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Beyond Critical Loss:
Tailoring Applications of the Hypothetical Monopolist Paradigm

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"Critical elasticity of demand" and "critical loss" analysis are now standard analytical tools for implementing the hypothetical monopolist paradigm for market delineation,[1] routinely used in the investigation and litigation phase of most merger cases.[2] Although these tools are highly useful, significant limitations must be recognized. This paper presents three scenarios in which standard critical elasticity and critical loss analysis can be highly misleading and explains how the hypothetical monopolist paradigm can be fully and faithfully implemented in those scenarios.[3]

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[1] There is a substantial and growing literature on these tools. *See* Michael G. Baumann & Paul E. Godek, *Could and Would Understood: Critical Elasticities and the Merger Guidelines*, 40 ANTITRUST BULL. 885 (1995); Kenneth Danger & H.E. Frech III, *Critical Thinking About "Critical Loss" in Antitrust*, 46 ANTITRUST BULL. 339 (2001); Barry C. Harris & Joseph J. Simons, *Focusing Market Definition: How Much Substitution Is Enough*, *in* 12 RESEARCH IN LAW AND ECONOMICS 207 (Richard O. Zerbe, Jr. ed., 1989); Frederick I. Johnson, *Market Definition under the Merger Guidelines: Critical Demand Elasticities*, *in* 12 RESEARCH IN LAW AND ECONOMICS 235 (Richard O. Zerbe, Jr. ed., 1989); James Langenfeld & Wenqing Li, *Critical Loss Analysis in Evaluating Mergers*, 46 ANTITRUST BULL. 299 (2001); Gregory J. Werden, *Demand Elasticities in Antitrust Analysis*, 66 ANTITRUST L.J. 363, 387–96, 410–12 (1998); Gregory J. Werden, *Four Suggestions on Market Delineation*, 37 ANTITRUST BULL. 107, 119–20 (1992).

[2] In *FTC v. Swedish Match*, opposing economic experts both used a critical elasticity analysis. The court found neither "persuasive" but relied on its own critical loss analysis in concluding that "it cannot be unprofitable for the hypothetical monopolist to raise price . . . because the hypothetical monopolist would lose only a small amount of business." 131 F. Supp. 2d 151, 160–61 & n.8 (D.D.C. 2001). Several courts have relied on critical loss analysis assessing the geographic scope of hospital markets: FTC v. Tenet Health Care Corp., 186 F.2d 1045, 1052–54 (8th Cir. 1999) (reversing grant of preliminary injunction in light of defendant's critical loss analysis), *rev'g*, 17 F. Supp. 2d 937 (E.D. Mo. 1998); California v. Sutter Health Sys., 130 F. Supp. 2d 1109, 1120, 1128–32 (C.D. Cal. 2001) (rejecting plaintiff's proposed market on the basis of a critical loss analysis); United States v. Mercy Health Servs., 902 F. Supp. 968, 980–83 (N.D. Iowa 1995), *vacated as moot*, 107 F.3d 632 (8th Cir. 1997) (basing the geographic scope of the relevant market on several alternative critical loss analyses).

[3] Although this paper addresses only market delineation, the discussion below also applies to critical elasticity and critical loss analysis when used to assess competitive effects of mergers.

Some Limitations of Standard Critical Elasticity and Critical Loss Analysis

The "profit-maximization" (PM) versions of critical elasticity (CE) and critical loss (CL) analysis use the economists' model of monopoly to compare the prevailing price with the price that maximizes the hypothetical monopolist's profits. The PMCE is the maximum elasticity of demand a hypothetical, profit-maximizing monopolist could face at premerger prices and still want to increase price by some significance threshold, e.g., 5%. The PMCL is the maximum reduction a hypothetical, profit-maximizing monopolist would be willing to tolerate in its quantity sold to sustain a given price increase.

The "breakeven" version of critical elasticity and critical loss analysis uses simple algebra to solve for the price increase equating the hypothetical monopolist's profits after the increase to those before it. The BECE is the maximum elasticity of demand a hypothetical monopolist could face at premerger prices and still not experience a net reduction in its profits from increasing price by a given amount, e.g., 5%. The BECL is the maximum reduction a hypothetical monopolist could experience in its quantity sold without suffering a net reduction in its profits, as it increases price by a given amount.

The widely published formulae for critical elasticity and critical loss indicate their dependence on the significance threshold for price increases, t, and the premerger price-cost margin (price minus marginal cost, all divided by price), m, for the candidate market.[4] Except for the BECL formula, they also depend on the assumed curvature of demand. Tables 1 and 2 present these formulae assuming linear or isoelastic demand curves, which are the only demand curves giving rise to simple formulae, except in the case of BECL. In these formulae, t and m are expressed a proportions, i.e., for a 5% price increase, $t = .05$.

Table 1. Critical Elasticity of Demand Formulae

Demand Curve	*Profit Maximization*	*Breakeven*
Linear	$\frac{1}{m+2t}$	$\frac{1}{m+t}$
Isoelastic	$\frac{1+t}{m+t}$	$\frac{\log(m+t)-\log(m)}{\log(1+t)}$

[4] For concise derivations, see Werden, *Demand Elasticities*, *supra* note 1, at 410–12.

Table 2. Critical Sales Loss Formulae

Demand Curve	*Profit Maximization*	*Breakeven*
Linear	$\frac{t}{m+2t}$	$\frac{t}{m+t}$
Isoelastic	$1-(1+t)^{\frac{-1-t}{m+t}}$	$\frac{t}{m+t}$

Discussed below are three scenarios in which the application of these formulae may suggest that the relevant market is significantly broader than a rigorous application of the hypothetical monopolist paradigm indicates that it really is. The first scenario involves a product with multiple uses having significantly differing elasticities of demand. The application of these formulae in that scenario may mislead mainly because it does not pose the critical question—whether the hypothetical monopolist would happily sacrifice its sales to users with elastic demand in order to exploit substantially users with inelastic demand.

Derivations of these formulae effectively assume the hypothetical monopolist has the same marginal cost at every level of output, and perhaps also a fixed cost of producing any output. In the latter two scenarios below, this assumption is unrealistic. In one, marginal cost changes with the level of output, and in the other, some fixed costs can be avoided by shutting down blocks of capacity. In both scenarios, the formulae can greatly underestimate the hypothetical monopolist's incentive to restrict output. In all three scenarios, the proper application of the hypothetical monopolist paradigm is one tailored to the facts of the case.

Scenario 1: Different Uses Have Significantly Differing Elasticities of Demand

Some of a product's consumers nearly always have more elastic demand than others. A common reason is that different consumers use the product in different ways, or in different places, presenting different substitution possibilities or relative prices for substitutes. Consider the simple case of a product with two distinct uses—one in which demand highly elastic, and one in which demand is highly inelastic. Assuming price discrimination is impossible, the hypothetical monopolist faces a single aggregate demand curve consisting of the sum of the separate demands in the two uses. Figure 1 illustrates how the hypothetical monopolist's demand curve might look.

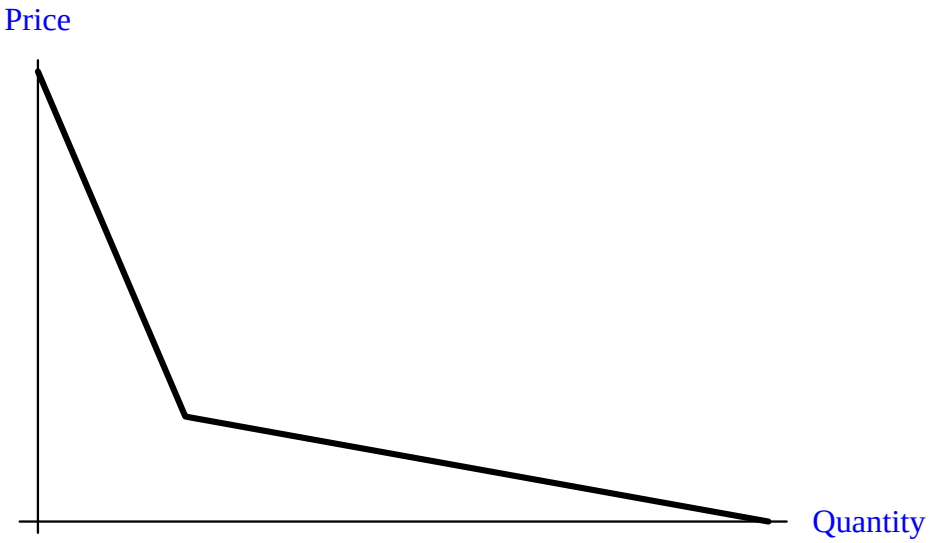

Figure 1. Aggregate Demand with Distinct Elastic and Inelastic Uses

This demand curve was constructed from assumptions about the premerger equilibrium: In one use for the product, the elasticity of demand is 0.25, and 20 units are purchased. In the other use, the elasticity of demand is 6, and 10 units are purchased. Price is $100, and demand in both uses is linear. In addition, marginal cost is assumed to $50 at every level of output, yielding a premerger margin of 50%, which is fairly typical in manufacturing industries.

A naive application of the hypothetical monopolist paradigm might be premised on the fact that this aggregate demand curve is linear in the neighborhood of the premerger equilibrium. At the premerger price, the elasticity of the aggregate demand curve is 2.17, which exceeds the 1.67 given by the linear demand PMCE formula for a price increase of 5% and margin of 50%. Since the actual demand elasticity exceeds the critical elasticity, this naive analysis concludes that the relevant market is larger than this one product. This conclusion is unwarranted, however, because demand is not linear globally and the hypothetical monopolist could raise price beyond the kink in the demand curve and hence outside the neighborhood in which demand is linear.

No assumption need be made about the shape of the demand curve if the BECL formula is applied instead. Figure 2 plots the BECL for the assumed 50% margin and price increases of up to 20%. For the same range of price increases, the figure plots the actual sales loss. Because the actual sales loss exceeds the BECL, it follows that all price increases near the 5–10% range are unprofitable. Note that the kink in the actual sales loss curve corresponds with the kink in the aggregate demand curve. A price increase of 16.7% reduces quantity to the point of the kink, and further increases beyond that point cause sales to be lost at a lower rate as price is increased.

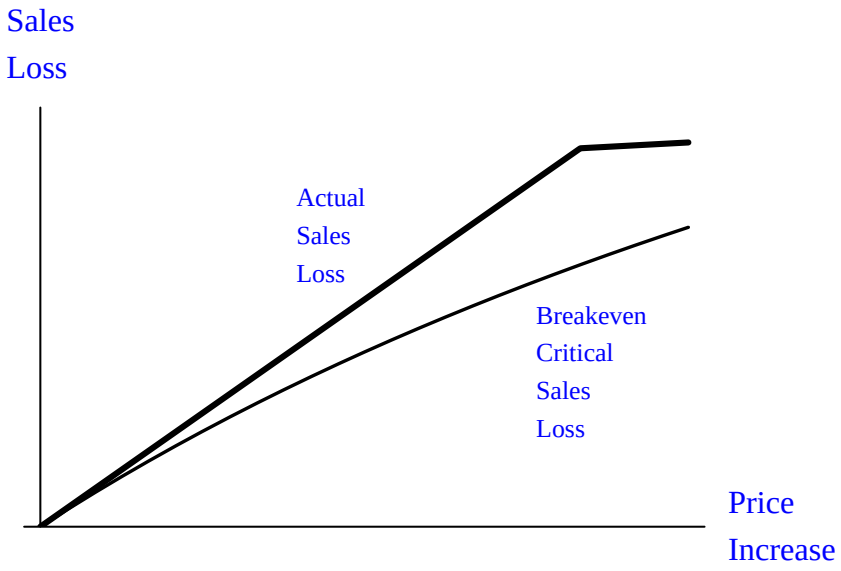

Figure 2. Breakeven Critical Loss for Price Increases Up to 20%

Naive applications of the PMCE and BECL formula fail to consider the possibility that the hypothetical monopolist would sacrifice all of its sales for the use with elastic demand in order to be able to exploit customers for the use with inelastic demand. That possibility is examined in Figure 3, which plots the hypothetical monopolist's profits as a function of its price. The kink in the demand curve causes a double peak in the profit function, and either peak could be the higher one. Under the assumptions made, higher-price peak clearly is the higher-profit peak, and the profit-maximizing strategy is to raise price 175%. Any analysis that fails to consider both peaks is an incomplete and inappropriate application of the hypothetical monopolist paradigm.

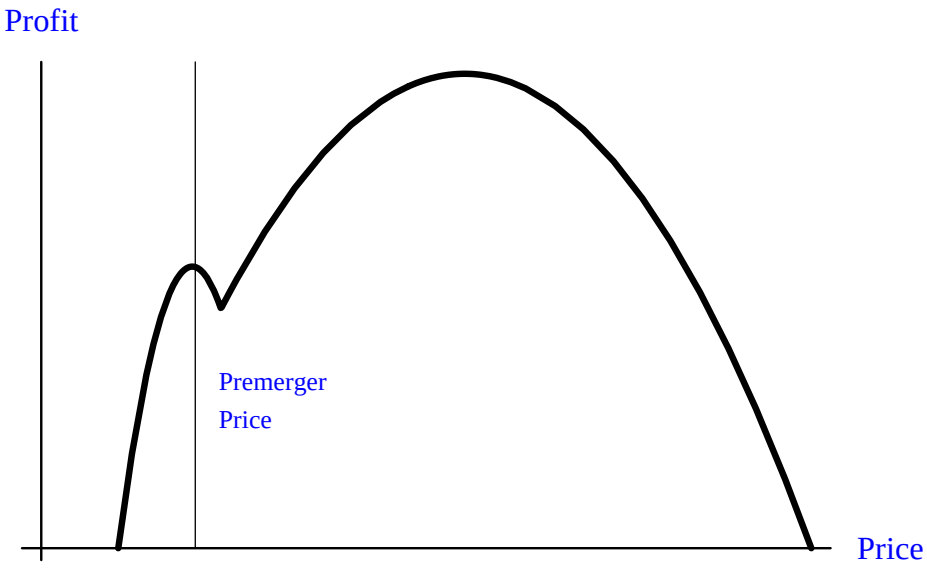

Figure 3. Hypothetical Monopolist's Profits as a Function of its Price

The foregoing application of the BECL formula neglects the possibility that a large price increase is profitable, even though a small price increase is not.[5] Figure 4 corrects this omission by extending the range of the BECL analysis to price increases of up to 400%. As the figure indicates, price increases of between 31% and 319% are profitable, although smaller price increases are not. This analysis does not pin down the amount by which a hypothetical monopolist would raise price, but in this case, it does show that amount would be at least 31%.

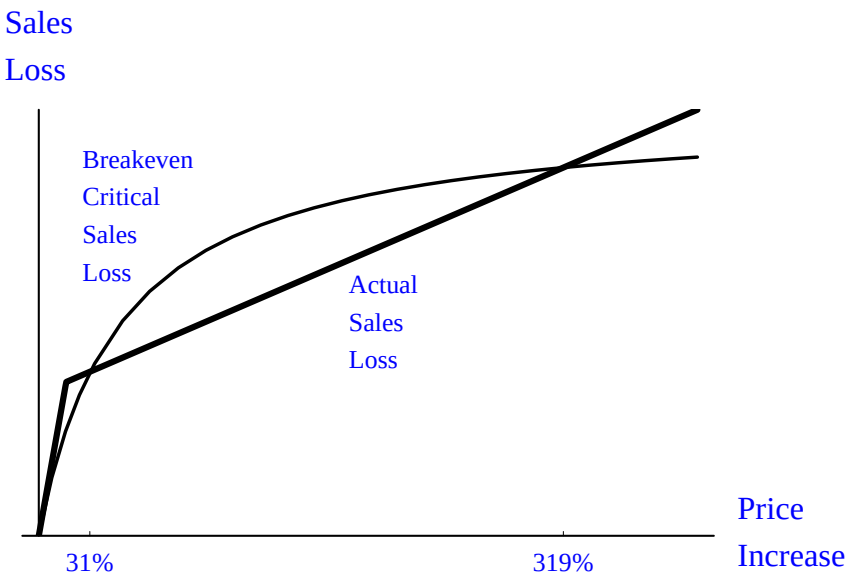

Figure 4. Critical Loss Analysis for Large and Small Price Increases

The mere existence of the kink in the aggregate demand curve does not imply that the hypothetical monopolist would raise price significantly. Depending on a variety of factors, especially the ratio of premerger quantity devoted to the use with elastic demand to the quantity devoted to the use with inelastic demand, the hypothetical monopolist may not find it profit maximizing to give up all sales of the product in the use with elastic demand. It would not, for example, if the elastic-to-inelastic use ratio was changed from 1:2 to 2:1. The lessons from this scenario are that multiple uses for a product can cause the application of critical elasticity and critical loss formulae to be misleading and that the potential to be misled can be avoided by explicitly modeling the hypothetical monopolist as facing demand from multiple sources with differing elasticities.

[5] It follows that the court erred in California v. Sutter Health Sys., 130 F. Supp. 2d 1109 (C.D. Cal. 2001). Purporting to follow the Horizontal Merger Guidelines, the court held that the only relevant price increase for the critical loss analysis is the Guidelines' 5%. *Id.* at 1129.

Scenario 2: Marginal Costs Significantly Differ Across Units of Capacity

It common for a product to be produced in many plants of differing scale and vintage, and hence with differing cost.[6] A likely consequence is an increasing marginal cost function for the industry and hence for the hypothetical monopolist. Industry-wide margins based on average variable cost would be relevant in the hypothetical monopolist paradigm only if it was assumed that the monopolist would reduce output proportionately from all facilities (as a cartel might). But the hypothetical monopolist is assumed to maximize profit, which dictates that output be restricted in the manner that minimizes cost for any given level of output.

Figure 5 presents a simplified illustration of heterogenous marginal cost, with five plants of equal capacity, all of which are assumed to be in use in the premerger equilibrium. To highlight the role of cost heterogeneity, it is assumed that marginal costs differ substantially: The five plants are assumed to have marginal costs of $80, $100, $120, $160, and $200.

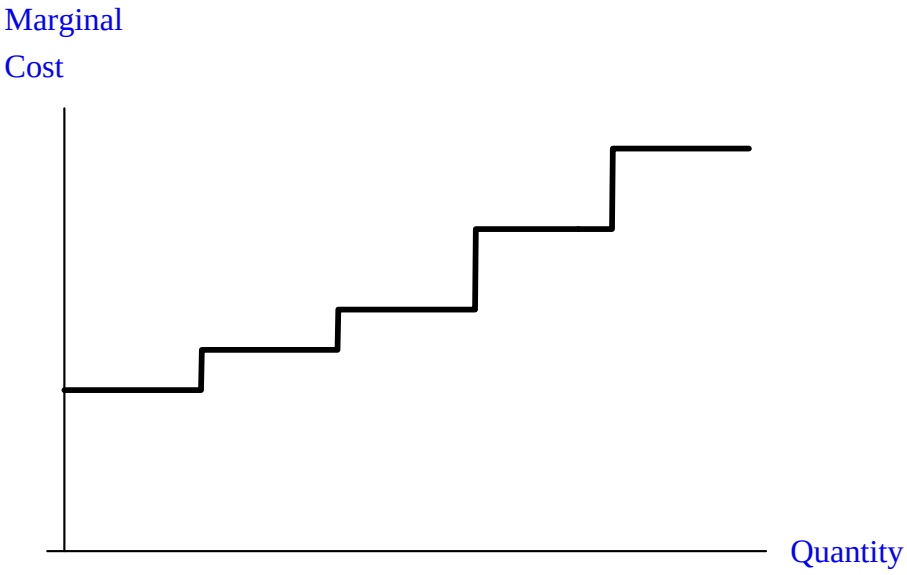

Figure 5. Industry Marginal Costs with Heterogeneous Capacity

As Figure 6 illustrates, the marginal cost function in Figure 5 causes the margin to vary dramatically with total output. Figure 6 is based on the linear (inverse) demand curve: $p = 360 - 20q$, which was constructed by assuming a premerger equilibrium with a relatively high elasticity of demand of 3, a relatively low margin of just under 30%, and in which output is at a level requiring exactly 4.5 plants to produce.

[6] This scenario was inspired by the facts of United States v. Georgia Pacific Corp., No. 00-2824 (D.D.C., filed Nov. 21, 2000); 66 Fed. Reg. 9,096 (Feb. 6, 2001) (complaint and competitive impact statement), in which different tissue-making machines had different marginal costs.

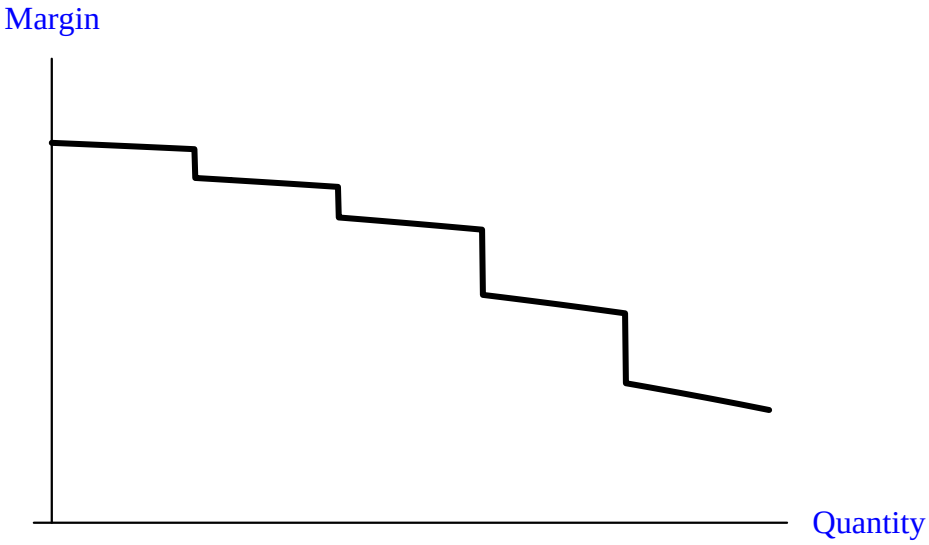

Figure 6. Margins as a Function of Quantity

The assumption of linear demand suggests the use of the PMCE formula in Table 1, which yields a critical elasticity of 2.78 for a 5% price increase and the premerger equilibrium margin in Figure 6. Although the assumed premerger demand elasticity of 3 is greater than the 2.78 critical value, the latter value is predicated on the false assumption of constant marginal cost.

Figure 7 plots the hypothetical monopolist's profits as a function of its quantity. Clearly, the profit-maximizing strategy is to restrict output to the point at which the third-highest-cost plant is fully utilized and the two highest cost plants are not used at all. The resulting monopoly price is exactly 10% above the premerger price, so the hypothetical monopolist test confirms that relevant market is limited to this one product.

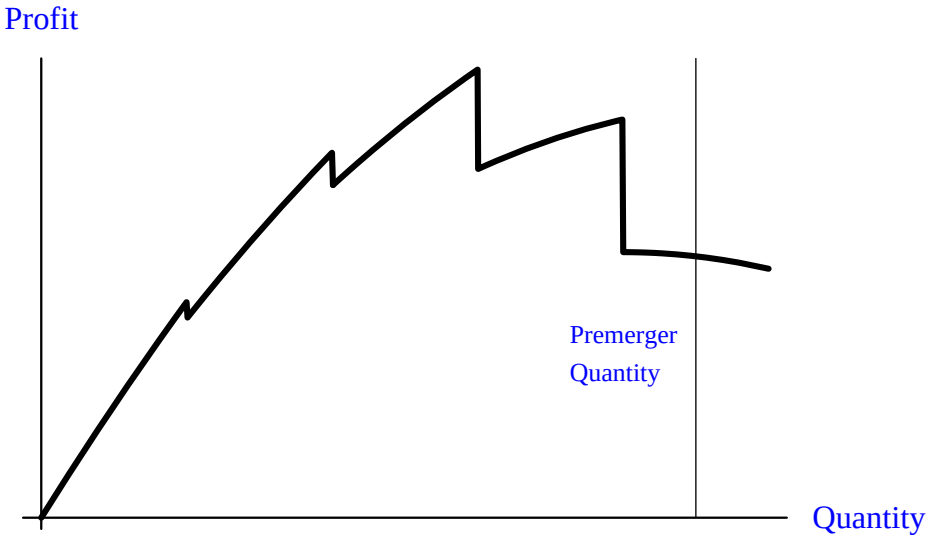

Figure 7. Profit as a Function of Quantity

Scenario 3: Fixed Costs Avoidable by Shutting Down Capacity

In the third scenario, there is a substantial fixed cost associated with operating each block of capacity (e.g., plant or machine) and minimal marginal costs of production from capacity once in operation. This scenario resembles the airline industry, in which there are significant costs of offering a flight but a low marginal cost of serving an additional passenger on a flight that is not full.[7] To illustrate, consider a production technology in which it costs $1,800 to turn on each block of capacity, plus $3 per unit produced up to a maximum of 100 units per block of capacity. Figure 8 plots the total costs of producing up to 500 units.

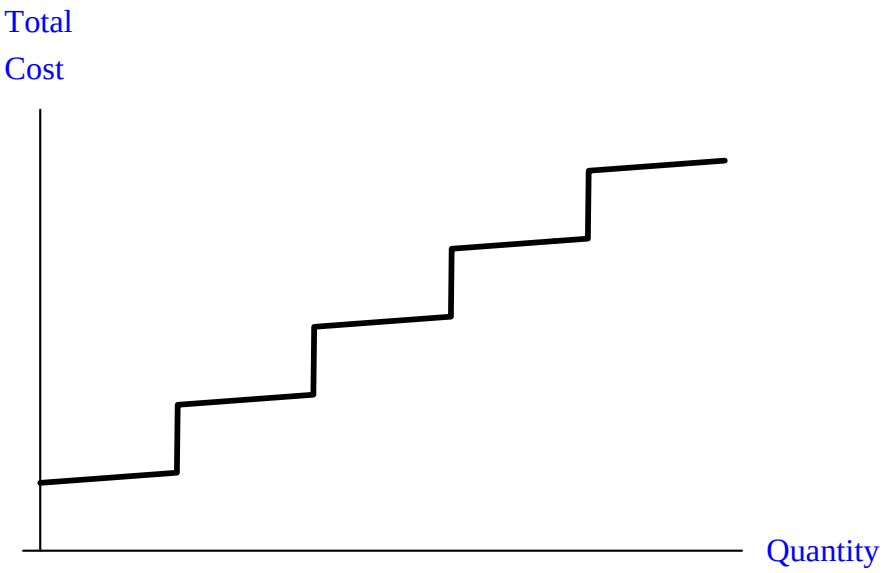

Figure 8. Total Costs with Large Fixed Costs of Operating Capacity

Also assume that demand is linear and that, in the premerger equilibrium, price is $30, quantity is 500, and the elasticity of demand is 1.2. The resulting (inverse) demand curve is $q = 55 - p/20$. With this demand curve and the cost curve in Figure 8, the hypothetical monopolists profits are as shown in Figure 9. The sawtooth pattern of the profit function is a direct consequence of the cost assumptions. As each block of capacity is turned on, profits fall because a substantial fixed cost is incurred, and as output from a block of capacity is increased, profits increase rapidly because the marginal cost of production is slight.

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[7] This scenario also was inspired by the facts of United States v. SunGard Data Sys., 172 F. Supp. 2d 172 (D.D.C. 2001). The defendants treated all costs associated with computer hardware and software as fixed, resulting in a marginal cost that was a tiny fraction of price. In this industry, however, hardware and software were replaced fairly frequently and services were sold though long-term contracts, so it might have been argued that the relevant price-cost margin actually was quite low.

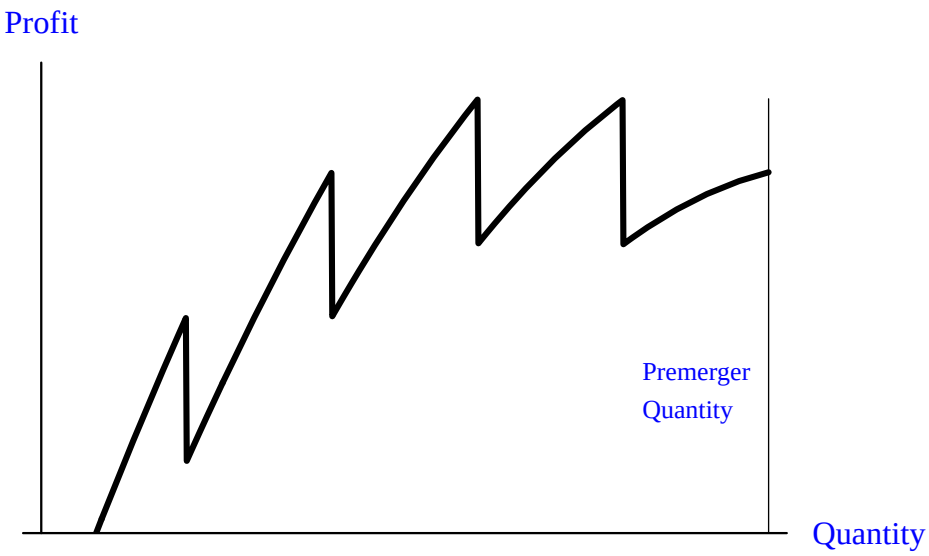

Figure 9. Profits as a Function of Quantity

While difficult to see from Figure 9, the third peak from the left in the profit function is the highest. The fixed cost of turning on each block of capacity is just enough to induce the profit-maximizing hypothetical monopolist to shut down two units of capacity. This implies a 40% reduction output and a 33.3% price increase. For a large range of smaller fixed costs per unit of capacity, the hypothetical monopolist would shut down one unit of capacity and raise price 16.7%. In either event, a properly tailored application of the hypothetical monopolist paradigm indicates that the relevant market is not broader than the one product under analysis.

Conventional analyses employing the formulae in Tables 1 and 2, however, suggest a contrary conclusion. With a price of $30 and a marginal cost of just $3, the margin is 90%. For a 5% price increase, the PMCE formula for linear demand gives a value of 1.0, which is less than the assumed premerger demand elasticity of 1.2. The formula is misleading because it ignores the cost savings from shutting down capacity. Essentially the same is true with the BECL formula. For every price increase (greater than zero), that formula indicates a critical sales loss that is less than the actual sales loss, thus wrongly indicating that no price increase is profitable.

If the fixed cost associated with turning on each unit of capacity is less important relative to the marginal cost of producing from capacity already turned on, a hypothetical monopolist might elect not to shut down any capacity. In that event, conventional analyses employing the formulae in Tables 1 and 2 would not mislead, but it is important to make sure that shutting down capacity is not the profit-maximizing strategy, and that requires going beyond these formulae.

Conclusions

Since the Supreme Court's decision in *Daubert*, a critical determinant of the admissibility of an expert's testimony has been its "fit" with the facts of the case.[8] In one notorious case and several others, expert economic testimony has been excluded for lack of fit.[9] Litigants can and should challenge proffered critical elasticity and critical loss analysis whenever such analysis is predicated on demand or cost assumptions demonstrably at variance with actual market conditions. Only a tailored application of the hypothetical monopolist paradigm meets the rigorous admissibility tests applied by the courts.

A properly tailored application of the paradigm is only slightly more demanding than conventional critical elasticity and critical loss analysis. All of the models presented above were calibrated from observable features of the premerger equilibrium. The demand curves were constructed from data on the equilibrium price and quantity and the equilibrium elasticity of demand. Identifying prices and quantities, even separately for several uses of a product, should pose no great difficulties, especially since that normally is required just to assign market shares. Identifying elasticities of demand is not so straightforward, but estimation is now commonly used, and market delineation inherently requires at least a rough assessment of demand elasticities based on qualitative evidence. The cost curves in the latter two scenarios were constructed from data on the capacities of machines or plants and their operating costs. Capacities often are both readily available and used to assign market shares, and costs must be ascertained in any event, to compute the margins on which critical elasticity and critical loss analysis depend. Once demand and cost curves are calibrated on the basis of real-world data, fully and faithfully applying the hypothetical monopolist paradigm is quite simple.

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[8] Daubert v. Merrill Dow Pharms., Inc., 509 U.S. 579, 591 (1993) (courts must consider whether the "expert testimony proffered in the case is sufficiently tied to the facts of the case"), quoting United States v. Downing, 753 F.2d 1224, 1242 (3d Cir. 1985). *See also* General Electric Co. v. Joiner, 522 U.S. 136, 146 (1997) ("A court may conclude that there is simply too great an analytical gap between the data and the opinion proffered."); Brooke Group Ltd. v. Brown & Williamson Tobacco Corp., 509 U.S. 209, 242 (1993) ("When expert testimony is not supported by sufficient facts to validate it in the eyes of the law, or when indisputable record facts contradict or otherwise render the opinion unreasonable, it cannot support a jury's verdict. Expert testimony is useful as a guide to interpreting market facts, but it is not a substitute for them.") (citation omitted).

[9] Concord Boat Corp. v. Brunswick Corp., 207 F.3d 1039, 1055–57 (8th Cir. 2000) (damages testimony excluded because the expert's model was inconsistent with facts of the industry); Bloomkest Fertilizer, Inc. v. Potash Corp. of Sask., 203 F. 3d 1028, 1038 (8th Cir. 2000) (econometric study not probative of collusion because it failed to account for events affecting prices); Johnson Elec. N. Am., Inc. v. Mabuchi Motor Am. Corp., 103 F. Supp. 2d 268, 280–87 (S.D.N.Y. 2000) (expert analysis excluded in part because it did "not 'fit' the facts of [the] case because it fail[ed] to take into account" key facts).