Portfolio Theory

Lecture 4

Luděk Benada

Department of Finance - 402, benada.esf@gmail.com

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Structure

Short selling

- The assumption for the technique: $P_{t+n} < P_t$
- Investor has to borrow securities, which will sell and use the sources for a portfolio construction
- When the portfolio will be realized, then the securities must be returned
- At that point the investor must purchase on the market the security
- The owner of the security has rights on all CF´s
- *An example...*

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The impact of SS on the EPF

- The Short Sell expands the feasible set of portfolios
- ...expands the Efficient Portfolio Frontier
- The SS could be used even the $\bar{r}_i > 0$
- ... but there will be a lost of the diversification offect.
- The graphical representation ...

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Calculation algorithm

- The purpos is to find an extrem of given function
- There are two feasible approaches:
  - ...to maximize the expected returne of the portfolio
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The minimum risk portfolio

- Two ways of problem solving:
  - ... to find an absolute minimum risk
  - ... to find a minimum risk for a given return

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The absolute minimum risk

- Solving the optimalization problem
- The weights are found by solving the objectie function, assuming the restriction...
- Objective function: $\sigma_p^2 \rightarrow min$
- Restrictive conditions: $\sum_{i=1}^{N} w_i = 1$

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Searching the extrems

- $f(\overrightarrow{W}) \rightarrow min$
- Lagrange function: $L(\overrightarrow{W}) = L(\overrightarrow{W},\overrightarrow{\lambda}) = \sum_{i=1}^{N} \lambda_i f_i(\overrightarrow{W})$
- For finding the solution will be used the Lagrange multipliers:
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Lagrange Multipliers

- $L(\overrightarrow{Y}) = \sigma_p^2(\overrightarrow{W}) + \lambda_1(\sum_{i=1}^{N} w_i - 1)$
- Deriving with respect to all variables we obtain n+1 equiations
- The first n equiations will be equal to zero
- $2C\overrightarrow{W} + \lambda_1 \overrightarrow{e} = 0$
- At the last equation transforme the "1" to the right site
- $\overrightarrow{W^T} \overrightarrow{e} = 1$
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Minimizing risk by given return

- Restrictions:
  - $\sum_{i=1}^{N} w_i = 1$
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- Lagrangian:
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