed that price elasticity was –1/3 at the peak anchor point. Residual demand is not constant throughout the year because must run production and the fringe's production fluctuates over the different months.

The Model

The model is basically the same than the 1-month model. I assumed that there are only *two* periods per month (rather than 6 as in the 1-month model): a high demand period and a low demand period (t=1 and t=2 respectively). Each of them lasts for 360/372 hours. Notice that by reducing the number of periods to only two, I am implicitly reducing the demand variability. Given that Firm 1 is allowed to store hydro flows one month and release them in a later period, a state variable, namely the reservoir level, is introduced. Water inflows are assumed to get to the reservoir at the beginning of each month.

Each period, Cournot producers face a residual demand given by:

$$D^R(P_t) = D(P_t) - S^f(P_t) - q^{MR}{}_t - q^{PS}{}_{ht}$$

where D(P) is *expected* market demand, D^R (P) is *expected* residual market demand, S^f (P) is the Fringe's (thermal) supply function (adjusted by transmission losses), q^{MR} is must-run units' generation and $q^{PS}{}_h$ is the Fringe's hydro production from reservoirs distributed across periods according to a Peak shaving strategy.

Firm 2's optimization problem remains basically the same than in the 1-year model. Firm 1's optimization problem is modified to take account of the reservoir level, and the corresponding minimum and maximum capacity constraints.

Firm 1's Optimization problem

$$\text{Max} \sum_t \{ P_t(q_t)*(q_{1ht} + q_{1Tht}) - CT_1(q_{1Th\,t}) \} \qquad (1)$$

s.t.

(2) $q_{1Th\ MIN} \leq q_{1Th\ t} \leq q_{1Th\ MAX}$ $\quad \forall\ t$ (thermal production min/max constraints)

(3) $q_{1h\ MIN} \leq q_{1h\ t} \leq q_{1h\ MAX}$ $\quad \forall\ t$ (hydro production min/max constraints)

(4) $S_m = S_{m-1} + F_m - q_{1hydro\ m}$ $\quad \forall\ m$ (Reservoir's level evolution)

(5) $S_{MIN} \leq S_m \leq S_{MAX}$ $\quad \forall\ m$ (Reservoir's min/max capacity constraint)

(6) $S_0 = S_{INI}$ (Reservoir's initial level)

(7) $S_{12} = S_{END}$ (Reservoir's level at the end of the year)

where:

m = month index; m = 1,2,..12. m=1 for the *first* month of the hydrological year assumed.

t = index for each period of 360/372 hours. Since I assumed that there are only two periods per month, t = 1,2, …24. The periods are set in chronological order (t=1 for the high period, month 1; t=2 for the low period, month 1; t=3 for the high period, month 2, and so on).

$P_t(q_t)$ = is the inverse function of the *expected* residual demand in period t

Q_t = is total production by firm 1 and 2 in period t, ($Q_t = q_{1t} + q_{2t}$),
$q_{it} = q_{iTh\ t} + q_{ih\ t}$ is total production by Firm i in period t,
$q_{iTh\ t}$ = total energy produced by Firm i out of thermal plants, period t
$q_{1h\ t}$= total energy produced by Firm 1 out of hydro-storage plants, period t
$CT_i(q_{iTh\ t})$ = Total Cost function, thermal plants, firm i
$q_{iTh\ MIN}$ = Minimum thermal production, Firm i, period t
$q_{1Th\ MAX}$ = Maximum thermal production, Firm i, period t
$q_{1h\ MIN}$ = Minimum hydro production, Firm 1, period t
$q_{1h\ MAX}$ = Maximum hydro production, Firm 1, period t
$q_{1h\ tot}$ = available hydro production for the whole period
S_m = reservoir level at the *end* of month m, measured in terms of energy.
S_{INI} = initial reservoir level.
S_{END} = end of year reservoir level.
F_m = water inflow *expected* to get to the reservoir at the beginning of month m. Measured in terms of energy.
$q_{1hydro\ m}$ = total energy produced by Firm 1 out of hydro storage plants, month m.

$$q_{1hydro\ m} = q_{1h(2m-1)} + q_{1h(2m)}$$ [63]

The objective function is basically the same than in the 1-month model except for the residual demand, which is in terms of expectation. Constraints (2) and (3) are the same than in the 1-month model and thus need no further explanation. Constraint (4) states that the hydro energy available in the reservoir at the end of each month is equal to the hydro energy available at the end of the previous month plus the water inflows (in terms of hydro energy) received at the *beginning* of the month and less the hydro energy that was produced during the month. Constraint (5) states the minimum and maximum bound for the reservoir level. Finally constraints (6) and (7) are initial and end conditions.[64]

This version of the model takes into the account the incentives Firm 1 has to re-allocate water from one month to another (but always within a year). This is a good assumption for all the Chilean reservoirs except for El Laja (the biggest one) that is so big that its water can be allocated from one year to another. For the moment it is not possible to estimate a model with such a long planning horizon because the information requirements cannot be fulfilled.

This model's treatment of uncertainty is certainly simplistic and incomplete. Only expected values of water inflows and demand per period are considered. Variability is not taken into account because there was no information available. This omission is important, especially with respect to hydro inflows, as hydro producers are likely to manage their reservoirs taking into account the probability of the hydrological year being normal, dry or rainy. Under the current regulation, generators are required to ensure the power supply even under the driest conditions. As a consequence it is reasonable to expect them to manage the reservoirs less aggressively. The exercise of market power is not likely to be the unique goal producers pursue as this model implicitly assumes.

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[63] $q_{1hydro\ m=1} = q_{1h(t=1)} + q_{1h(t=2)}$, $q_{1hydro\ m=2} = q_{1h(t=3)} + q_{1h(t=4)}$, and so on.
[64] The model will be estimated under two versions: with and without constraint (7).

I assumed that the discount rate was equal to zero.

Model estimation

The model was estimated under two different market assumptions: i) the market is perfectly competitive and ii) producers are able to exercise market power.

Market demand was assumed to be linear. Each month's slope was calculated such that the elasticity at the peak demand level is $-1/3$. Anchor quantity to parameterize demand was set to the average load (augmented by spinning reserves) of the highest/lowest 360/372 hours. Anchor price was given by each month's nodal price. Table A2.1 reports the data used to parameterize the market demand and to estimate the residual demand.

Capacity and marginal cost functions are the same than in the 1-month model.

Hydro inflows are given by the average hydro generation per month reported by the CNE for "normal" hydrological year (See Table A2.2). In this sense, water inflows can be interpreted as "expected flows". These inflows were used to allocate hydro production according to a peak shaving strategy, when needed. In the model where Firms 1 and 2 are supposed to be able to exercise market power, I allocated the Fringe's hydro production according to this approach. In this case, I assumed that its water inflows were not large enough to store a portion of them given the min and max production constraints. As a consequence, its hydro production per period was calculated by allocating hydro resources over a 1-month time period. When perfect competition is assumed, water inflows of the entire system are allocated according to the peak shaving strategy.

As expected, results of the model turned out to be sensitive to the starting month of the one-year period (i.e. hydro scheduling for the year Jan – Dec was slightly different than for April – March, and so on). The longer the planning horizon, the less important the starting point is. Unfortunately, my data set is too short to further increase the time horizon. In addition I do not have information that could be used to simulate water inflows for a longer period. In an attempt to overcome this problem, I estimated the model and the competitive equilibrium for the 12 possible starting points. I will report results for each estimation and will look for general trends.

I assumed that minimum and maximum capacity of the reservoirs were not restrictive, as I did not have information for many of them. This assumption should not introduce too many distortions as in the past years the reservoir level reached its maximum or minimum capacity only a few times, and most of them took place in very rainy or dry years, and thus in periods in which inflows were different from the ones used to estimate this model.[65]

[65] The problem to this argument is that all the observations are from periods in which the management of hydro resources is supposed to have been competitive.

The reservoir level at the end of the year may be important too, being this another consequence of the short time period used. In order to analyze how important this constraint is, I estimated two versions of the model:

- Reservoir level at the end of the year restricted to be equal to the initial level; in this way, Firm 1 is constrained to use all the water available in the year ("bounded estimation").
- Reservoir level at the end of the year restricted to be within min/max bound limits (Constraint (7) is not included). Firm 1 is implicitly allowed to use only a fraction of the water available for the whole year ("unbounded estimation")

It is worth to mention that even though results from these two versions of the model differ, the differences are rather small. In particular, any change in the amount of hydro production is usually compensated at least in part by a change in Firm 2's thermal production.

Results

I do not intend to exhaustively analyze the results of this model's estimation, as many of the results are similar, in qualitative terms, to what was found in the 1-month model and additional insights are beyond this paper's reach. I will focus on the question that motivated the estimation of this model: Would Firm 1 store water in low price-elasticity of demand months and release it in high price-elasticity of demand months as it could be extrapolated from the 1-month model? The analysis will be made in comparative terms. In particular, I will compare each month's rate of use of water inflows that results in the Cournot and in the competitive model. Results are reported in Tables A2.3, A2.4, A2.5 and A2.6

The assumption for the reservoir level at the end of the year turned out not to be important, as results from the "bounded" and "unbounded" almost coincide.

The starting month assumption is more important for the competitive equilibrium than for any of the Cournot models estimated. In the latter, the pattern of hydro scheduling is almost always the same (but magnitudes differ), no matter what month was assumed to be the first. In particular, water is stored in June and July (winter months) and October and November. Water from June and July is completely shifted to August and September, while water stored in October and November is allocated to the first half of the year. In the case of the competitive equilibrium, when the model was estimated assuming that the initial month was any of the first 5 months, all the water available was exhausted in every 1-month period. Results start to change as June and the months in the second half of the year are assumed to be the "initial month", as there is enough water to be stored and used in a later period.[66] In a perfectly competitive industry, water would be stored in the second half of the year, the rainy and melting season, and released in the first half of the next year. This allocation is in line with the availability of hydro production relative to demand as power is "saved" in the periods when it is relatively abundant and it is released when it is relatively scarce.

[66] In line with this result is the greater standard deviation that resulted in the competitive model.

As I mentioned, a Cournot producer would store power in June, July, October and November. According to these results, it could not be argued that water is allocated from low elasticity months to high elasticity months. Indeed, as Table A2.6 reports, the correlation coefficient between the average price elasticity of demand and the rate of use of available hydro flows is positive, as expected, but relatively small (0.37). Firm 1's hydro scheduling strategy is closer to the conclusions of the traditional competitive supply-demand analysis, where water is stored in those periods in which it is relatively abundant and released when it is relatively scarce. This conclusion is supported by the correlation coefficient between the rate of use of available hydro flows and the inflows/demand monthly ratio (-0.89). See charts A2.1 and A2.2 for a graphic support of the relative importance of each of the alternative explanations for the hydro scheduling strategy.

In order to understand why Firm 1 does not inter-month differences in demand elasticity, two alternative hypotheses are formally analyzed.

1. *Intra-month differences in demand elasticity are big enough* and water can be allocated within a month in order to exercise market power. In order to analyze this hypothesis, I estimated the model assuming no intra-month differences in demand elasticity and keeping constant inter-month differences. A priori, this explanation does not sound plausible as I assumed that there are only two periods in each month and therefore, demand variability is considerably reduced. If this hypothesis is true, then we should observe that given that producers cannot exploit intra-month demand variability, greater inter-month reallocation should be observed. As it is reported in Table A2.6, column (2), the resulting correlation coefficient between the use rate of available inflows and price-elasticity of demand is still positive, but considerably smaller. At the same time, the correlation with the inflows/demand ratio is still high and thus the relationship strong. Accordingly, intra-month variability seems not to be a good explanation.
2. *Inter-month differences in demand elasticity are relatively small. As a result, the incentive to exploit them is also small.* In order to test for this hypothesis, I estimated the model assuming that price elasticity of market demand was not –1/3 at the peak anchor point, as I did in the previous exercise, but could take different values (E=-0.1, -1/3, -1/2, -2/3, -1.0) which were randomly allocated to each month. Consequently, inter-month variability of price elasticity of demand was 7 times higher. Results are fully supportive of this hypothesis. Indeed, the correlation coefficient is very high (0.95) and it is almost unaffected when no intra-month variability is assumed (column (4)).

Observe that the larger are inter-month differences in price elasticity, the more incentives Firm 1 has to allocate its hydro resources in order to exercise market power and the less relevant is the traditional competitive supply-demand analysis in order to understand the hydro scheduling strategy (See Charts A2.3 and A2.4). Preliminary evidence from a 1-year model is in line with the MVW analysis: producers' incentive to exercise market power by shifting water from one month to another depends on how large inter-month differences in price elasticity are. The smaller the difference, the closer is the hydro scheduling strategy to the traditional supply-demand analysis' conclusions (i.e. more water to periods in which water is relatively scarce).

I would like to conclude this analysis by emphasizing that this model is just an attempt to address the issue of hydro scheduling over a longer planning horizon when firms have market power. It is clear that an additional modeling effort is needed in order to have more definitive answers. In particular, it is necessary to improve the treatment of uncertainty and to further increase the planning horizon (in order to reduce the importance of the starting and final points).

Table A2.1 : Data to estimate residual demand (E=-1/3)

Month	t	Average Load (MW)	Price (US$/MW)	A	B	qPS	qMR
Jan	1	4169.4	31.14	5559.2	44.6	180.7	590.9
Jan	2	3178.6	31.14	4238.1	34.0	137.8	590.9
Feb	3	4008.4	31.14	5344.6	42.9	148.6	566.2
Feb	4	3492.2	31.14	4656.3	37.4	144.0	566.2
Mar	5	4386.8	31.14	5849.1	47.0	489.2	509.5
Mar	6	3439.9	31.14	4586.6	36.8	372.3	509.5
Apr	7	4176.0	31.10	5568.1	44.8	259.9	394.5
Apr	8	3300.8	31.10	4401.1	35.4	137.8	394.5
May	9	4321.9	31.10	5762.5	46.3	406.9	399.6
May	10	3433.8	31.10	4578.4	36.8	137.9	399.6
June	11	4398.8	31.10	5865.1	47.1	489.2	455.7
June	12	3364.4	31.10	4485.9	36.1	387.6	455.7
July	13	4286.3	31.10	5715.0	45.9	489.2	447.2
July	14	3295.8	31.10	4394.4	35.3	313.0	447.2
Aug	15	4224.6	31.10	5632.7	45.3	489.2	431.5
Aug	16	3302.0	31.10	4402.7	35.4	396.4	431.5
Sept	17	4179.9	32.74	5573.2	42.6	489.2	473.9
Sept	18	3178.8	32.74	4238.4	32.4	368.9	473.9
Oct	19	4228.5	37.26	5637.9	37.8	489.2	498.7
Oct	20	3219.6	37.26	4292.9	28.8	322.0	498.7
Nov	21	4298.8	37.26	5731.7	38.5	489.2	587.0
Nov	22	3344.6	37.26	4459.4	29.9	317.4	587.0
Dec	23	4334.2	37.26	5778.9	38.8	489.2	603.1
Dec	24	3353.1	37.26	4470.8	30.0	249.7	603.1

Table A2.2 : Expected water inflows (GWh)

Month	Firm 1	Fringe	Total
January	940.5	118.5	1059.0
February	898.6	98.3	996.9
March	1012.6	320.5	1333.1
April	974.9	143.2	1118.0
May	1091.5	202.7	1294.3
June	1245.4	315.7	1561.1
July	1177.5	298.4	1475.9
August	1054.8	329.4	1384.2
September	953.7	308.9	1262.5
October	1177.1	301.8	1478.9
November	1094.7	290.4	1385.1
December	1004.9	274.9	1279.8

Table A2.3: Cournot model, "bounded version"

Hydro production / hydro production available

Starting month ↓	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	100%	99%	99%	102%	100%	90%	97%	107%	109%	93%	98%	111%
Feb	117%	100%	98%	102%	100%	89%	95%	105%	108%	90%	95%	108%
Mar	115%	113%	98%	102%	100%	88%	94%	103%	106%	89%	94%	107%
Apr	114%	112%	110%	100%	100%	87%	93%	102%	105%	88%	93%	106%
May	113%	110%	109%	112%	100%	86%	92%	101%	103%	87%	92%	105%
Jun	112%	109%	108%	112%	106%	85%	91%	100%	103%	87%	91%	104%
Jul	111%	108%	106%	110%	104%	100%	90%	98%	101%	86%	90%	103%
Aug	109%	106%	105%	109%	103%	97%	104%	97%	99%	85%	89%	102%
Sep	109%	106%	105%	108%	103%	93%	99%	109%	99%	84%	89%	101%
Oct	109%	106%	105%	108%	103%	90%	97%	107%	109%	84%	89%	101%
Nov	106%	103%	102%	106%	100%	90%	97%	107%	109%	100%	87%	99%
Dec	103%	100%	99%	103%	100%	90%	97%	107%	109%	97%	103%	96%

Ratio > 100% → use > monthly inflow, ratio < 100% → use < monthly inflow (water is stored).

Table A2.4: Cournot model, "Unbounded version"

Hydro production / hydro production available

Starting month ↓	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	100%	99%	99%	102%	100%	90%	96%	106%	108%	91%	96%	109%
Feb	117%	100%	98%	102%	100%	89%	95%	105%	108%	90%	95%	108%
Mar	115%	113%	98%	102%	100%	88%	94%	103%	106%	89%	94%	107%
Apr	114%	112%	110%	100%	100%	87%	93%	102%	105%	88%	93%	106%
May	113%	110%	109%	112%	100%	86%	92%	101%	103%	87%	92%	105%
Jun	112%	110%	108%	112%	106%	85%	91%	100%	103%	87%	91%	104%
Jul	111%	108%	106%	110%	104%	90%	90%	98%	101%	86%	90%	103%
Aug	109%	106%	105%	109%	103%	90%	96%	97%	99%	85%	89%	102%
Sep	109%	106%	105%	108%	103%	90%	96%	106%	99%	84%	89%	101%
Oct	109%	106%	105%	108%	103%	90%	96%	106%	108%	84%	89%	101%
Nov	106%	103%	102%	106%	100%	90%	96%	106%	108%	91%	87%	99%
Dec	103%	100%	99%	103%	100%	90%	96%	106%	108%	91%	96%	96%

Ratio > 100% → use > monthly inflow, ratio < 100% → use < monthly inflow (water is stored).

Table A2.5: Competitive model

Hydro production / hydro production available

Starting month ↓	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Jan	100%	100%	100%	100%	100%	92%	96%	101%	102%	92%	100%	113%
Feb	124%	100%	100%	100%	100%	90%	94%	99%	100%	91%	99%	112%
Mar	121%	121%	100%	100%	100%	89%	92%	97%	98%	89%	97%	110%
Apr	120%	120%	108%	100%	100%	88%	92%	96%	97%	88%	96%	109%
May	118%	118%	107%	118%	100%	87%	90%	95%	96%	87%	95%	107%
Jun	117%	117%	106%	118%	107%	86%	90%	94%	95%	86%	94%	106%
Jul	117%	117%	106%	106%	100%	100%	90%	94%	95%	86%	94%	106%
Aug	117%	117%	100%	100%	100%	98%	102%	94%	95%	86%	94%	106%
Sep	117%	108%	100%	100%	100%	96%	100%	105%	95%	86%	94%	106%
Oct	117%	102%	100%	100%	100%	94%	98%	104%	105%	86%	94%	106%
Nov	100%	100%	100%	100%	100%	95%	100%	105%	102%	100%	94%	106%
Dec	100%	100%	100%	100%	100%	95%	99%	104%	104%	96%	104%	100%

Ratio > 100% → use > monthly inflow, ratio < 100% → use < monthly inflow (water is stored).

Table A2.6: Rate of use of Inflows under different price elasticity assumptions.

	Hydro production / Inflows (Bounded version)					Average Elasticity of residual demand				St dev intra-month elasticity				Inflows/ net demand
	PC	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	
Jan	114%	110%	111%	66%	70%	0.46	0.46	0.14	0.14	0.10	0.0	0.03	0.0	0.39
Feb	110%	106%	106%	138%	137%	0.42	0.42	1.22	1.22	0.05	0.0	0.14	0.0	0.40
Mar	102%	104%	105%	76%	75%	0.47	0.47	0.14	0.14	0.09	0.0	0.03	0.0	0.46
Apr	104%	106%	96%	118%	119%	0.52	0.52	1.02	1.02	0.12	0.0	0.23	0.0	0.40
May	101%	102%	104%	95%	96%	0.43	0.43	0.43	0.43	0.06	0.0	0.06	0.0	0.45
Jun	92%	91%	91%	79%	78%	0.48	0.48	0.48	0.48	0.10	0.0	0.10	0.0	0.56
Jul	95%	95%	96%	80%	81%	0.47	0.47	0.47	0.47	0.09	0.0	0.09	0.0	0.52
Aug	99%	103%	104%	101%	99%	0.47	0.47	0.70	0.70	0.09	0.0	0.14	0.0	0.49
Sep	99%	105%	106%	113%	110%	0.45	0.45	0.87	0.87	0.10	0.0	0.18	0.0	0.48
Oct	90%	89%	90%	98%	97%	0.37	0.37	0.68	0.68	0.07	0.0	0.12	0.0	0.53
Nov	96%	92%	93%	129%	133%	0.38	0.38	0.95	0.95	0.07	0.0	0.14	0.0	0.50
Dec	107%	104%	105%	113%	110%	0.37	0.37	0.69	0.69	0.06	0.0	0.10	0.0	0.45
Average						0.44	0.44	0.65	0.65	0.08	0.0	0.11	0.0	0.47
stdev						0.05	0.05	0.34	0.34	0.02		0.06		0.06
Corr Elasticity	0.11	0.37	0.12	0.95	0.94									
Corr Inflows / Net demand	-0.92	-0.89	-0.73	-0.23	-0.25									

Ratio > 100% → use > monthly inflow, ratio < 100% → use < monthly inflow (water is stored).

Figure A2.1

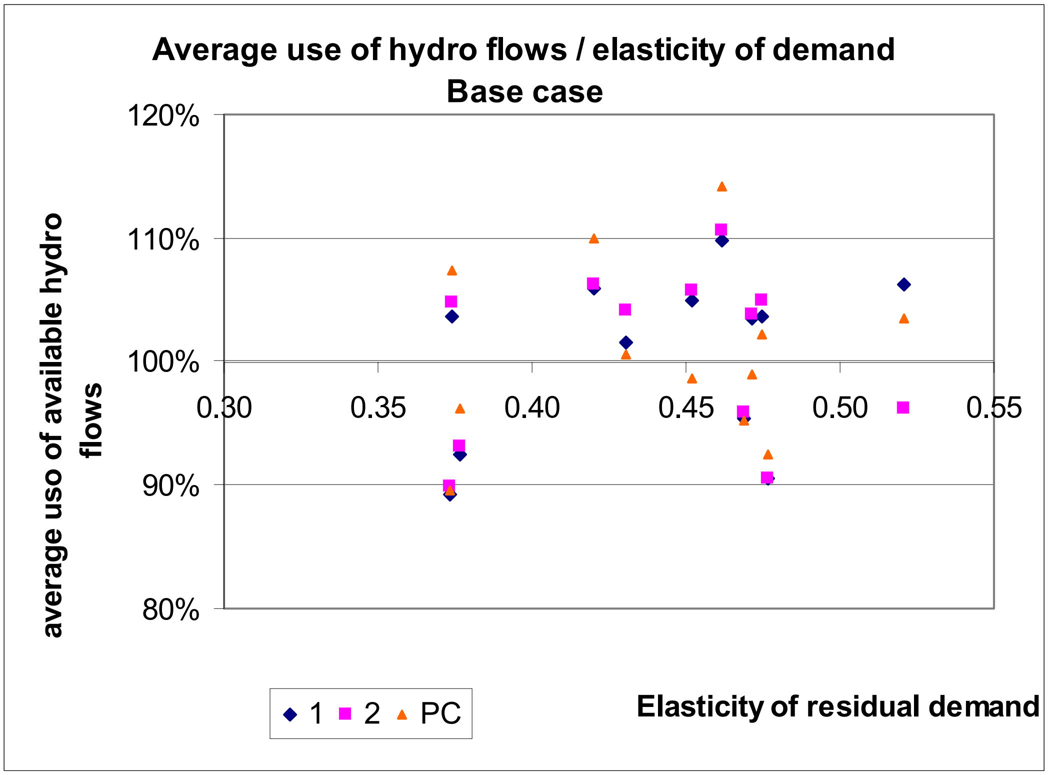

Figure A2.2

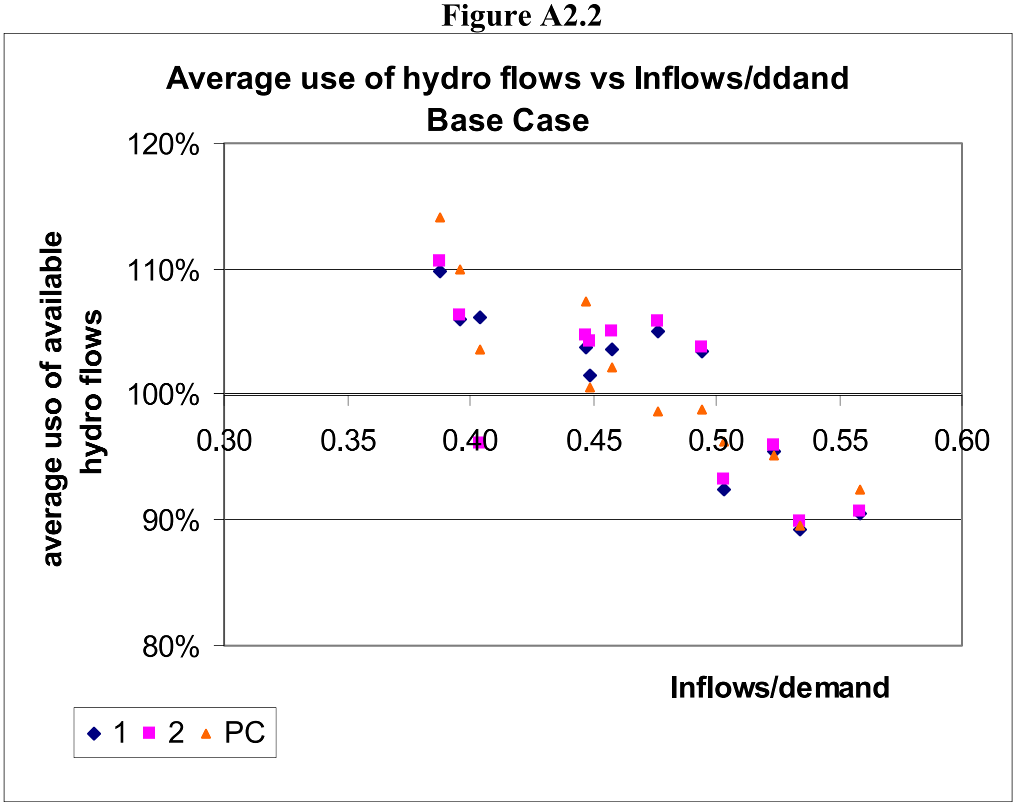

Figure A2.3

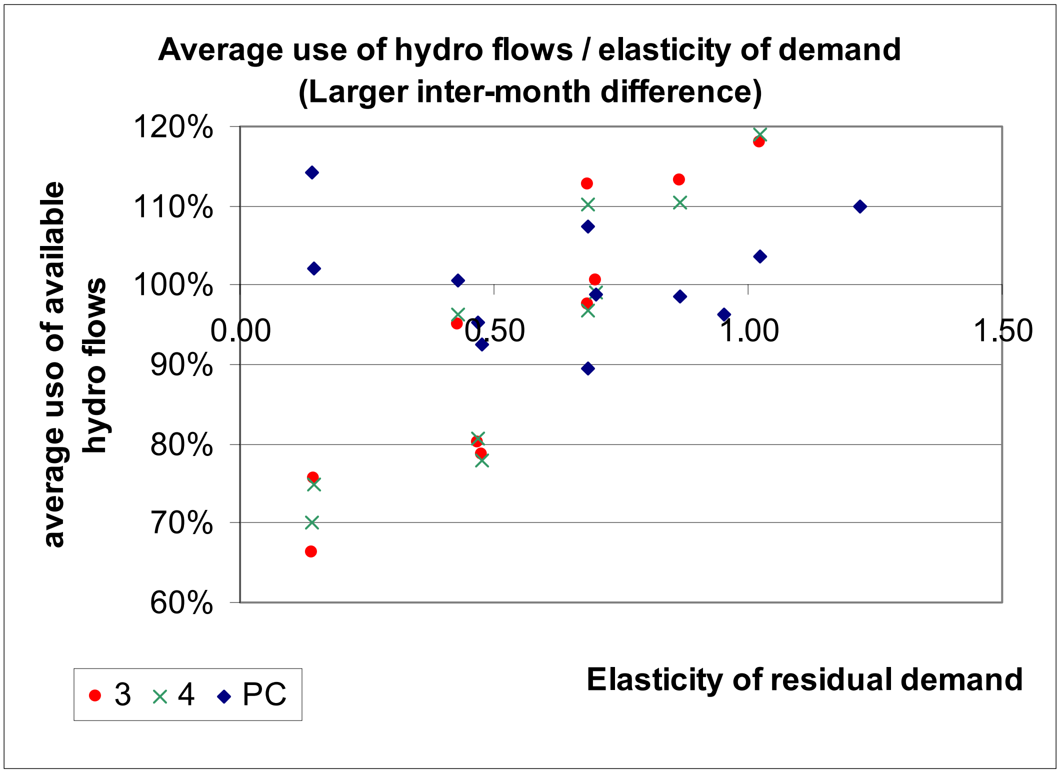

Figure A2.4