Portfolio Theory

Lecture 1

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Structure

Instructions

- 1.. Active work at seminar (max. 3 absence)
- 2.. Bloomberg 5 Stocks→Covar and Correl Matrix
- 3.. Two tests (Σ30 p., each 15 p., vØ min. 60 %)
- No satisfy condition 1-3→"F"
- 1st test - 4/04/2016, 2nd test - 16/05/2016
- Correction test (30 points)
- Literature:**ELTON, E.; Modern portfolio theory and investment analysis**

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Marks

- Prerequisite $1 + 2\surd$
- Score of both tests:
- A: [27,30)
- B: [25,27)
- C: [23,25)
- D: [21,23)
- E: [18,21)
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  - HICKS, J.;*Application of Mathematical Methods of the Theory of Risk* (1934)
  - MARKOWITZ, H.;*Portfolio Selection* (1952) -**Founder of MPT**(inovative r$\wedge\sigma\Longrightarrow$Efficient frontier)
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- Portfolio:$\sum_{i=1}^{n} w_i A_i$;$\sum_{i=1}^{n} w_i = 1$; $X w_i \ldots weigh$;$A_i \ldots asset$
- Conditions of assets - identifiability, mesuarability (price)
- Investment:$f(r,\sigma,l)$
- Assuming arational investor. The aim of building a portfolio is to find such a composition of assets that corresponds his needs.

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Return

- Return of an asset:
  - $r_i = ln(P_{t+k}) - ln(P_t)$
  - $r_i = \frac{P_{t+k} - P_t}{P_t}$
  - $r_{id} = \frac{Di}{P_i}$
  - $r_{iTotal} = \frac{P_{t+k} - P_t + D_i}{P_t}$

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Return as random variable

- Uncertainty in the future development $\Longrightarrow$ random variable X (discreet random variable)

$\Longrightarrow$ Characteristic of RV E(X), σ^2(X) $\Longrightarrow$ **Mean Variance Portfolio**

- Mean
    - $\mu = \frac{1}{N}\sum_{i=1}^{N} X_i$,
    - $E(X) = \frac{1}{n}\sum_{i=1}^{n} X_i$,
    - $E(X) = \sum_{i=1}^{n} x_i * p(x_i)$
- *Characteristics of mean:*
    - $E(c) = c$, where c is a constant
    - $E(c*X) = c.E(X)$
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- **Variace**(discete case... (D, var,σ^2, s^2)
  $D(X) = E[X - E(X)]^2 = E(X)^2 - [E(X)]^2$
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- *Property of var:*
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  - $D(X+Y) = D(X) + D(Y) + 2*cov(X,Y)$... dependent RV
- **! Statistical population !** $\Longrightarrow n > 30 \Longrightarrow \sigma^2 = \frac{1}{n}\sum_{i=1}^{n} E[X_i - E(X)]^2$,
  $n < 30 \Longrightarrow$ **Sample variance** $\sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n} E[X_i - E(X)]^2$

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Risk

- ... the change in expected return - standard deviation $\sqrt{D(X)}$ (σ,s)
  - $\sigma_i = \sqrt{\frac{1}{n} * \sum_{i=1}^{n} (r_i - \bar{r})^2}$... n > 30,
  - $\sigma_i = \sqrt{\frac{1}{n-1} * \sum_{i=1}^{n} (r_i - \bar{r})^2}$... n < 30,
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The relation between RVs

- **Covariance…(cov(X,Y),$\sigma_{X,Y}$)**
  cov (X,Y) = E{[X-E(X)][Y-E(Y)]};
  - $\sigma_{X,Y} = \frac{1}{n}\sum_{i=1}^{n}[X_i - E(X)]*[Y_i - E(Y)]$
- $\sigma_{ij} = \frac{1}{n}\sum_{i=1}^{n}[r_i - \bar{r}_i]*[r_j - \bar{r}_j]\ldots n > 30,$
- $\sigma_{ij} = \frac{1}{n-1}\sum_{i=1}^{n}[r_i - \bar{r}_i]*[r_j - \bar{r}_j]\ldots n < 30,$
- *Property of covariance:*
  - cov(X,Y) = 0; E (X+Y) = 0∧E(X) = 0 = E(Y) = 0
  - cov(X,Y) = cov(Y,X)
  - cov(X+a,Y+b) = cov(X,Y)
  - cov(X*a,Y*b) = a*b*cov(X,Y)
  - cov(X,X) =σ^2(X)

  range of covar (-∞;∞)
  - ⟹standardization

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- The absolut dimension of covar is relativized
- $\rho_{XY} = \frac{cov(X,Y)}{\sigma_X * \sigma_Y}$
- Reflect the degree of **linear** dependence
- Interval for correlation <-1;1> (falling/rising)
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