Dominated and non-dominated variants (dominance analysis)

(model example)

Local police department wants to purchase new cars. Following table contains data about several considered models. Decide which models are dominated, determine basal and ideal variants and full solution set (the models that should be considered for further choice).

Model	Acceleration	Top speed	Fuel consum.	Trunk size	Price
Forman	17	141	8.1	450	220
Felicia combi	16	148	7.6	450	250
Lada 1500	15	153	7.6	480	210
Trabant	30	110	8.1	380	180

With increasing amount of available options and evaluated criteria in multi criteria evaluation it can get very difficult to orientate. One way to simplify it is to reduce the available set of options, let's call it a **full solution**, in which we do not consider further those options that are not relevant (dominated ones) – the options that practically cannot be chosen over some other available option.

Irrelevant variant is in this case the one to which there exists at least one other variant that is not worse in any of the considered criteria while being better in at least one criterion. Such variant is then considered as a **dominated** variant, and is being dominated by all other variants that fulfill the condition of being better in at least one criterion while not being worse in any other.

Available variants in most cases do not dominate each other, meaning that one is better in some criteria, while worse in other. Then they are considered to be non-dominated.

Sometimes it might help to determine theoretically worst and best variant. The worst variant is the one with the worst available values from the set, it is called **basal** variant (**B**), and contains basal values. On contrary, with the best available values from the set we get the **ideal** variant (**I**) with the ideal values of criteria.

Solution: **Basal variant** has acceleration of 30, top speed of 110, fuel consumption 8.1, trunk size 380 and costs 250. **Ideal variant** has acceleration of 15, top speed of 153, fuel consumption 7.6, trunk size 480 and costs 180. Forman and Felicia are dominated (by Lada), Lada and Trabant are not dominated. For further evaluation we would consider Lada and Trabant (**full solution**).

*In case of more complex problem you can use available tools, like SANNA from PSE.

Transformation of minimizing criteria to maximizing

(model example)

Police department wants to purchase new cars. Following table contains data about considered models. Some of the criteria are minimizing. Transform all to maximizing criteria.

Model	Acceleration	Top speed	Fuel consum.	Trunk size	Price
Forman	17	141	8.1	450	220
Felicia combi	16	148	7.6	450	250
Lada 1500	15	153	7.6	480	210
Trabant	30	110	8.1	380	180

In practice of multicriteria evaluation we often encounter situation where some criteria are desired to be maximized (like output level), while other minimized (like price). Transformation to the one type can reduce possibility of making a mistake due to such difference and can be also useful later.

We can transform the values of minimizing criteria to maximizing using the following transformation:

$$y_{ij}(max) = B(min) - y_{ij}(min)$$

Where *y(max)* means transformed value from min criterion to max criterion, *B(min)* means basal value of given min criterion (in such case the highest value of such criterion), and *y(min)* means original value of min criterion.

*in case we have a fixed available interval of values, like using grades 1-5, we use 5 as basal value independently from the fact that none of the evaluated variants actually got grade 5 in the criterion.

Solution:

Model	**T-Acceleration**	Top speed	**T-Fuel consum.**	Trunk size	**T-Price**
Forman	**13**	141	**0**	450	**30**
Felicia combi	**14**	148	**0,5**	450	**0**
Lada 1500	**15**	153	**0,5**	480	**40**
Trabant	**0**	110	**0**	380	**70**

WSA – weight sum approach

(model example)

Police department wants to purchase new cars. Following table contains data about considered models. Use WSA to select the best variant (weight of criteria are 30%, 10%, 30%, 30%).

Model	Acceleration	Top speed	Fuel consum.	Price
Octavia	9	200	6.8	410
Rapid	10	190	6.5	360
Fabia	11	180	6.3	330

WSA means that individual evaluated criteria are assigned with certain weights, that represent their level of importance in the final evaluation. Significantly worse parameters in one less important criterion therefore do not mean that the variant will automatically not be selected, if it has better parameters in more important criteria. For correct use we need to transform original values to the appropriate form. We transform values to the same type and then normalize them, so we would have comparable values. Final score for each variant is then scalar product of normalized values of criteria and their weights.

Solution steps: Transformation formulae for normalizing maximizing criteria:

$$y_{ij}(normalized) = \frac{y_{ij}max - B_j}{I_j - B_j}$$

Resp. transformation formulae for normalizing minimizing criteria:

$$y_{ij}(normalized) = \frac{B_j - y_{ij}min}{B_j - I_j}$$

Using these transformation we get normalized matrix of values between 0 and 1 and then we multiply the values with the weights. Normalized matrix looks like this:

Model	N-Acceleration	N-Top speed	N-Fuel consum.	N-Price
Octavia	1	1	0	0
Rapid	0.5	0.5	0.6	0.625
Fabia	0	0	1	1

Solution: Octavia gets 40%, Rapid 56.75% and Fabia 60%. Best model is **Fabia**.

Scales and ranges – assigning points within a scale

(model example)

Region is deciding between projects of several water plants on different rivers. Three projects were submitted, with criteria of building costs, running costs, output and safety level (range 0-10). You as an expert should evaluate criteria of projects on a scale of 0-100 and choose the best.

River	Building costs	Running costs	Output	Safety
Bobrava	170	73	67	9
Ponávka	132	38	45	7
Želetavka	99	41	33	5

Method of scales requires ability of quantitative evaluation of given parameters within evaluated criteria, meaning that the evaluator assigns values based on his expert opinion. Unlike with strictly mathematical methods this allows the consideration of other factors as well, like the experience of the evaluator, preferences or other aspects. The better the value of a parameter, the more points are assigned. Thanks to that we do not have any more issues with min/max criteria and their transformations or normalization.

On the other hand, the disadvantage of this method is the dependency on the subjective evaluation of parameters. For reducing the risk of making incorrect decision, multiple independent expert evaluation are often used. Final score is then a result of sum of individual evaluations, or their weighted sum, if opinions of different experts are weighted differently.

Analogically we can assign different weight to different criteria based on their importance. Assigned points for each parameter are then weighted and summed together afterwards. Such method is then called scoring method.

Example of a possible solution, a subjective assignation of points to the parameters (0-100):

River	Building costs	Running costs	Output	Safety	Sum
Bobrava	40	50	100	90	280
Ponávka	61	100	77	70	**308**
Želetavka	80	95	50	50	275

Lexicographic method

(model example)

Region wants to build a bridge over the river Jevišovka. Estimated parameters of variants are in the following table. For decision use lexicographic method with criteria preferences C→B→D→A and requirements for criteria being A ≥ 440, B ≥ 6, C ≥ 7 a D ≤ 50.

Bridge	A) Capacity	B) Looks	C) Place	D) Costs
of victory (1)	406	7	8	50
Red (2)	444	8	8	39
of friendship (3)	505	4	9	44
of labor (4)	568	5	10	32
of proletariat (5)	541	8	6	52

Lexicographic method evaluated available variants sequentially based on the importance of individual criteria and limiting requirements. In the first step we take the most important criterion and discard the variants that do not meet the requirements for this criterion. In the second step we continue analogically with the reduced set of remaining variants. The evaluation process is finished when only one variant remains, and this variant is chosen as the best. In case we are left with multiple variants even after the last criterion, we need to use some additional method for choosing a compromise variant (for instance selecting one with the best parameter in the last evaluated criterion).

The disadvantage of this method is that we are practically taking into account only the last evaluated criterion and do not consider previous as long as the minimal requirements were met. Moreover, the result can be notably biased by the criteria preferences selection. In cases with no non-dominated variants, with "appropriate" preference order and minimal requirements it is sometimes possible to secure any of the variants as the winning one.

Solution steps: In the first step we evaluate according to the C criterion – we discard variant 5, remaining set is {1, 2, 3, 4}. In the second step we evaluate according to the B criterion – we discard variants 3 and 4, remaining set is {1, 2}. In the third step we evaluate according to the D criterion – we do not discard any variants, the set remains {1, 2}. In the fourth step we evaluate according to the A criterion – we discard variant 1, remaining set is {2}.

Solution: We choose the **Red bridge (2).**