Portfolio Theory

Lecture 9

Luděk Benada

Department of Finance - 402, benada.esf@gmail.com

April 25, 2016

Structure

Single index model

- There is empirical evidence: $\uparrow M \Rightarrow \uparrow S$
- Therefore the (excess) return of a security is represented in relation to the market:
  - $r_i = a_i + b_i * r_M$
- The return of a security consists of two parts:
  - Dependent on the market
  - Independent on the market

Single index model

- There is empirical evidence: $\uparrow M \Rightarrow \uparrow S$
- Therefore the (excess) return of a security is represented in relation to the market:
  - $r_i = a_i + b_i * r_M$
- The return of a security consists of two parts:
  - Dependent on the market
  - Independent on the market

Single index model

- There is empirical evidence: $\uparrow M \Rightarrow \uparrow S$
- Therefore the (excess) return of a security is represented in relation to the market:
  - $r_i = a_i + b_i * r_M$
- The return of a security consists of two parts:
  - Dependent on the market
  - Independent on the market

Single index model

- There is empirical evidence: $\uparrow M \Rightarrow \uparrow S$
- Therefore the (excess) return of a security is represented in relation to the market:
  - $r_i = a_i + b_i * r_M$
- The return of a security consists of two parts:
  - Dependent on the market
  - Independent on the market

Single index model

- There is empirical evidence: $\uparrow M \Rightarrow \uparrow S$
- Therefore the (excess) return of a security is represented in relation to the market:
  - $r_i = a_i + b_i * r_M$
- The return of a security consists of two parts:
  - Dependent on the market
  - Independent on the market

Single index model

- There is empirical evidence: $\uparrow M \Rightarrow \uparrow S$
- Therefore the (excess) return of a security is represented in relation to the market:
  - $r_i = a_i + b_i * r_M$
- The return of a security consists of two parts:
  - Dependent on the market
  - Independent on the market

The indipendent component of the model

- The model could be splited into:
  - Estimate
  - Ramdom error
    - $r_i = \alpha_i + \beta_i * r_M + \varepsilon_i$
- The return of the market and the error are random variable $\Rightarrow (\mu, \sigma^2)$
- Model must garantee:
  - $cov(\varepsilon_i, r_m) = 0$
  - $cov(\varepsilon_i, \varepsilon_j) = 0$
  - $\sigma_{i,j} = \beta_i * \beta_j \sigma_M^2$

The indipendent component of the model

- The model could be splited into:
  - Estimate
  - Ramdom error
    - $r_i = \alpha_i + \beta_i * r_M + \varepsilon_i$
- The return of the market and the error are random variable $\Rightarrow (\mu, \sigma^2)$
- Model must garantee:
  - $cov(\varepsilon_i, r_m) = 0$
  - $cov(\varepsilon_i, \varepsilon_j) = 0$
  - $\sigma_{i,j} = \beta_i * \beta_j \sigma_M^2$

The indipendent component of the model

- The model could be splited into:
  - Estimate
  - Ramdom error
    - $r_i = \alpha_i + \beta_i * r_M + \varepsilon_i$
- The return of the market and the error are random variable $\Rightarrow (\mu, \sigma^2)$
- Model must garantee:
  - $cov(\varepsilon_i, r_m) = 0$
  - $cov(\varepsilon_i, \varepsilon_j) = 0$
  - $\sigma_{i,j} = \beta_i * \beta_j \sigma_M^2$

The indipendent component of the model

- The model could be splited into:
  - Estimate
  - Ramdom error
    - $r_i = \alpha_i + \beta_i * r_M + \varepsilon_i$
- The return of the market and the error are random variable $\Rightarrow (\mu, \sigma^2)$
- Model must garantee:
  - $cov(\varepsilon_i, r_m) = 0$
  - $cov(\varepsilon_i, \varepsilon_j) = 0$
  - $\sigma_{i,j} = \beta_i * \beta_j \sigma_M^2$

The indipendent component of the model

- The model could be splited into:
  - Estimate
  - Ramdom error
    - $r_i = \alpha_i + \beta_i * r_M + \varepsilon_i$
- The return of the market and the error are random variable $\Rightarrow (\mu, \sigma^2)$
- Model must garantee:
  - $cov(\varepsilon_i, r_m) = 0$
  - $cov(\varepsilon_i, \varepsilon_j) = 0$
  - $\sigma_{i,j} = \beta_i * \beta_j \sigma_M^2$

The indipendent component of the model

- The model could be splited into:
  - Estimate
  - Ramdom error
    - $r_i = \alpha_i + \beta_i * r_M + \varepsilon_i$
- The return of the market and the error are random variable $\Rightarrow (\mu, \sigma^2)$
- Model must garantee:
  - $cov(\varepsilon_i, r_m) = 0$
  - $cov(\varepsilon_i, \varepsilon_j) = 0$
  - $\sigma_{i,j} = \beta_i * \beta_j \sigma_M^2$

The indipendent component of the model

- The model could be splited into:
  - Estimate
  - Ramdom error
    - $r_i = \alpha_i + \beta_i * r_M + \varepsilon_i$
- The return of the market and the error are random variable $\Rightarrow (\mu, \sigma^2)$
- Model must garantee:
  - $cov(\varepsilon_i, r_m) = 0$
  - $cov(\varepsilon_i, \varepsilon_j) = 0$
  - $\sigma_{i,j} = \beta_i * \beta_j \sigma_M^2$

The indipendent component of the model

- The model could be splited into:
  - Estimate
  - Ramdom error
    - $r_i = \alpha_i + \beta_i * r_M + \varepsilon_i$
- The return of the market and the error are random variable $\Rightarrow (\mu, \sigma^2)$
- Model must garantee:
  - $cov(\varepsilon_i, r_m) = 0$
  - $cov(\varepsilon_i, \varepsilon_j) = 0$
  - $\sigma_{i,j} = \beta_i * \beta_j \sigma_M^2$

The indipendent component of the model

- The model could be splited into:
  - Estimate
  - Ramdom error
    - $r_i = \alpha_i + \beta_i * r_M + \varepsilon_i$
- The return of the market and the error are random variable $\Rightarrow (\mu, \sigma^2)$
- Model must garantee:
  - $cov(\varepsilon_i, r_m) = 0$
  - $cov(\varepsilon_i, \varepsilon_j) = 0$
  - $\sigma_{i,j} = \beta_i * \beta_j \sigma_M^2$

An optimal portfolio in SIM

- Suppose that the SIM is the best method of predicting a covariace structure of returns
- For creating a portfolio basket it will be useful to have a tool to select assets
- If SIM holds, then the decision making criteria:
  - $\frac{\bar{r}_i - r_f}{\beta_i}$
- Ranking expresses favourableness of every assets included into the portfolio

An optimal portfolio in SIM

- Suppose that the SIM is the best method of predicting a covariace structure of returns
- For creating a portfolio basket it will be useful to have a tool to select assets
- If SIM holds, then the decision making criteria:
  - $\frac{\bar{r}_i - r_f}{\beta_i}$
- Ranking expresses favourableness of every assets included into the portfolio

An optimal portfolio in SIM

- Suppose that the SIM is the best method of predicting a covariace structure of returns
- For creating a portfolio basket it will be useful to have a tool to select assets
- If SIM holds, then the decision making criteria:
  - $\frac{\bar{r}_i - r_f}{\beta_i}$
- Ranking expresses favourableness of every assets included into the portfolio

An optimal portfolio in SIM

- Suppose that the SIM is the best method of predicting a covariace structure of returns
- For creating a portfolio basket it will be useful to have a tool to select assets
- If SIM holds, then the decision making criteria:
  - $\frac{\bar{r}_i - r_f}{\beta_i}$
- Ranking expresses favourableness of every assets included into the portfolio

An optimal portfolio in SIM

- Suppose that the SIM is the best method of predicting a covariace structure of returns
- For creating a portfolio basket it will be useful to have a tool to select assets
- If SIM holds, then the decision making criteria:
    - $\frac{\bar{r}_i - r_f}{\beta_i}$
- Ranking expresses favourableness of every assets included into the portfolio

Implication of decision criteria

- If a security with its ratio is in the portfolio included, then all securities with higher ratio should be included as well
- If a security with its ratio is not in the portfolio, then all securities with lower ratio must be excluded to the portfolio

Implication of decision criteria

- If a security with its ratio is in the portfolio included, then all securities with higher ratio should be included as well
- If a security with its ratio is not in the portfolio, then all securities with lower ratio must be excluded to the portfolio

Portfolio selection with ban on short sell

- It is necessary to establish the threshold C*
- Subsequently selection is done:
    - Securities to the portfolio
    - Securities out of the portfolio

Portfolio selection with ban on short sell

- It is necessary to establish the threshold C*
- Subsequently selection is done:
  - Securities to the portfolio
  - Securities out of the portfolio

Portfolio selection with ban on short sell

- It is necessary to establish the threshold C*
- Subsequently selection is done:
  - Securities to the portfolio
  - Securities out of the portfolio

Portfolio selection with ban on short sell

- It is necessary to establish the threshold C*
- Subsequently selection is done:
  - Securities to the portfolio
  - Securities out of the portfolio

Procedure by the selection

- Ranking of every security by $\frac{\bar{r}_i - r_f}{\beta_i}$
- Include securities with:
  - $\frac{\bar{r}_i - r_f}{\beta_i} > C^*$

Procedure by the selection

- Ranking of every security by $\frac{\bar{r}_i - r_f}{\beta_i}$
- Include securities with:
  - $\frac{\bar{r}_i - r_f}{\beta_i} > C*$

Procedure by the selection

- Ranking of every security by $\frac{\bar{r}_i - r_f}{\beta_i}$
- Include securities with:
  - $\frac{\bar{r}_i - r_f}{\beta_i} > C*$

Determining of cut-off

- $C_i = \dfrac{\sigma_M^2 \sum_{i=1}^{N} \frac{(\bar{r}_i - r_f) * \beta_i}{\sigma_{\varepsilon_i}^2}}{1 + \sigma_M^2 * \sum_{i=1}^{N} \left(\frac{\beta_i^2}{\sigma_{\varepsilon_i^2}} \right)}$
- Securities are included to the portfolio if:
  - $\frac{\bar{r}_i - r_f}{\beta_i} > C_i$
- C* corresponds to the last securities holding this condition

Determining of cut-off

- $C_i = \frac{\sigma_M^2 \sum_{i=1}^{N} \frac{(\bar{r}_i - r_f) * \beta_i}{\sigma_{\varepsilon_i}^2}}{1 + \sigma_M^2 * \sum_{i=1}^{N} \left(\frac{\beta_i^2}{\sigma_{\varepsilon_i}^2} \right)}$
- Securities are included to the portfolio if:
  - $\frac{\bar{r}_i - r_f}{\beta_i} > C_i$
- C* corresponds to the last securities holding this condition

Determining of cut-off

- $C_i = \frac{\sigma_M^2 \sum_{i=1}^{N} \frac{(\bar{r}_i - r_f) * \beta_i}{\sigma_{\varepsilon_i}^2}}{1 + \sigma_M^2 * \sum_{i=1}^{N} \left(\frac{\beta_i^2}{\sigma_{\varepsilon_i}^2} \right)}$
- Securities are included to the portfolio if:
  - $\frac{\bar{r}_i - r_f}{\beta_i} > C_i$
- C* corresponds to the last securities holding this condition

Determining of cut-off

- $C_i = \frac{\sigma_M^2 \sum_{i=1}^{N} \frac{(\bar{r}_i - r_f)*\beta_i}{\sigma_{\varepsilon_i}^2}}{1+\sigma_M^2 * \sum_{i=1}^{N} \left(\frac{\beta_i^2}{\sigma_{\varepsilon_i}^2} \right)}$
- Securities are included to the portfolio if:
  - $\frac{\bar{r}_i - r_f}{\beta_i} > C_i$
- C* corresponds to the last securities holding this condition

Weights in portfolio

- $w_i = \frac{Z_i}{\sum_{i=1}^{N} Z_i}$
- $Z_i = \frac{\beta_i}{\sigma^2_{\varepsilon_i}}$

Weights in portfolio

- $w_i = \frac{Z_i}{\sum_{i=1}^{N} Z_i}$
- $Z_i = \frac{\beta_i}{\sigma_{\varepsilon_i}^2}$

Portfolio selection if SS is allowed

- In this case the C* $\ldots C_n$

Short sell is not allowed

- ...

Short sell is allowed

- . . .