Portfolio Theory

Lecture 1

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Structure

Expert estimates

- Estimates of market prices of assets at the time of realization
- N experts will provide estimates for all actives (considered for investment)
- In the calcultation is used the probability structure
- No dividend payment considered
- The price of asset(s) is know at the point of buying (selling)

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Return and risk of a security

- If the probability of price development is known
- The mean of the security could be determined
- $r_i = \sum_{i=1}^{N} r_i * p_i$
- ... and thus the risk of security
- $\sigma_i = \sqrt{\sum_{i=1}^{N} (r_i - \overline{r})^2 * p_i}$

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Indexation

- P_{it}...Market price of an asset in the point of portfolio formation
- N_{ij}...The number of total number of estimates for the future price (of i-th assets, done by j-th expert)
- N_{ij}... The probability according of j-th expert´s estimates of the return during the period
- !In accordance with the condition: $\sum_{i=1}^{N} \mathrm{p}_{ijk} = 1$!
- ... then must be applied $p_{ij} = \frac{1}{N_e} * \sum_{j=1}^{N_e} p_{ijk}$
- $r_{ijk} = \frac{P_{it+n} - P_{it}}{P_{it}}$

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- Return
- Normalization of the probability
- The return of portfolio
- The risk of portfolio

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Portfolio - expected return

- A portfolio based of n assets
- i-th asset has $w_i$and r_i
- ...thus the return of this portfolio will be: $r_p = \sum_{i=1}^{N} w_i * r_i$
- if the expected return of i-th asset will be $\bar{r}_i$
- ...thus the expected return of the portfolio will be: $\bar{r}_p = \sum_{i=1}^{N} w_i * \bar{r}_{ii}$
- under condition: $\sum_{i=1}^{n} = 1$
- Note: It is posible to have $w_i < 0$, then sale short is executed

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The risk of an expected return

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- $\sigma_p = \sum_{i=1}^{N} w_i^2 * \sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1/i\neq j}^{N} w_i * w_j * \sigma_{ij}$
- *A special case (equal weights)*
- $\sigma_p^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{\sigma_i^2}{N} + \frac{N-1}{N}\sum_{i=1}^{N}\sum_{j=1/i\neq j}^{N}\frac{\sigma_{ij}}{N*(N-1)} \Rightarrow \sigma_p^2 = \frac{1}{N} * \bar{\sigma}_i{}^2 + \frac{N-1}{N} * \sigma_{ij}$

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Comment to risk

- The contribution of a partial risk to the total risk of the portfolio is decreasing to zero with growing number of securities
- The contribution to the portfolio risk flowing from covariance is with the growing number of assets approaching an average covariance
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