The Quadratic Iterator

- Verhulst model: Describes the population development of a species in time

  $$x_{n+1} = a\, x_n\, (1-x_n)$$

  x_n ... population at time n

  a ... growth parameter which depends on ecological conditions

- Well known dynamic system with chaotic behavior

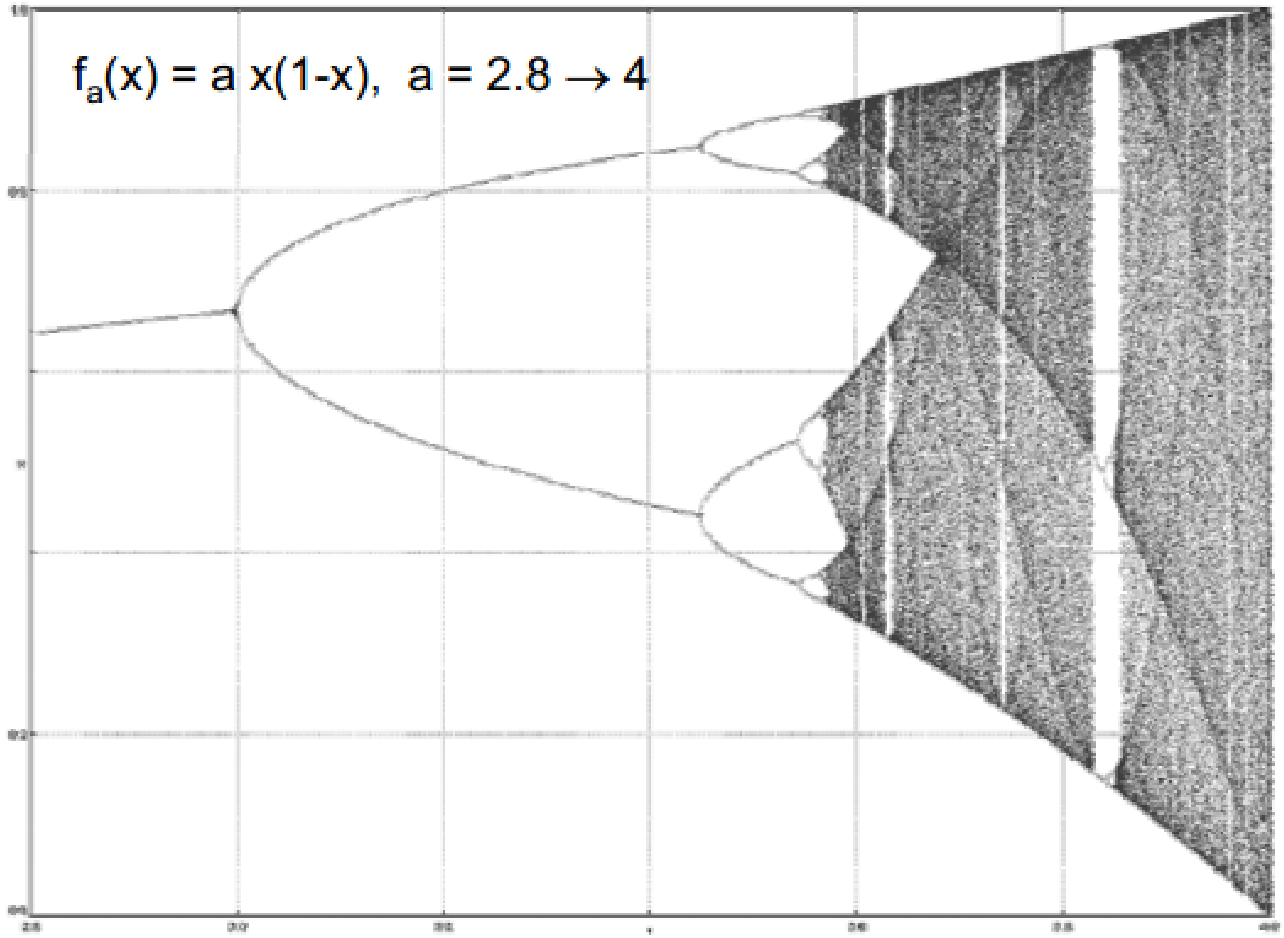
http://www.ibiblio.org/e-notes/MSet/Logistic.htm

Final State Diagram

- Shows route from order to chaos
- Examination of the long time behavior of a dynamic system $\{X, f_a\}$
- Given: A function $f_a(x)$ with one maximum
  - Calculate the time series for an interval of parameter values
  - Plot the last n points of the time series for each parameter value as final state

Final State Diagram

- Route from order to chaos



Feigenbaum Constant

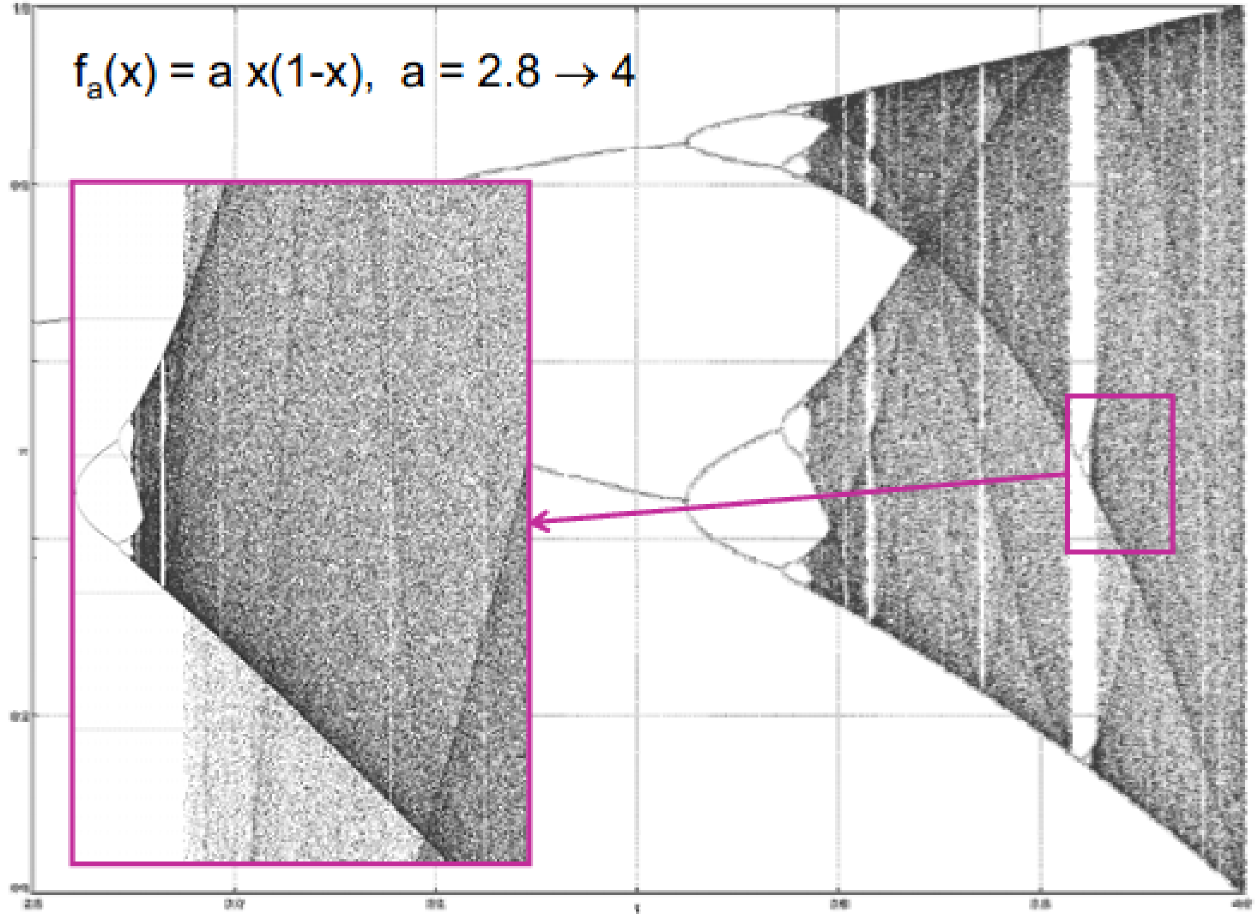
- Calculation of the limit ratio F of the distance between two successive bifurcation points:

$$F = \lim_{n\to\infty} \frac{a_n - a_{n-1}}{a_{n+1} - a_n} = 4.6692016091029...$$

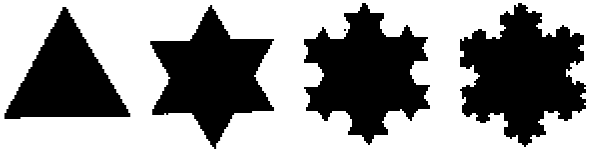
- F is called Feigenbaum constant
- It describes the entrance into chaos

Zoom Into the Diagram

- Self similar fractal



On to Fractals – Now let's consider *Scale*



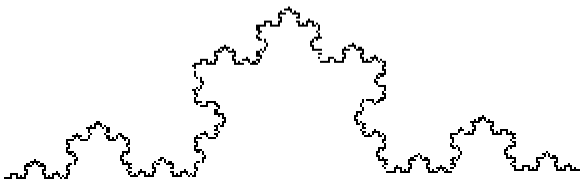
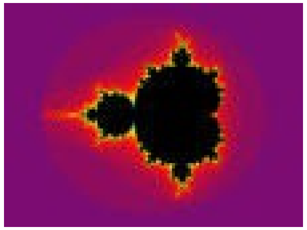
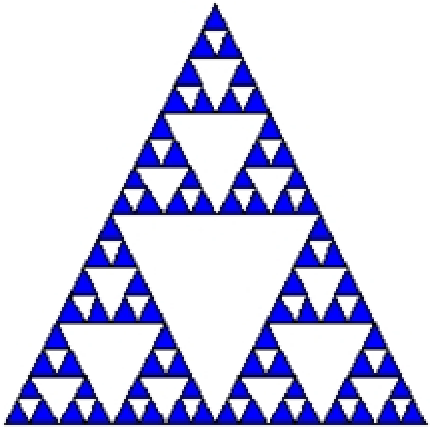


Figure 4: the Koch curve (James Gleick, Chaos - Making a New Science, pg. 99)





It's all about scales and its invariance (not just space though – can also time

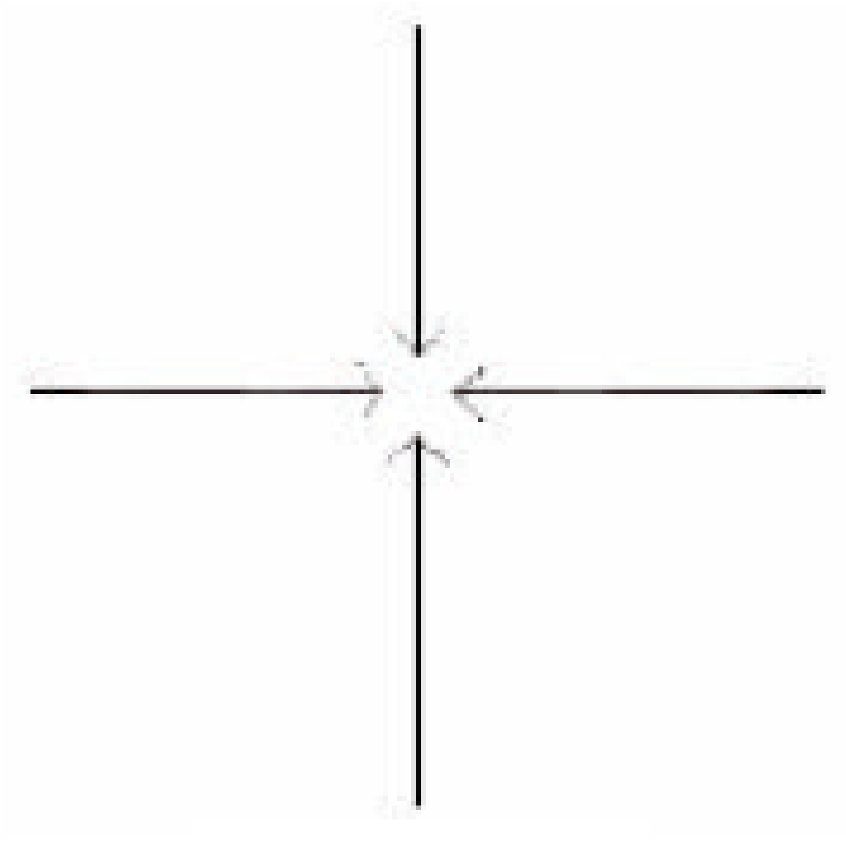
And self-organized similarity (scale invariance) a rather new term coined these days

- Another manifestation of Chaos theory in the form of scales

Types of Attractors

- Point Attractors
- Limit Cycles
- Strange or Fractal Attractors

Point Attractors

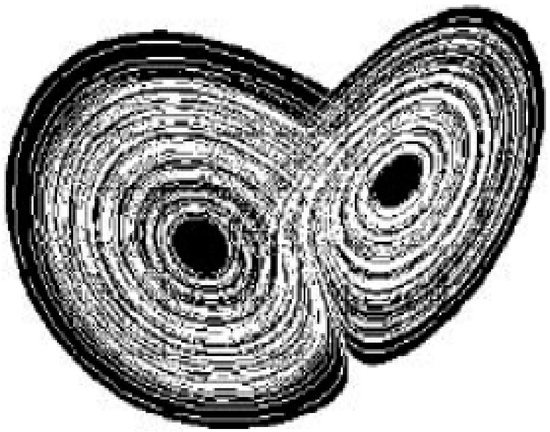


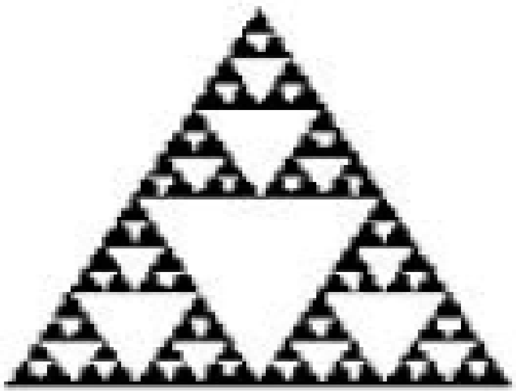
- Simplest form of attractor
- Point at which a pendulum's swing reverses
- Theoretically comparable to an economic supply/demand balance or market equilibrium

Limit Cycle Attractors

- Friction and mechanical limitations are removed
- Pendulum freely swings a full 360° indefinitely
- This represents the market volatility around equilibrium or "market noise"

Chaotic or Fractal Attractors



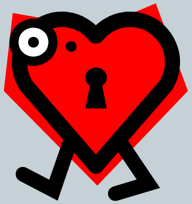


- Maps multiple pendulum swings of various magnitudes but never completing a full 360° circle
- These random swings are always within a subset of the range of the Limit Cycle attractor called a "phase space"
- Phase Space is comparable to the perimeter of the Sierpinski triangle
- These attractors have fractal dimensions

The Human Heart

Healthy heart

- Has slight variations in the time between one beat to the next.
- Has a heart-rate that is a chaotic pattern that is self-similar.



Interesting Fact:

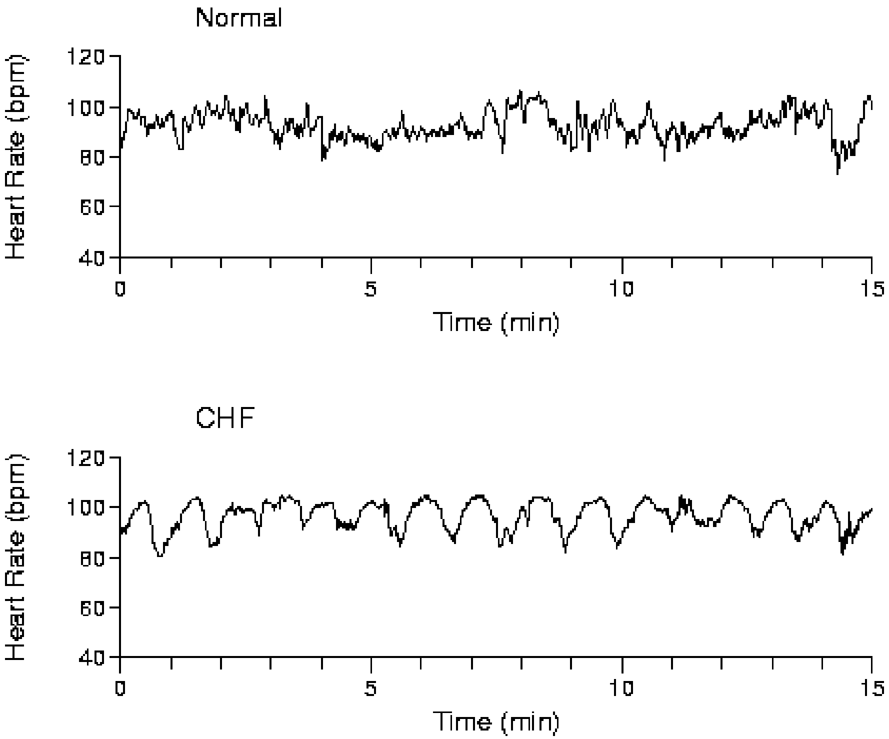
This concept also applies to human walking. When humans walk with variation in their step they are normal. With the onset of a disease, such as Parkinson’s disease, the human stride is more constant.

Diseased heart

- Doesn’t exhibit slight variations in time between each beat
- Has a heart-rate that is steady, constant, and predictable or either extremely random

Evidence for chaotic healthy hearts

As you can see the top graph shows a normal healthy heart. The graph of the healthy heart has more complexity then the bottom graph. Complexity = healthy in many physiological aspects. The bottom time scale graph shows a heart with CHF (congestive heart failure). CHF is just one of the many diseases that causes the heart rate to lose it's chaotic property. Click on the image for a further view of the normal heart-rate.



Picture from:http://www.physionet.org/tutorials/ndc/

Overview

- Chaos Theory – What is it really?
- Fractals – What is it really?
- Relevance of Chaos Theory and Fractals to Management

Former conception of chaos

- **Chaos** and **order** have long been viewed as antagonistic in the sciences
  - Traditional notion of chaos – *unorganized, disorderly, random* etc.
- Natural laws (Newton, Kepler, …) represent the domain of order
- Chaos was understood as a different state of nature where natural laws are not valid

Examples of Chaos

Not everything can be observed or predicted perfectly or in a deterministic (very accurate) fashion:

- Lightning
- Weather Patterns
- Earthquakes
- Financial Markets
- Social and Natural Systems
- Governmental and Financial Institutions

Chaos Theory

- But Chaos Theory has nothing do with the traditional notion per se
- Not all that ‘chaos’ you see is due to chance, or random or caused by unknown factors
- Uncertainties in various sources causes our observations and predictions to behave randomly.
- But then Chaos Theory comes in and bridges the gap
  - a mathematical approach to modeling patterns of non-linear, non-independent behaviors of dynamical systems. It is not, per se, a philosophical system or paradigm.

Properties of Non-Linear Systems

- They do not follow the principle of superposition (linearity and homogeneity).
- They may have multiple isolated equilibrium points.
- They may exhibit properties such as limit-cycle, bifurcation, chaos.
- Sensitivity to initial conditions – causing large divergence in the prediction. But this divergence is not infinite, it oscillates within bounds.
- Discovered by Edward Lorenz in Weather Modeling

Contd.

- **Butterfly Effect:** The flapping of a single butterfly's wing today produces a tiny change in the state of the atmosphere. Over a period of time, what the atmosphere actually does diverges from what it would have done. So, in a month's time, a tornado that would have devastated the Indonesian coast doesn't happen. Or maybe one that wasn't going to happen, does. (Ian Stewart, *Does God Play Dice? The Mathematics of Chaos*, pg. 141)
- Or life itself – more chaotic. One tiny decision you take today (apparently tiny), you have no idea where it might take you in the long after an accumulation of the triggering effects.

Why is chaos theory important?

- Organisations operate in turbulent and dynamic environments.
- This means uncertainty, unease and feelings of powerlessness with people in and around organisations.
- This is unfortunate as it often is on the outskirts of chaos that creativity flourishes.

Why is chaos theory important?

”Understanding chaos theory is important because of its significant implications for world systems design, organization design and administrative behaviour, and public policy analysis and implementation.” (Farazmand 2003:341)

Implicationess for organisational theory

Long-term planning is very difficult

- compare weather forecasting
- we cannot learn too much about the future by studying the past
- strategic planning should work with possible scenarios
- to focus on flexibility

Industries do not reach a stable equilibrium

- changes in industry structures can be endogenous

Implicationeness for organisational theory

Dramatic change can occur unexpectedly

- managers might underesti-mate the potential for large changes in industry conditions or competitors' behavior
- Small exogenous disturbances can lead to unexpectedly large changes

Short-term forecasts and predictions of patterns can be made

- chaotic systems trace repetitive patterns

In a complex system the best strategies might achieve goals indirectly and even appear counter-intuitive.

Systems theory and chaos theory

”Systems theory is concerned with stability and equilibrium whereas chaos and transformation theories are characterized by chaotic changes that lead to order and vice versa.” (Farazmand 2003:351)