Portfolio Theory

Lecture 5

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Structure

Risky and Risk free Asset

- $r_f \ldots$ treasury bills, deposits at a bank
- Model example (r_i, r_f, r_p)
- Return and risk
- Graphycal representation

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The Shape of EPF

- All combination of risky and risk free asset $\Rightarrow$a line
- The mean of the security could be determined
- $r_p = r_f + \left(\frac{\bar{r}_A - r_f}{\sigma_A} \right) \sigma_p$
- The impact on the permissible and effective set (EPF)
- Which line to choose?

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Lending and Borrowing

- A part of our fund to $r_f \Rightarrow$borrow sources to someone
- We can invest more than we own ...we must borrow
- The boundary between lending and borrowing represents the situation when all funds are invested in r_i
- !In accordance with the condition: $\sum_{i=1}^{N} p_{ijk} = 1$!
- The hypothetical shape of EPF and the real shape of EPF

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Finding a portfolio with respect to r_f

- **Four possible scenarious:**
  - The Short Sell is allowed and there is r_f
  - The Short Sell is allowed, but there is not r_f
  - The Short Sell is not allowed and there is r_f
  - There is neither SS allowed, nor r_f exists

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Short Sell allowed with existance of r_f

- Maximalization of an objective function with restrictions
- The objective function is tg of the angle (r_f, T)
- The restrictions are weights ...
- $f(\overrightarrow{X}) = \frac{\bar{r}_p - r_f}{\sigma_p}$

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Diversification of assets

- Meaning of diversification
- The Central Limit Theorem ...
- A portfolio of N assets created with same weights:
  - $r_p = \frac{1}{N}\sum_{i=1}^{N} w_i * r_i$
  - Variance of portfolio on conditions ($N(\mu, \sigma^2, \sigma_{i,j} = 0, N \to \infty)$ $\Rightarrow 0$
- If the distribution deviates from gaussian, then the mean-variance approach exhibits shortcomings

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Alternative approaches to risk

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