Storing Wind for a Rainy Day

What kind of electricity does Denmark export?

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Storing wind for a rainy day:

What kind of electricity does Denmark export?

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Abstract

Physical laws mean that it is generally impossible to identify which power stations are exporting to another country, but economic logic offers strong clues. On windy days, Denmark tends to export electricity to its neighbours, and to import power on calm days. Storing electricity in this way thus allows the country to deal with the intermittency of wind generation. We show that this kind of behaviour is theoretically optimal when a region with wind and thermal generation can trade with one based on hydro power. However, annual trends in Denmark's trade follow its output of thermal generation and are inversely related to Nordic production of hydro power and the amount of water available to Scandinavian generators, with no correlation with wind generation. We estimate the cost of volatility in Denmark's wind output to equal between 4% and 8% of its market value.

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1. Introduction

The last decade has seen a remarkable increase in the number of wind installations throughout the world, as part of a coordinated effort to shift towards a higher share of renewable generation. In Europe, Denmark is amongst the leading countries in wind generation, in terms of installed capacity relative to population and demand (Cossent et al, 2009; Gøransson and Johnsson, 2008; Eriksen et al., 2005). It is therefore not surprising that the country has often been used as a case study to investigate the consequences and challenges of high rates of wind generation, from both a technical and economic perspective. Proponents of wind power point out that Denmark successfully obtains a high proportion of its energy from wind; opponents suggest that this is only possible because of good interconnections to neighbouring countries, and that the Danish experience could not be replicated in countries without these (Sharman, 2005). Denmark's trade in electricity has been controversial, with some critics asserting that much of the electricity from wind power is exported, which apparently means Danish consumers get no benefit from it (Bach, 2009, CEPOS, 2009). In an interconnected electricity system, it is of course impossible to identify any one power station as the physical source of exports. Mignard et al (2004) have shown that there is a stronger correlation between the production of electricity from (so-called) local Combined Heat and Power plants and exports from West Denmark than between wind output and exports, but do not draw on any underlying model of the system's behaviour.

This paper re-examines Denmark's patterns of electricity production and trade from 2001 to 2009, in the light of the economic theory underlying the optimal operation of a power system with a mix of hydro, thermal and wind resources. One of the great benefits of the Nord Pool market is that it allows countries with different types of power stations to benefit from trade. We show that short-term fluctuations in wind output are highly correlated with short-term fluctuations in net exports of electricity, which is exactly the efficient pattern of operation dictated by our model. On a longer timescale, however, using data from 1996 to 2008, there is no correlation between Denmark's annual production of electricity from wind and its net exports. There is a strong correlation between Denmark's net exports and thermal production and (inversely) the level of hydro generation in the Nordic countries. Once again, this fits the pattern of efficient operation in our model. Denmark is effectively using its neighbours for short-term storage to offset fluctuations in its output of wind energy, and is being used by them for a longer-term response to fluctuations in their availability of hydro power.

We go on to estimate the cost of intermittency in Denmark's wind output. The cost of intermittency is only one part of the much broader analysis needed to assess the costs and benefits of wind power, but it is a part which has received attention in its own right (Gross et al, 2006). First, we compare the value of Denmark's wind output, evaluated at the actual Nord Pool prices when it is produced, with the value of a smoothed output series, substituting the average output for each hour and each month. Prices tend to be lower in hours with above-average wind output, and so the actual output series are worth 8% and 4.1% less than the smoothed series would have been, in West and East Denmark respectively. We also show that the market price in Denmark responds much more to additional wind output in hours when transmission lines to the north of Denmark are congested than in those when extra power can be exported.

There would also be a cost of storing wind power in Denmark's neighbours if Denmark has relatively higher net exports at times when the prices at which it trades are relatively low. This is not to suggest that Denmark actually loses from exporting at these times – quite the reverse. Denmark exports electricity when it is more valuable abroad than consumed at home, and imports power when the cost of doing so is less than the cost of local production. Nonetheless, Denmark tends to export more power on days when its price is relatively low for the time and season, and this can be seen as a storage charge that it pays its neighbours because of the fluctuations in its wind output. Our second calculation gives this cost, again summing the deviations from average output levels, but now valuing them at a trade-related price (either in Nord Pool, or between Nord Pool and the German market EEX). For West Denmark, this is 4.3% of the value of its wind output, and for East Denmark it is 3.1%. Note that these two measures of the cost of intermittency are based on alternative ways of viewing the problem (and the response to it) and should not be added together.

The rest of the paper is organised as follows: Section two gives background information on the Danish electricity system, and summarises the related literature. Section three develops a theoretical model to explain the trade flows between a wind- and hydro generating region. The model extends Førsund's (2007) generalised bathtub framework, making easier to account for the impact of varying wind output on the trade flows between the two regions of interest. Section four tests the theoretical implications of the model for the volumes of production and trade, using data for Denmark and its neighbours. The fifth section assesses the cost of fluctuations in wind output, in terms of differences between average prices and those actually received for wind output and Danish exports. Finally, section six concludes.

2. The Context

The Danish electricity system is divided between two separate zones, East and West, which are synchronized to the NORDEL (Nordic) and UCTE (Western European) grid systems, respectively. Both zones are connected to Germany and to Sweden; West Denmark is also connected to Norway. The two zones were not directly connected until the Storebaelt cable started operation in September 2010.

In 2009, 22.5% of the electricity generated in West Denmark, and 13% of that generated in East Denmark, came from wind power (19.6% for the country as a whole). The total wind capacity at the end of 2009 was 3,482 MW, compared to a maximum demand during the year of 6,287 MW. There are targets for a significant increase in wind capacity by 2020 – ENTSOe (2010) cites a figure of 5,635 MW. The country's interconnections are strong in proportion to its demand and generation – West Denmark can export up to 3.2 GW of power or import 2.6 GW, and East Denmark can export 2.3 GW or import 1.9 GW. Although East Denmark is the more densely populated zone of the two, it is West Denmark that exhibits the higher rates of wind generation (21% as opposed to 8% for East Denmark, Østergaard 2008).

Many studies on the integration of wind-generated output with other forms of generation appear in the engineering literature and focus on the technical challenges entailed in the integration and storage of large amounts of highly variable and intermittent wind output. In the economics literature, a number of studies analyse the possible impact of renewables on costs and prices for a particular country or region. For Denmark, Østergaard (2008) models the impact of the geographic dispersion of wind generation on the variability of wind output and, consequently, the need for operational reserve capacity. His findings suggest an explicit negative correlation between the two, implying that local fluctuations in wind output can be partly offset by spreading wind turbines over a sufficiently large area[1].

The benefits of interconnection in the integration of wind output are also discussed in Oswald et al (2008) for the UK, who, however, find that UK wind speeds can be highly and positively correlated with these in neighbouring countries during times of low output. As a result, wind output can be low during periods of high demand, implying again the need for sufficient available reserve capacity.

[1] However, the argument cannot be readily generalised to reduce the maximum reserve capacity needed in the area, as Østergaard acknowledges the presence of times with zero wind production in all interconnected areas.

Rosen et al (2007) present a model analysis of the short- and long-term effects of high levels of wind generation on output composition for Germany. Their results suggest that wind output acts mainly as a substitute for intermediate- and base-load plants (such as coal, lignite and nuclear plants), whereas gas-fired plants are used as stand-by capacities to balance fluctuations in wind output. Similar findings are reached in Weigt and Hirschhausen (2009), who use hourly wind generation data for Germany over the period 2006 to 2008, to assess the potential of wind energy as a substitute of installed conventional generation capacities. They also discuss the impact of wind generation on market prices, finding it to lead to significant reductions, especially during peak hours. Twomey and Neuhoff (2010) point out, however, that since the impact of wind generation in reducing prices is greatest in the hours with the highest levels of wind output, the market price weighted by the actual level of wind output will be relatively low. Wind generators who are remunerated on the basis of those market prices (as opposed to those receiving a feed-in-tariff) will thus be at a disadvantage. Green and Vasilakos (2010) simulate this effect for the case of Great Britain in 2020, with around a quarter of generation coming from wind, finding that onshore wind generators might receive 8% less than the time-weighted average price.

3. The System Model

The model we use is an extended version of the multi-period "bathtub" framework presented in Førsund (2007) and Førsund et al (2008), adjusted to allow for the inclusion of thermal, wind and hydro generation over several time periods, indexed by $t = 1 \ldots T$. In its simplest form, the original model provides a stylised way to analyse the effect of competing hydro and thermal generators on prices and demand patterns for each type of plant[2]. The quantities of generating output for each type of plant (and, the resulting price of electricity) can be obtained

[2] In a more recent article, Førsund et al (2008) extend the original model to include wind and hydro generation (but not thermal).

as solutions to a constrained optimisation problem that maximises a social welfare function subject to a series of capacity constraints.

We assume that generation takes place within two regions, A and B. Region A produces electricity from thermal and wind power stations, with outputs of e_t^{Th} and e_t^{W} respectively. The stations' capacities are fixed at $\bar{e}^{Th}$ and $\bar{e}^{W}$. Region B is equipped with hydro plants of total fixed capacity $\bar{e}^{H}$, producing output of e_t^{H}. The thermal generators have a collective variable cost function given by $c_t\left(e_t^{Th}\right)$. Hydro and wind generators have no variable costs. While the wind capacity is fixed, its ability to generate electricity depends on the strength of the wind, represented by the variable $u_t \in [0,1]$. Note that we measure u_t in electrical, rather than meteorological units: MW of electrical output per MW of installed generating capacity. Similarly, the water inflow to the hydro reservoirs, w_t, is measured in MWh rather than in cubic metres of water or mm of rainfall.[3] The amount of water in the reservoir at the end of period t is equal to R_t and the maximum capacity of the reservoir is $\bar{R}$ (both measured in MWh). All generators behave competitively.

There is demand in each region, $D_t^A(p_t^A)$ and $D_t^B(p_t^B)$, which varies between periods and is sensitive to the wholesale price in that region in that period, p_t^A or p_t^B as relevant. Retail prices (which we do not model) include the wholesale price (for smaller consumers, this will be averaged over many periods), transmission and distribution charges, and various taxes, which differ between countries. We can invert the demand functions to obtain the marginal (social) value of power, given by $p_t^A(D_t^A)$ and$p_t^B(D_t^B)$.

[3] Note that u_t will be non-monotonic in the wind speed as measured in metres per second – generators cannot produce any power when there is too little wind, and have to shut down when the wind is too high. Similarly, the amount of electricity that can be generated from a cubic metre of water actually depends on the state of the reservoir it flows into – the fuller the reservoir, the higher the head, and the greater the power that can be achieved – a complication we ignore here.

The two regions are connected to each other by an interconnector of finite capacity $\bar{x}$. We denote country A's exports by x_t^A and country B's by x_t^B; we will follow the convention that at most one of these is non-zero in any period. It should be obvious that A's exports are B's imports, and vice versa. Since production must equal consumption of electricity, we can substitute for the demand in each country:$D_t^A = e_t^{Th} + e_t^w - x_t^A + x_t^B$ and $D_t^B = e_t^H - x_t^B + x_t^A$. Alternatively, we could find the export volumes from demand and production (and our convention that at most one of them is non-zero).

If the generators act as price takers and choose how to allocate their outputs between the periods in such a way as to maximize their total profits subject to their capacity constraints, a competitive market with no uncertainty will replicate the solution to the social planner's problem. From a social planner's point of view, therefore, the optimisation problem at hand takes the form:

$$\max_{D_t^A,\, D_t^B, e_t^{Th},\, e_t^w, e_t^H} \quad \sum_{t=1}^{T} \int_0^{D_t^A} p_t^A(q)\partial q + \int_0^{D_t^B} p_t^B(q)\partial q - c_t\left(e_t^{Th}\right) \qquad (1)$$

subject to

$$D_t^A = e_t^{Th} + e_t^w - x_t^A + x_t^B \qquad (1.1)$$

$$D_t^B = e_t^H - x_t^B + x_t^A \qquad (1.2)$$

$$e_t^{Th} \leq \bar{e}^{Th} \qquad (1.3)$$

$$e_t^w \leq u_t \bar{e}^w \qquad (1.4)$$

$$e_t^H \leq \bar{e}^H \qquad (1.5)$$

$$R_t \leq R_{t-1} + w_t - e_t^H \qquad (1.6)$$

$$R_t \leq \bar{R} \qquad (1.7)$$

$$x_t^A \leq \bar{x} \qquad (1.8)$$

$$x_t^B \leq \bar{x} \qquad (1.9)$$

The two integrals in the objective function represent the value of electricity consumption in the two regions, and the variable cost of thermal output is subtracted from this. The first two constraints, (1.1) and (1.2), ensure that consumption in each country is equal to the supply available to it – local production, plus imports minus exports. The physics of electricity requires these constraints to be equalities, but all our constraints on production are inequalities. Constraints (1.3) to (1.5) require that the output of each type of plant is no greater than its capacity – in the case of wind plants, this is adjusted for the strength of the wind to give the maximum generation in each period. We write this as an inequality, since it is always possible to "spill" wind, although doing so would obviously not be optimal as long as the marginal costs of other generators were positive. Thermal generators can have negative marginal costs if they cannot reduce output without shutting down and (expensively) starting up again later, although we do not formally consider such issues here.[4]

Constraint (1.6) shows how the stock of water in the reservoirs varies from period to period – the stock at the start of period t+1 is equal to the stock at the start of period t, plus inflows in period t, less generation in that period. Constraint (1.7) shows that the amount of stored water cannot exceed the capacity of the reservoir – if inflows in a period are too high, relative to the amount of water already stored and the demand for power (even at a wholesale price of zero), then it may be necessary to spill water. The possibility of doing so (which might be an alternative to spilling wind) is why constraint (1.6) is written as an inequality. The two final constraints, (1.8) and (1.9), require that exports are less than the capacity of the transmission line between the two countries. We assume that this is the same in both directions, although in practice, constraints within a country can mean that its export and import capacities differ. The Lagrange function (adapted from equation 6.30 in Førsund (2007)) takes the form:

[4] An additional complication is that EU rules for priority access can be interpreted as giving the system operator no right to constrain off wind generators in this way.

$$
\begin{aligned}
\mathrm{L} = & \sum_{t=1}^{T} \int_0^{e_t^{Th}+e_t^{w}-x_t^A+x_t^B} p_t^A(q)\partial q + \int_0^{e_t^{H}-x_t^B+x_t^A} p_t^B(q)\partial q - c_t\left(e_t^{Th}\right) \\
& - \sum_{t=1}^{T} \theta_t\left(e_t^{Th} - \bar{e}^{Th}\right) - \sum_{t=1}^{T} \nu_t(e_t^{w} - u_t\bar{e}^{w}) - \sum_{t=1}^{T} \xi_t(e_t^{H} - \bar{e}^{H}) \\
& - \sum_{t=1}^{T} \lambda_t(R_t - R_{t-1} - w_t + e_t^{H}) - \sum_{t=1}^{T} \gamma_t(R_t - \bar{R}) \\
& - \sum_{t=1}^{T} \alpha_t(x_t^A - \bar{x}) - \sum_{t=1}^{T} \beta_t(x_t^B - \bar{x})
\end{aligned} \tag{2}
$$

The first line gives the objective function, the second the constraints on available capacity, the third the constraints relating to the availability of water and the fourth the constraints on transmission capacity between the countries. The first order necessary conditions are:

$\frac{\partial L}{\partial e_t^{Th}} = p_t^A(D_t^A) - c'^{(e_t^{Th})} - \theta_t \leq 0 \quad \left(= 0 \text{ for } e_t^{Th} > 0\right)$ (2.1)

$\frac{\partial L}{\partial e_t^{w}} = p_t^A(D_t^A) - \nu_t \leq 0 \quad (= 0 \text{ for } e_t^{w} > 0)$ (2.2)

$\frac{\partial L}{\partial e_t^{H}} = p_t^B(D_t^B) - \lambda_t - \xi_t \leq 0 \quad (= 0 \text{ for } e_t^{H} > 0)$ (2.3)

$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \leq 0 \quad (= 0 \text{ for } R_t > 0)$ (2.4)

$\frac{\partial L}{\partial x_t^A} = p_t^B(D_t^B) - p_t^A(D_t^A) - \alpha_t \leq 0 \quad (= 0 \text{ for } x_t^A > 0)$ (2.5)

$\frac{\partial L}{\partial x_t^B} = p_t^A(D_t^A) - p_t^B(D_t^B) - \beta_t \leq 0 \quad (= 0 \text{ for } x_t^B > 0)$ (2.6)

$\theta_t \geq 0 \quad \left(= 0 \text{ for } e_t^{Th} < \bar{e}^{Th}\right)$ (2.7)

$\nu_t \geq 0 \quad (= 0 \text{ for } e_t^{w} < u_t\bar{e}^{w})$ (2.8)

$\lambda_t \geq 0 \quad (= 0 \text{ for } e_t^{H} < R_{t-1} + w_t - R_t)$ (2.9)

$\xi_t \geq 0 \quad (= 0 \text{ for } e_t^{H} < \bar{e}^{H})$ (2.10)

$\gamma_t \geq 0 \quad (= 0 \text{ for } R_t < \bar{R})$ (2.11)

$\alpha_t \geq 0 \quad (= 0 \text{ for } x_t^A < \bar{x})$ (2.12)

$\beta_t \geq 0 \quad (= 0 \text{ for } x_t^B < \bar{x})$ (2.13)

Equations (2.1) and (2.7) show that the price in country A will equal the marginal cost of thermal power, unless the thermal capacity constraint is binding. Equations (2.2) and (2.8) show that the shadow value of wind power will also equal the price in country A, unless some wind power is being spilled. (We reiterate that this would only be optimal if the marginal cost of thermal power was negative.) Equations (2.3), (2.9) and (2.10) show that the price in country B will equal the shadow value of water, unless the hydro generation capacity constraint is binding or water is being spilled. Equations (2.4) and (2.11) show that the shadow value of water will be the same in adjacent periods, unless the reservoir is at full capacity or is empty. Finally, equations (2.5), (2.6), (2.12) and (2.13) show that the prices in the two countries will be the same, unless the transmission line between them is being fully used.

The shadow value of water will be the same in each period within a group in which the reservoir is neither full nor empty.[5] If a group of periods ends with sizeable inflows, the reservoir may become full and the shadow price of water will need to be relatively low to maximise the amount of generation and avoid (wastefully) spilling water. Once the inflows fall below the level of generation and the reservoir level starts to fall, we are in a new group

[5] Note that this model does not allow for fluctuations in the price of hydro power between peak and off-peak periods – we could have created these if we had assumed that there were two hydro plants, one with much lower storage capacity than the other. In off-peak periods, demand (net of any exports) can be met by the plant with a large reservoir (and hence a low water value) but in peak demand periods the plant with a small storage capacity must be used as well. If the second plant has a small enough storage capacity, even relative to its lower utilisation, then its water value, and hence the price of power in peak periods, will be higher than that of the other plant.

of periods, with the water value determined by the need to avoid running out of water before the next time inflows are large. In the absence of transmission constraints, wind output should be accepted whenever it is available, and water should be allocated across periods to equalise the level of thermal output in each period, and hence its marginal cost, which will equal the shadow water value. If the export or import of power required to achieve this is too great (in other words, fluctuations in demand or in wind output are large relative to transmission capacity), then the prices will differ between the two countries, and the country with the lower price (and marginal cost of power) should export as much as possible, subject to the transmission constraint.

To analyse the impact of variations in wind and water availability, we will consider a set of consecutive periods, W, in which the reservoir constraint is not binding and the water value is therefore identical. Within this set there are three subsets of (not necessarily consecutive) periods: W_U in which the transmission lines are unconstrained, W_A in which country A is exporting as much as possible, and W_B in which country B is exporting as much as possible. Adding up the constraints from equations (1.1) to (1.6), denoting the (common) water value by λ, the inverse of the thermal generators' marginal cost function by $g_t(\lambda)$, and the water stored in the first and last periods of the set by R_0 and R_W respectively, we get:

$$R_W - R_0 + \sum_{t\in W} D_t^B(\lambda) - w_t + \sum_{t\in W_U} D_t^A(\lambda) - g_t(\lambda) - u_t\bar{g}^w - \sum_{t\in W_A} \bar{x} + \sum_{t\in W_B} \bar{x} = 0 \tag{3}$$

Positive terms denote a demand for electricity in country B, and negative terms a source of supply. When the interconnector is unconstrained, demand and generation in country A count directly, whereas in congested hours, we count only the constrained level of exports or imports. Water available at the start of the period is a supply of energy; water to be stored at the end (maybe a full reservoir, maybe none) is effectively a demand. To save on notation, without fundamentally altering our results, we will assume that the constraints on hydro and thermal generation capacity never bind, so that ξ_t and θ_t can be ignored. We differentiate equation (3) totally to obtain:

$$\frac{\partial \lambda}{\partial w_t} = \frac{-1}{-\sum_{t \in W} \frac{\partial D_t^B}{\partial \lambda} + \sum_{t \in W_U} \left(\frac{1}{c''_t\left(e_t^{Th}\right)} - \frac{\partial D_t^A}{\partial \lambda} \right)} = \bar{g}^w \left. \frac{\partial \lambda}{\partial \nu_t} \right|_{t \in W_U} \leq 0 \tag{4}$$

$$\bar{g}^w \left. \frac{\partial \lambda}{\partial \nu_t} \right|_{t \in W_A} = \bar{g}^w \left. \frac{\partial \lambda}{\partial \nu_t} \right|_{t \in W_B} = 0 \tag{5}$$

Equation (4) shows us that a change in the available water, or in the wind output[6] in a period when the interconnector is unconstrained, leads to changes in consumption in the hydro country in every period, and to changes in consumption and thermal generation in country A in every period with unconstrained transmission. Concentrating on country A, we get an equation for the change in net exports:

$$\begin{aligned} \bar{g}^w \left. \frac{\partial x_{At}}{\partial \nu_t} \right|_{t \in W_U} &= 1 + \left(\frac{\partial g_t}{\partial \lambda} - \frac{\partial D_t^A}{\partial \lambda} \right) \bar{g}^w \left. \frac{\partial \lambda}{\partial \nu_t} \right|_{t \in W_U} \\ &= 1 - \frac{\frac{1}{c''_t\left(e_t^{Th}\right)} - \frac{\partial D_t^A}{\partial \lambda}}{-\sum_{t \in W} \frac{\partial D_t^B}{\partial \lambda} + \sum_{t \in W_U} \left(\frac{1}{c''_t\left(e_t^{Th}\right)} - \frac{\partial D_t^A}{\partial \lambda} \right)} > 0 \end{aligned} \tag{6}$$

The upper line of equation (6) shows that the change in net exports equals the change in output from wind and thermal generation, less the change in demand – what is not consumed locally must be exported. The lower line shows that the changes in generation and consumption in this period will be small, unless there are very few periods in W or the

[6] We multiply the derivative with respect to the output per unit of wind capacity by the amount of capacity.

variables are particularly sensitive to price in this time period. In other words, the optimal solution requires an increase in exports almost as large as the increase in wind output, if the transmission lines are unconstrained.

Equation (5) tells us that a change in the amount of wind output in a period in which the lines are constrained has no impact on the value of water. Instead, the price in Country A must change to ensure that consumption, plus the (constrained) level of net exports, equals demand:

$$\bar{g}^w \left.\frac{\partial p_t^A}{\partial v_t}\right|_{t\in W_A} = \bar{g}^w \left.\frac{\partial p_t^A}{\partial v_t}\right|_{t\in W_B} = -\frac{1}{\frac{1}{c''_t\left(e_t^{Th}\right)} - \frac{\partial D_t^A}{\partial p_t^A}} < 0 \tag{7}$$

It should be clear that the optimal price in country A will fall by more in response to an extra MWh of wind output in periods when the transmission lines are congested than when the country is free to increase its net exports. When the lines are congested, the country can only accommodate wind output by reducing thermal output and increasing demand; when transmission capacity is available, it is possible to export power and increase net consumption later. The next section looks into the trade patterns between Denmark and its neighbours and investigates to what extent the flows of output between the two countries could be justified in the context of the model that has been outlined in this section.

4. Patterns of production and trade

In this section we assess the practical implications of our model, using data to analyse the trade flows of thermal-, hydro- and wind-generated output between Denmark and its neighbours. Our dataset compiles information from the Danish Transmission System Operator, Energinet (www.energinet.dk), for hourly observations on the production, consumption and trade for East and West Denmark, from 2001 to 2009; and from Nordel (www.entso.eu), for monthly data on Nordic reservoir levels, output and trade from 1996 to 2008. Both sources give Danish production from three types of generating plant: wind, local CHP plants (which tended to follow their heat demand rather than participating actively in the electricity market) and primary plants, also usually CHP, but larger, and active participants in the wholesale market. The sources also contain separate trade flows (which we aggregate) for each of the five interconnectors: Norway to West Denmark, and Sweden and Germany to (both) East and West Denmark.

Figure 1 goes about here

We start with the big picture, trends in production and exports on an annual scale. Our hypothesis is that thermal production will tend to be inversely correlated with the amount of water available to hydro generators. Figure 1 gives twelve-month moving totals for thermal and wind production in Denmark (left hand scale) and hydro inflows and production in Norway, Sweden and Finland (right hand scale), derived from Nordel's annual statistical yearbooks.[7] At the top of the figure, the hydro inflows are clearly volatile, even after taking 12-month totals. The output levels are smoother, but also closely follow the inflows, with a bit of a lag (the industry cannot take advantage of a wet year until after it has occurred).

In the middle of the diagram, Denmark's production of thermal electricity follows the inverse pattern to Nordic hydro generation – when the latter is high, Denmark generates relatively less power. Note that the scales of the two series are different, and an equal distance on the page implies two and a half times as much hydro generation as thermal: the change in Denmark's generation is only part of the response to hydro conditions further north.

[7] The Nordel yearbooks give the reservoir levels in each country at regular intervals through the year, together with monthly hydro production, from which it is straightforward to derive the inflows.

At the bottom of the diagram, the 12-month average of Denmark's net exports clearly follows the same pattern as the country's thermal generation. When thermal generation is high, so are exports. There is no apparent correlation between the level of exports and of wind generation, however. Wind output has been gradually increasing throughout the period. Exports and thermal output have cycled around a trend which appears essentially flat since 2000.

These visual impressions are confirmed by analysis of the correlations between the variables, shown in table 1. The top panel gives correlation coefficients between annual values for the whole sample (1996 to 2008), while the bottom panel drops the first four years, in which wind generation was very low (the industry was in its infancy) but exports were (for this period) unusually high. Correlation coefficients do not give information on causation, of course, but it should be obvious that any causation will run from the weather to the electrical variables, rather than the other way round.

Table 1: Correlation coefficients between Danish power and Nordic hydro variables

1996-2008	Water inflows	Hydro output	Danish exports	Danish thermal	Danish wind
Water inflows	1	0.824	-0.825	-0.781	0.351
Hydro output		1	-0.887	-0.815	0.337
Danish exports			1	0.980	-0.580
Danish thermal				1	-0.726
Danish wind					1
2000-2008	Water inflows	Hydro output	Danish exports	Danish thermal	Danish wind
Water inflows	1	0.783	-0.776	-0.788	0.144
Hydro output		1	-0.902	-0.831	-0.096
Danish exports			1	0.976	-0.045
Danish thermal				1	-0.242
Danish wind					1

Source: authors' calculations from Nordel data, using calendar year totals

It is clear that water inflows and hydro generation are highly correlated, and that there is a strong negative correlation between these variables (especially the latter) and Denmark's exports. There is a very high correlation between Denmark's thermal generation and its exports, and basically no correlation between wind generation and any of the other variables in the period after 2000.

Another prediction of the theory is that Denmark should tend to export power in off-peak hours, and import at peak times. Figure 2 shows the short-term pattern for Denmark's production, consumption and trade during winter months, taking the average of each hour of the day from November to February. Net exports are inversely correlated with consumption, so that average output varies less than average demand. This implies that Danish generators are producing more at times when the marginal cost of power in the country is relatively low, and less at times when the marginal cost is relatively high, reducing the overall cost of meeting demand. Wind output is very slightly higher (on average) in the daytime hours, when Denmark's net exports are below average.

Figure 2 about here

The analysis so far has been in terms of averages, but how does the volatility of wind generation affect Denmark? We represent this volatility by the deviation of hourly wind output (and other variables) from the average level for that hour in that month. Indexing hours by h, days by *d* and months by *m*, and with D_m days in a month, we calculate these average levels by $\bar{q}^i_{mh} = \sum_{d=1}^{D_m} \left(\frac{1}{D_m} \sum_{d=1}^{D_m} q^i_{mdh} \right)$, where the output in hour *h* of day *d* of month *m* from plants of type *i* is given by q^i_{mdh}. The deviations, $\tilde{q}^i_{mdh}$, are simply given by $\tilde{q}^i_{mdh} = q^i_{mdh} - \bar{q}^i_{mh}$.

Table 2 shows that deviations from the normal level of net exports are strongly and positively correlated with deviations in wind output from its mean value. This in turn suggests that Danish exports are on average higher when there is "excess wind". Although the correlation of net exports with thermal output is positive, it is significantly weaker. Finally, deviations in Danish exports are found to be uncorrelated with deviations in consumption or the output from local CHP plants.

Table 2: Correlation coefficients between deviations from normal levels

	Net Exports	Primary Stations	Local CHP	Wind	Consumption
Net Exports	1	0.280	0.046	0.673	0.079
Primary Stations		1	0.149	-0.342	0.636
Local CHP			1	-0.038	0.192
Wind				1	0.033
Consumption					1

There is a strong correlation between deviations in consumption and in the production from primary stations, suggesting that variations in demand are mostly met by changing the output from large power stations, with much weaker responses from local stations or exports. The deviations in output of primary stations are also negatively correlated with those of the wind stations, implying that the primary stations respond to high levels of wind production, making room for this in the market. In our model, short-term variations in wind output and in demand should all be dealt with mainly by changing the level of hydro output (and hence net exports), but if the transmission system is constrained, then the output of thermal plants will have to respond instead.[8]

5. The cost of storing wind abroad

The theoretical model and data analysis presented in the earlier sections make clear that there is (and should be) a negative correlation between the level of thermal output in Denmark and the amount of water available in Norwegian reservoirs. In the short term, there is also a clear relationship between the amount of wind generation and the net exports of electricity from

[8] Where the increase in electricity demand is linked to cold weather which has led to a greater number of CHP plants being turned on to meet a heat demand, this would tend to increase the amount of electricity available.

Denmark. Effectively, Denmark is storing some of its wind in its neighbours' power systems. Does it have to pay for the privilege? When the wind is high, power prices will be relatively low (for the time of day and time of year), whereas they will tend to be relatively high when there is little wind. In this section we use data on the value, volume and prices of power flows between the two Danish markets (East and West) and its neighbouring trade partners (Norway, Sweden and Germany) to recover estimates of this cost of "storing wind".

First, we confirm that short-term variations in wind power do affect prices, and that these variations have a greater effect in hours when the interconnectors to the north are constrained. We define uncongested hours as those in which the relevant part of Denmark has the same Nord Pool price as other price areas, for this implies that there is spare capacity on the interconnectors to Norway or Sweden. We regressed deviations in the hourly price ($\tilde{p}^i_{mdh}$, defined in the same way as $\tilde{q}^i_{mdh}$ above) against deviations in wind output ($\tilde{q}^i_{mdh}$ itself) and a constant, allowing both parameters to depend on whether or not the lines were congested:

$$\tilde{p}^i_{mdh} = \hat{\alpha}_C Cong + \hat{\alpha}_U(1 - Cong) + \hat{\beta}_C Cong\ \tilde{q}^i_{mdh} + \hat{\beta}_U(1 - Cong)\ \tilde{q}^i_{mdh} + \varepsilon_{mdh}$$

where *Cong* is an indicator variable equal to one when the interconnectors are constrained. We used a GLS estimator with Cochrane-Orcutt corrected errors to remove auto-correlation in the time series, and used Durbin-Watson tests to confirm that the residuals were free from auto-correlation. The results are given in Table 3:

Table 3: The impact of wind output on prices, hourly deviations from averages

	Denmark West		Denmark East	
Year	Uncongested	Congested	Uncongested	Congested
2001	-0.0436*	-0.0470*	-0.0099	0.0178
	(0.0078)	(0.0081)	(0.0060)	(0.0127)
2002	-0.0732*	-0.1152*	-0.0145*	-0.0940*
	(0.0113)	(0.0120)	(0.0056)	(0.0111)
2003	-0.0741*	-0.1367*	-0.0136*	-0.1532*
	(0.0138)	(0.0133)	(0.0052)	(0.0114)
2004	-0.0514*	-0.0959*	-0.0104*	-0.0744*
	(0.0059)	(0.0063)	(0.0047)	(0.0070)
2005	-0.0528*	-0.0952*	-0.0144	-0.0524*
	(0.0096)	(0.0098)	(0.0082)	(0.0118)
2006	-0.0530*	-0.0847*	-0.0168*	-0.1102*
	(0.0074)	(0.0075)	(0.0063)	(0.0081)
2007	-0.0794*	-0.1154*	-0.0284*	-0.1102*
	(0.0117)	(0.0120)	(0.0085)	(0.0100)
2008	-0.0596*	-0.0862*	-0.0077	-0.0339*
	(0.0078)	(0.0079)	(0.0062)	(0.0069)
2009	-0.0690*	-0.0999*	-0.0133	-0.0417*
	(0.0081)	(0.0085)	(0.0088)	(0.0110)
All years	-0.0641*	-0.0986*	-0.0143*	-0.0779*
	(0.0032)	(0.0033)	(0.0023)	(0.0032)

Standard errors in parentheses; * implies significant at 5%.

The results are clearer for West Denmark, where the larger proportion of wind generation means that it has a more noticeable impact on prices, than they are for East Denmark. However, the signs and magnitudes are consistent for 17 out of 18 region-year pairs and for both of the regressions across all years - an increase in wind output leads to a reduction in the local Nord Pool price, measuring both relative to their averages for the month and time of day, and this is much stronger when the transmission lines to the north of Denmark are congested, as the theory implies.

We next compare the average price that would be received per MWh of wind power in Denmark if sold in Nord Pool at the prevailing local price,[9] to a series based on the same prices, but weighted by the average amount of generation in each hour in each month. Indexing hours by h, days by d and months by m, and with D_m days in a month, this smoothed series (calculated for a year at a time) is given by

$\overline{p}^s = \sum_{m=1}^{12}\sum_{h=1}^{24}\sum_{d=1}^{D_m} p_{mdh}\left(\frac{1}{D_m}\sum_{d=1}^{D_m} q^w{}_{mdh}\right) \div \sum_{m=1}^{12}\sum_{h=1}^{24}\sum_{d=1}^{D_m} q^w{}_{mdh}$, where the price and wind output in hour h of day d of month m are given by p_{mdh} and $q^w{}_{mdh}$ respectively. This smoothed series captures variations in the average pattern of wind output over the year and over the day, but excludes day-to-day variation around this pattern. For the series giving the actual output-weighted price, $\overline{p}^a$, replace the bracketed term by $q^w{}_{mdh}$. We calculate these prices for each year from 2001 to 2009, and the results are shown in table 4.[10]

[9] Note that most Danish wind generators receive a feed-in tariff with pre-set rates, rather than the wholesale market prices we present here.

[10] We use Danish Kroner, noting that 1Euro was worth 7.45 DKK on average during our sample period, with minimal variation The highest monthly average exchange rate from January 1996 to December 2009 was less than 1.5% above the lowest.

Table 4: Prices for wind generation in Denmark, 2001-2009, DKK/MWh

	West Demark				East Denmark			
Year	Time-weighted	Wind: smoothed	Wind: actual	penalty	Time-weighted	Wind: smoothed	Wind: actual	penalty
2001	176.87	178.22	169.34	5.0%	175.44	177.16	173.91	1.9%
2002	189.22	191.45	171.48	10.6%	212.36	214.21	207.35	3.2%
2003	250.27	252.26	214.07	15.3%	273.45	275.69	261.70	5.1%
2004	214.28	213.52	201.86	5.4%	210.94	210.84	206.92	1.9%
2005	277.44	267.29	245.04	8.0%	251.89	250.71	235.85	5.9%
2006	329.54	320.37	302.43	5.4%	361.97	349.71	334.59	4.2%
2007	241.37	238.33	213.49	10.3%	245.94	243.63	226.95	6.8%
2008	420.70	409.31	381.62	6.6%	422.28	411.55	396.02	3.7%
2009	268.41	272.03	256.49	5.8%	296.90	301.48	295.68	2.0%
All years	263.15	265.49	244.38	8.0%	272.38	278.52	267.42	4.1%

Source: energinet data and authors' calculations

We report results for West and East Denmark separately, since the former has a higher proportion of wind generation. In some years, the smoothed average price for wind generation is slightly above the time-weighted average price of power. This is not surprising, since wind output is higher in the winter months, when we would expect also to see higher prices. In all years, the price weighted by the actual pattern of output is below the smoothed series. The difference captures the effect of the co-variance between the level of wind output and the wholesale price – when wind generation is above average (for the time of day and year) the price is likely to be below average, as noted by Twomey and Neuhoff (2010). The effect is much greater for West Denmark than for East Denmark, where there is proportionally less wind generation. This financial cost of intermittency in West Denmark varies between 5% and 15.3% of the time-weighted average price, while in East Denmark it varies between 1.9% and 6.8%. Over the eight year period, the average cost in West Denmark is 8% of the time-weighted average price, and that in East Denmark is 4.1%. These costs are noticeable but not unbearable.

Third, we estimate the cost of storing wind power in Denmark's neighbours. We will assume that all of the variation in wind output (relative to the hourly-monthly mean, as before) leads to an equal variation in net exports.[11] Over the course of a month, these quantity variations sum to zero, by definition. If Denmark received the same price for the positive variations as it paid for the negative ones, then these variations would be costless to the country. If, however, the price is lower at times when output is high, then coping with short-term fluctuations in wind output by trade may be expensive.

We value each trade flow at the average of the price in (the relevant part of) Denmark and that in the foreign market – either Nord Pool (for Norway and Sweden) or EEX (for Germany). We use Bialek's (1996) tracing algorithm to identify the flows that are transits, simply passing through Denmark from one neighbouring country to another. This algorithm simply apportions the power leaving a point on a network in proportion to the power entering it, so that if Denmark is importing 300 MW of power from Germany, but exporting 400 MW to Norway and 400 MW to Sweden, we will divide the flow from Germany equally between the two receiving countries. Denmark itself would thus be exporting 250 MW to each country. The price we apply to value the deviation in wind output for that hour is the 50-50 mix of the prices for trade with Norway and with Sweden. (Overall, in this case, the final value will be 50% of the Danish price, 25% of the Norwegian, and 25% of the Swedish.) We

[11] To the extent that not all of the variation in wind output is exported, this over-estimates the cost of storing wind, but it ignores any impacts from variations in the monthly averages.

denote this price as p^t_{mdh}. The estimated cost of dealing with intermittency through trade is then given by $c^t = \sum_{m=1}^{12}\sum_{h=1}^{24}\sum_{d=1}^{D_m} p^t_{mdh}\tilde{q}^w_{mdh} = \sum_{m=1}^{12}\sum_{h=1}^{24}\sum_{d=1}^{D_m} p^t_{mdh}\left(q^w_{mdh} - \bar{q}^w_{mh}\right)$ where the quantities are those of the wind power.

Table 5: The cost of variations in exports, 2001-2009

	West Demark			East Denmark		
Year	trade cost, m DKK	value of wind output, m DKK	proportion	trade cost, m DKK	value of wind output, m DKK	proportion
2001	17.9	601	3.0%	2.3	161	1.5%
2002	45.0	725	6.2%	5.8	223	2.6%
2003	117.3	1,092	10.7%	15.2	327	4.7%
2004	22.1	1,045	2.1%	3.5	360	1.0%
2005	19.7	1,393	1.4%	13.7	401	3.4%
2006	51.4	1,521	3.4%	13.2	539	2.4%
2007	79.9	1,342	6.0%	23.5	396	5.9%
2008	88.7	2,184	4.1%	22.1	754	2.9%
2009	38.6	1,375	2.8%	12.3	471	2.6%
Totals	480.6	11,279	4.3%	111.6	3,633	3.1%

Source: energinet data and authors' calculations

Table 5 shows the results, giving the value of c^t in each year from 2001 to 2009, separately for East and West Denmark. It also gives the value of wind output, measured at the time-weighted average price for each year (as in table 4). This shows that the cost of exporting power at times when prices are relatively low, and re-importing it at higher prices (on average) was between 1.4% and 10.7% of the annual value of the wind output in West Denmark, and between 1.0% and 5.9% of the value of the wind output in East Denmark. The average across the entire data sample in West Denmark was 4.3%, and 3.1% in the east. The high cost for 2003 is an outlier, likely to have been affected by exceptionally high prices in Norway in the early months of that year, following a drought the previous autumn. If we exclude that year, the average cost for West Denmark falls to 3.6% of the value of its wind output, measured at the time-weighted annual average prices.[12] A cost of almost DKK 600 million over ten years is clearly noticeable, but it is, again, small compared to the overall value of the wind output. With 54.9 TWh of wind generation from 2001 to 2009, the cost comes to DKK 10.78/MWh (€1.45/MWh) of wind output.

6. Conclusions

The cost of dealing with intermittency is only one aspect of wind power. A full cost-benefit analysis of Denmark's investment in wind generation would need to consider the direct cost of the country's power system, compared to that of a feasible counter-factual, together with a range of indirect benefits and costs. These include reduced carbon emissions, reduced dependence on (imported) fossil fuels and the creation of a successful turbine manufacturing industry – although the comparison should also consider what might have been achieved if the resources involved had been put into some other part of the Danish economy. This paper should be viewed as contributing some of the evidence needed to perform that analysis.

The starting point for our analysis is a theoretical model, adapted from the work of Førsund (2007), in which we show how production by thermal and hydro generators should be optimised to take advantage of wind power. A thermal producer should generate more, relative to its demand, at off-peak times, when the level of water available to the hydro producers is low, and when wind output is low. The last is a transient effect, given the variability of the wind.

[12] The cost in the East also falls, but by less, to 2.9%, as the cost of trade fluctuations in 2003 was less exceptional for that half of the country.

Our analysis of the Danish experience shows that this model works on all scales. First, we have shown that Danish exports of power respond to the hydrological situation in Norway, Sweden and Finland, and that exports are greatest when hydro generation is relatively low, because inflows of water have been low. This year-to-year pattern of exports is driven by the amount of power that Denmark's fossil-fuelled thermal power stations produce, and not by the growth of wind output.

At a daily level, Denmark's net exports of power are higher at off-peak times, showing that the country takes advantage of its neighbours' hydro storage to effectively flatten its load curve. It exports more overnight, and less during the day-time peak, thereby reducing the fluctuations in its own power output. There is a tendency for wind generation to be higher during the day, when Denmark's net exports are relatively lower.

However, the critics of the Danish experience are correct to say that Denmark relies heavily on its neighbours to absorb short term fluctuations in its wind output. We have shown that the correlation between short-term deviations (from the norm for that time of day and year) in wind power and in net exports are much higher than for thermal generation. When it is windy in Denmark, the country's exports rise. What the critics seem not to appreciate, however, is that much of this power is stored in the countries further north, which reduce their own generation to absorb it, and then bought back at times of relatively low wind output. In other words, Denmark tends to store its wind in the form of water.

This would not be possible without the cooperation of its neighbours, and, indeed, the presence of neighbours with large systems and (to the north) a lot of storage hydro. Furthermore, the Danes do have to pay for this storage, in that the positive deviations in net exports are traded at lower prices, on average, than the negative ones. We estimate the cost of this effect at just under DKK 11/MWh of wind output over the period between 2001 and 2008. This is 4% of the value of that output, based on annual time-weighted average market prices. Denmark is therefore able to deal with the problem of intermittency in a reasonably cost-effective way.

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Danish power and Nordic hydro

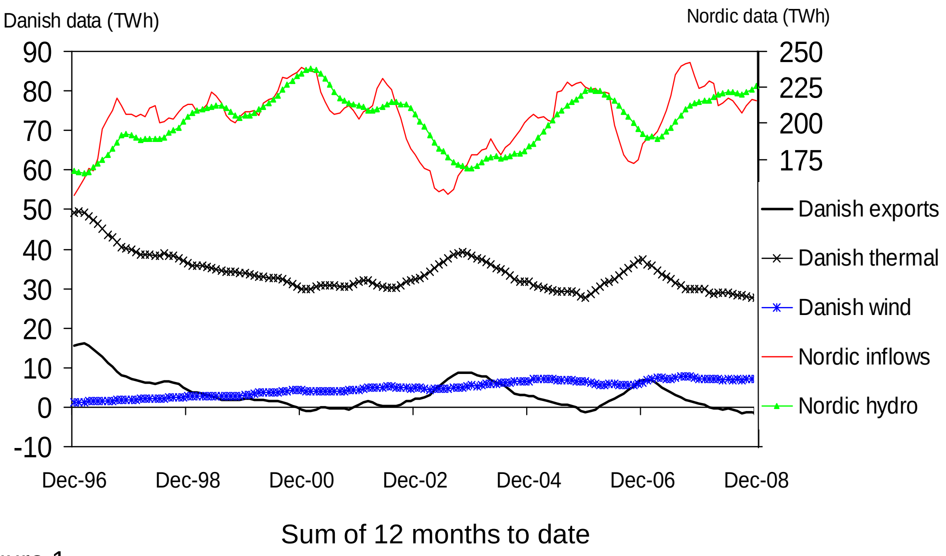

Figure 1

Daily pattern of output and consumption: winter

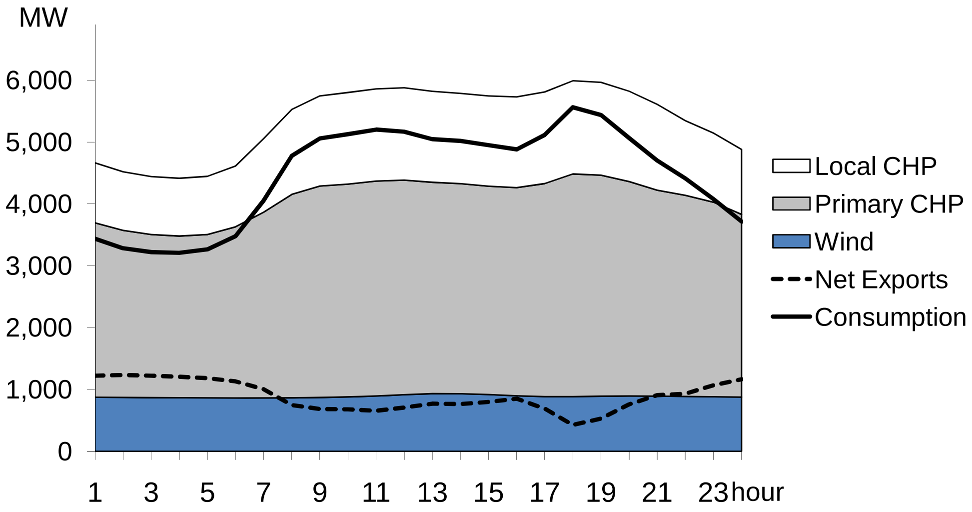

Figure 2