Discounting in time

(model example)

We have net annual income of 100. Calculate discounted value in t = 6 if r = 5% compared to today?

Value of financial means changes in future depending on the time horizon (*t)* and discount rate (*r)* – analogically to the inflation.

When considering cash flow in various years we discount with discount factor for given year (*t*), what depends on discount rate (*r*) according to the general formulae:

$$\frac{cashflow_t}{(1+r)^t}$$

where cash flow (CF_t) means cash flow in period *t*, or the sum of costs and incomes (sum of individually discounted costs and incomes yields the same results).

Note: Alternatively the $-CF_0$ is indicated as *I* – investment costs (costs with positive sign).

For the discounting rate *r* = 5% (alternatively *r* = 0.05) is the discounting factor equal to the following values:

Time period	0	1	2	3	4
Formulae	$\frac{x}{(1+0.05)^0}$	$\frac{x}{(1+0.05)^1}$	$\frac{x}{(1+0.05)^2}$	$\frac{x}{(1+0.05)^3}$	$\frac{x}{(1+0.05)^4}$
Factor	$\frac{x}{1.00}$	$\frac{x}{1.05}$	$\frac{x}{1.10}$	$\frac{x}{1.16}$	$\frac{x}{1.22}$
Multiplier	$x*(1.00)$	$x*(0.95)$	$x*(0.91)$	$x*(0.86)$	$x*(0.82)$

With the discounting rate *r* = 5% will be the value of financial means in the individual terms as shown below:

Term	0	1	2	3	4	5	6
Value	100	100	100	100	100	100	100
Discounted value	100.00	95.24	90.70	86.38	82.27	78.35	**74.62**

Solution: The income of 100 will have the value (compared to today) in *t* = 6 with *r* = 5% of **74.62**.

Payback Period – PB

(model example)

We have a project with investment costs of 300 and annual net income of 80. Project lifetime is 6 periods. What is simple and real payback period of this projects if *r* = 5%?

Payback period the period from the beginning of the project until the time when the cumulative income of the project with repay the costs, or the period when cumulative cash flow of the project will become non-negative. If payback period is longer than the project lifetime the project is rejected.

In case of simple payback time we do not consider discounting in time.

In case of real payback time we discount the cash flow for each period.

Calculations are shown below:

Period	0	1	2	3	4	5	6
Cash flow (simple)	-300	80	80	80	80	80	80
Cumulative cash flow (simple)	-300	-220	-140	-60	**20**	100	180
Cash flow (with r = 5%)	-300.00	76.19	72.56	69.11	65.82	62.68	59.70
Cumulative cash flow (with r = 5%)	-300.00	-223.81	-151.25	-82.14	-16.32	**46.36**	106.06

Note: cash flow of given period is always discounted individually with the discount factor for given period – discounting of the total sum of simple cash flows leads to the incorrect results.

Solution: Simple payback period of considered project is **4 periods**. Real payback period of considered project with *r* = 5% is **5 periods**.

NPV – Net Present Value

(model example)

We have project with investment costs of 200 and net income during 3 years of 70, 80 a 90. What is the NPV of the project with *r* = 5%? Is the project acceptable? What is the NPV with *r* = 10% and would the project still be acceptable?

NPV means the final value of costs and incomes of the project with considering the effect of discounting on the cash flow. It is basically in advance calculation of cumulative cash flow of the project for a specified time period and with specified discount rate (in this case investment period plus 3 years of operation with provided net incomes).

Note: Cumulative discounted cash flow in time *t* is the same as NPV in time *t*.

Note: PV indicator – present value, does not include investment costs, thus NPV = PV + CF_0.

General formulae for calculating NPV is:

$$NPV = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t}$$

In our case are steps for calculating NPV following:

Period	0	1	2	3
Cash flow	-200	70	80	90
Discounted cash flow (*r* = 5 %)	-200.00	66.67	72.56	77.75
NPV (*r* = 5 %)	-200.00	-133.33	-60.77	**16.97**
Discounted cash flow (*r* = 10 %)	-200.00	63.64	66.12	67.62
NPV (*r* = 10 %)	-200.00	-136.36	-70.25	**-2.63**

Note: When comparing project with different lifetime we need to transform these projects to the same lifetime – repeat the projects until their lifetimes become equal (finding their smallest common multiplier) and compare the NPV calculated from these "repeated" projects. However, if the investments costs are different as well, it is generally more appropriate to use a different evaluation criterion, such as Rentability index.

Solution: NPV of considered project with discounting rate 5% is **16.97**. NPV of the project becomes non-negative in the considered period, so the project **is acceptable**. With discount rate of 10% is the NPV of the project **-2.63** and from the NPV perspective the project **is not acceptable**.

Rentability Index – R_i

(model example)

We have project with investment costs of 200 and net incomes over 3 years of 70, 80 a 90. What is R_i of this project with discounting rate of 0.05?

Rentability Index is basically the ratio of the sum of project's total net income minus the investment costs to the investment costs. NPV value is divided by the costs from period 0 (with positive sign).

$$R_i = \frac{NPV}{-CF_0}$$

Alternative indicator is ROI (*Return of Investment)*. The idea is identical, the difference is that in the denominator we use cumulated cash flow without the cash flow from period 0 (the investment costs). Therefore logically ROI = R_i + 1. However, ROI usually does not use discounted cash flow.

For the calculation we use identical values as with NPV and then we calculate their ratio:

Period	0	1	2	3
Cash flow	-200	70	80	90
Discounted cash flow (r = 5%)	-200.00	66.67	72.56	77.75
NPV (r = 5%)	-200.00	-133.33	-60.77	16.97

R_i is then:

$$R_i = \frac{16.97}{200} = \mathbf{8.49\%}\ \boldsymbol{or}\ \mathbf{0.0849}$$

In case of ROI (without discounting) is the calculation:

$$ROI = \frac{240}{200} = 120\%\ or\ 1.2$$

Positive value of R_i means that the NPV is positive as well (or the sum of discounted net incomes after subtracting initial costs is non-negative) and project as such is acceptable. Analogically the project is acceptable if ROI ≥ 1.

Solution: Rentability index of considered project is **8.49%**, resp. 0.0849.

Internal Rate of Return – IRR

(model example)

We have project with investment costs of 10 and net income over 2 years of 7 and 8. What is the IRR of this project? Is the project acceptable if the standard project discount rate is 5%?

IRR (*Internal rate of return*) is calculated in the "reversed" way as NPV indicator. With NPV we have the discount rate and calculate the cumulative cash flow value. With IRR on the contrary we set the NPV = 0 and then calculate the corresponding "discount" rate. Calculated IRR value is then usually compared with the discount rate, where it usually holds that if IRR ≥ *r* the project is acceptable (this also means that the NPV of the project using standard value of *r* is non-negative), and on the opposite if IRR < *r* then the project is not acceptable (NPV is in such case negative).

IRR calculation is basically solving the polynomial of *n* degrees, where *n* = amount of project periods (without the investment period). Solving this for 1 year period is trivial, for 2 years it becomes a quadratic equation (our case), and for longer periods can be solved using numerical methods – usually it is sufficient to use appropriate spreadsheet function like "=IRR()" in MS Excel. A formula is:

$$\sum_{t=0}^{T} \frac{CF_t}{(1+IRR)^t} = 0$$

In some cases the IRR solving become problematic, because polynomial of *n* degree can have 0 up to *n* solutions. With multiple solutions it is up to our consideration us to, which solution makes the most sense. Also if it is not possible to calculate IRR, it does not exists in given case.

Solving our case in MS Excel is following:

	A	B	C
1	-10	7	8
2	=IRR(A1:C1)		31,05%

In our case we can solve the problem also as a quadratic equation:

- we create a substitution, e.g. *z* = (1+IRR) and then adjust our equation into

$$-10 + \frac{7}{z} + \frac{8}{z^2} = 0, resp. \; -10z^2 + 7z + 8 = 0$$

- and then solve as a standard quadratic equation

$$\frac{-7 \pm \sqrt{7^2 - 4 * (-10) * 8}}{2 * (-10)} = \frac{-7 \pm \sqrt{49 + 320}}{-20} = \frac{-7 \pm 19.21}{-20} = 1.31 \; resp. -0.61$$

- finally we adjust the substitution *z* = (1+IRR) and get the solution for IRR = **31%**, resp. -39%, and choose the first value as it makes more sense
- for illustration see graphical solution of the problem http://goo.gl/nS7BZU

Solution: IRR of considered project is IRR = **31%**. Compared to usual discount rate of 5% it is higher, therefore the **project is acceptable**.