Shadow pricing – deriving prices from the demand curves

(model example)

Region plans to build a new bridge that would lower the travelling costs between two cities by 100 CZK/trip. This costs reduction includes shorter travelling time, lower vehicle depreciation and lower fuel consumption of 120 CZK/trip in total minus 20 CZK toll per bridge crossing – a Region's income. Before the bridge there were total of 1 million trips per year, an estimate after the bridge is built is 1.5 million trips per year. Calculate annual benefits of the new bridge for drivers and for the Region.

Shadow pricing represents a way how to estimate the value of goods and services using a ***market-based approach***. In an optimal situation it is possible to find an alternative to the evaluated goods/service and use this price. If not, we can try to use shadow prices.

We use *shadow pricing* in situations where the actual price of goods/services does not exist or does not reflect the true value (often due to the regulations, temporary fluctuations, or some other market failures). A shadow price then can be perceived as a price that would be achieved in a perfect competition market. Shadow price represents a *proxy* value, usually considered as a value of what one has to sacrifice in order to get an additional unit. An estimation of a shadow price is usually done by observing subjects' behavior. The two main approaches to estimated shadow prices are based on benefits or on costs.

Benefits approach includes estimation methods like *change in productivity*, or *forgone earnings*. We can calculate these prices using a CBA for the specific goods/service. Costs approach includes methods like *replacements/restoration costs*, *preventive expenditure*, *damage avoidance costs*, etc.

In practice these methods can be used in a form of a natural experiment or pilot projects (small scale experiments). Impacts of such projects are classified, quantified and monetized. Using an extrapolation, shadow prices can be estimated for the whole project.

Alternative way of estimating shadow prices is by using pseudo- or quasi-demand curves. We use the initial state and the expected state after the project's realization and based on that estimate the demand curve. We then calculate consumer's benefit resulting from the change of the state.

Solution steps: In our case we have data about demand before and after the project realization, thus we derive shadow prices from the relevant demand curve. We can start with benefits to the Region, which are basically number of trips multiplied by the bridge crossing toll (*de facto* revenues).

In case of drivers we need to separate the group that would travel regardless of the new bridge – they would enjoy benefits of the full cost saving, and the remaining group with decreasing benefits due to the slope of the demand curve. In other words, benefit for the driver no. 1,000,001 is almost the full costs savings, while the benefit for the driver no. 1,500,000 is basically zero – this is the very last driver that decided to do the trip, as it was practically on par with the alternative of, for instance, staying at home or taking an alternative route.

Solution: Benefit for the drivers is **125 mil. CZK/year**, benefit for the Region is **30 mil. CZK/year**.

WTP – Willingness to Pay

(model example)

Municipality is considering introducing a new waste collection system – instead of central container (fully paid by the municipality) use bins for the individual households. Estimate the social benefits of such new system using WTP. This could tell the municipality at what level should it set the fee for the new system in order to cover the additional expenditures.

Population (200 total) has been sorted based on their willingness to pay for the new system from the lowest fee to the highest and divided into the quartiles. Assume that the willingness to pay grows linearly within each quartile. First person is not willing to pay anything, person at the first quartile can pay 300 CZK, at the second quartile 380 CZK, at the third quartile 440 CZK, and last one 560 CZK.

WTP method represents the second approach how to estimate shadow prices and is based on ***stated*** (declared) ***preferences*****.** It is a kind of a ***Contingent Valuation Method (CVM)*** that uses questionnaires/surveys. In contrast with the market-approach based on observation of the subjects, this approach estimates the values based on subjective opinions (theoretical preference), not from actual behavior, that has been confirmed at the market.

The method itself consists of collecting data from a survey, in which a relevant sample of subjects state the maximum price they are willing to pay for some goods or service. Survey can start with lower amounts and continue higher until maximum price is stated. An alternatively is a directly question about the highest price. Based on collected data we derive a pseudo-demand curve.

The benefits estimated by WTP method is, as with other methods based on preferences, calculated as the consumer surplus – the area under the derived pseudo-demand curve.

Analogical method is ***WTA – Willingness to Accept***, where you estimate the price, resp. equivalent that would make subject accept the presence of some negative factor.

Main problem of WTP and WTA methods is in often observed difference between stated and revealed (real, market) preferences, and the correct use of this method requires taking this into the consideration. Respondents in case of WTP and WTA often overstate their willingness to pay, resp. their minimal accepted equivalent for sustaining some negative effect. Another problem of these methods is acquiring the data from a representative sample during the survey.

Solution steps: Based on the collected data we derive a pseudo-demand curve of the willingness to pay and calculate the area under it. Total willingness to pay in 4th quartile is (50*440+(560-440)*50/2) = 25 000 CZK, in 3rd quartile is (380*50+(440-380)*50/2) = 20 500 CZK, in 2nd quartile is (50*300+(380-300)*50/2) = 17 000 CZK, and in the first quartile is (300*50/2) = 7 500 CZK.

Solution: Social benefit from new waste collection system using bins would be **70 000 CZK**.

Hedonic method

(model example)

You are asked to evaluate benefits of an anti-flood dam that would reduce the probability of a flood in a residential area of 80 houses. Use hedonic method for the evaluation if you know that the price of houses in given area can be estimated using linear regression model:

$$p_i = \alpha + \beta(prob) + \gamma(x_i) + \varepsilon_i$$

where p_i means estimated price of the house, $prob$ probability of a flood occurrence in some time horizon, and x_i some other parameters of house that we do not need to consider in this case.

Based on the survey of house prices the coefficient have values $\alpha = 75$, $\beta = -200$ and $\gamma = 1$. Dam construction is estimated to reduce the risk of flood in some time horizon from 0.4 to 0.05.

Hedonic method represents an approach used for estimating the values of certain measures, good or services that directly affect the market prices of some other goods. Usual application area is the realities market, which reflects the impacts of changes of local environmental attributes.

Hedonic method assumes that the price of a certain market good is directly influenced by its characteristics and our considered measure (resp. good, service). Steps of this method consist of collecting sufficient data regarding prices and characteristics of considered market goods and their subsequent statistical analysis. The result is then a price function of a market good consisting of effects of individual characteristics. Based on the impact of changing individual parameters we estimate the social benefits of introducing evaluated measure (good, service). In contrast with the stated preferences, here we use the real data, ***revealed preferences***.

When applying this method in a project evaluation we examine the impact on the welfare (e.g. change in the prices of realities) when changing selected variables that we can influence (deciding whether to undergo the project). Final monetized value of the project realization (resp. measure introduction) is then calculated as a difference between the original state and the new state after the project realization.

Solution steps: In our case we calculate the difference the value of houses before and after the construction of the anti-flood dam. Specifically we compare the total value of houses using regression model using the original and new parameters. Numerically this is a difference between $p_i = 75 - 200 * (0.4) = -5$, and $p_i = 75 - 200 * (0.05) = 65$, resp. simplified into $\Delta p_i = -200 * (0.05 - 0.4) = 70$ for a single house. Other parameters than flood occurrence probability in this case do not change, therefore we do not need to consider them in the calculation.

Change in the value of a single house in this case is a value increase by 70.

Solution: Total benefit of building an anti-flood dam for the residential area has a value of **5600**.

TCM – Travel Cost Method

(model example)

A municipality owns a lookout tower. Annual maintenance costs including personnel are 300,000 CZK (material, repairs, etc.). Municipality wants to know the value of the social benefits of the tower. Calculate them using TCM. Consider travel costs of 2 CZK/km, travel pace of 30 km/hour, average hourly wage 120 CZK, and an entrance fee of 10 CZK/person. Further parameters are in the table. Assume discontinuous changes in demand for visits (each zone has same demand level), and irrelevant visits from additional zones. Also consider that visits are not cumulative (only the tower).

Zone	Distance	Population	Probability of visit per year
0	1	200	50%
1	10	10,000	5%
2	30	100,000	2%

TCM represents a method of evaluating nonmarket goods based on observing the market behavior of subject (***revealed preferences***). It is used mostly when appraising attractive locations (points of interest, POI) or ecosystems, where it is naturally difficult to set a direct market monetary value.

This method assumes that the value of evaluated goods is equal to the loss (costs), that subjects are willing to accept by travelling to the destination with the POI, specifically the costs for travelling (fuel, public transportation), value of the lost time due to the travelling, and the entrance fees, if there are any. These costs are different for each of the considered zones based on the distance.

Initially we identify the demand zones in the form of amounts of visits and related total costs per visit for each considered zone. From these zones we derive a pseudo-demand curve for visiting considered POIs. The value of POI is then represented by the area under the pseudo-demand curve.

Disadvantage of this method is, for instance, the necessity of sufficient data, as well as inaccuracies due to the combined visits to the multiple POIs (costs are then shared between the visits), or the perceived benefits from the travelling itself.

Solution steps: We have data about three zones, we do not consider more. We also do not consider standard demand curve shape, but only three homogenous groups of visitors based on the zones. We calculate costs per visit for each zone: 0) 100*(2CZK*2*1km+120CZK*2*1km/30+10CZK); 1) 500*(2CZK*2*10km+120CZK*2*10km/30+10CZK); 2) 2000*(2CZK*2*30km+120CZK*2*30km/30 +10CZK).

Solution: Benefits of the lookout tower are 2200 CZK + 65000 CZK + 740000 CZK = **807 200 CZK/year**.

Dominated and non-dominated variants (dominance analysis)

(model example)

Local police department wants to purchase new cars. Following table contains data about several considered models. Decide which models are dominated, determine basal and ideal variants and full solution set (the models that should be considered for the further choice).

Model	Acceleration	Top speed	Fuel consum.	Trunk size	Price
Forman	17	141	8.1	450	220
Felicia combi	16	148	7.6	450	250
Lada 1500	15	153	7.6	480	210
Trabant	30	110	8.1	380	180

With increasing amount of available options and evaluated criteria in a multicriteria evaluation it can become very difficult to stay oriented. One way to simplify it is to reduce the available set of options, let's call it a **full solution**, in which we do not consider further those options that are not relevant (dominated ones) – the options that practically cannot be chosen over some other available option.

Irrelevant variant is in this case the one to which there exists at least one other option that is not worse in any of the considered criteria while being better in at least one criterion. Such variant is then considered as a **dominated** variant, and is being dominated by all other variants that fulfill the condition of being better in at least one criterion while not being worse in any other.

Available variants in most cases do not dominate each other, meaning that one is better in some criteria, while worse in other. Then they are considered to be mutually non-dominated.

Sometimes it might help to determine theoretically worst and best variant. The worst variant is the one with the worst available values from the set, it is called a **basal** variant (**B**), and contains basal values. On contrary, with the best available values from the set we get the **ideal** variant (**I**) with the ideal values of criteria from the evaluated set of options.

Solution: **Basal variant** has an acceleration of 30, top speed of 110, fuel consumption 8.1, trunk size 380 and costs 250. **Ideal variant** has an acceleration of 15, top speed of 153, fuel consumption 7.6, trunk size 480 and costs 180. Forman and Felicia are dominated (by Lada), Lada and Trabant are not dominated. For further evaluation we would consider Lada and Trabant (**full solution**).

*In case of more complex problems you can use available tools, like SANNA from PSE.

Transformation of minimizing criteria to maximizing ones

(model example)

Police department wants again to purchase new cars. Following table contains data about considered models. Some of the criteria are minimizing. Transform all such to maximizing criteria.

Model	Acceleration	Top speed	Fuel consum.	Trunk size	Price
Forman	17	141	8.1	450	220
Felicia combi	16	148	7.6	450	250
Lada 1500	15	153	7.6	480	210
Trabant	30	110	8.1	380	180

In practice of multicriteria evaluation we often encounter situations where some parameters are better if maximized (like output level), while other minimized (like price). Transformation to the one type can reduce possibility of making a mistake due to such difference and can be also useful later.

We can transform the values of minimizing criteria to maximizing using the following transformation:

$$y_{ij}(max) = B(min) - y_{ij}(min)$$

Where *y(max)* means transformed value from a min criterion to a max criterion, *B(min)* means basal value of given min criterion (in such case the highest value of such criterion), and *y(min)* means original value of min criterion.

*if we have a fixed available interval of values, like using grades 1-5, we use 5 (worst) as a basal value independently from the fact that none of the evaluated variants actually got grade 5 in the criterion.

Solution:

Model	**T-Acceleration**	Top speed	**T-Fuel consum.**	Trunk size	**T-Price**
Forman	**13**	141	**0**	450	**30**
Felicia combi	**14**	148	**0,5**	450	**0**
Lada 1500	**15**	153	**0,5**	480	**40**
Trabant	**0**	110	**0**	380	**70**

WSA – weight sum approach

(model example)

Police department still wants to purchase new cars. Following table contains data about considered models. Use WSA to select the best variant (weights of criteria are 30%, 10%, 30%, 30%).

Model	Acceleration	Top speed	Fuel consum.	Price
Octavia	9	200	6.8	410
Rapid	10	190	6.5	360
Fabia	11	180	6.3	330

WSA means that individual evaluated criteria are assigned with certain weights that represent their level of importance in the final evaluation. Significantly worse parameters in one less important criterion therefore do not mean that the variant will automatically not be selected, as long as it has competitive parameters in other more important criteria. For correct use os WSA we need to transform original values to the appropriate form. We transform values to the same type and then normalize them, so we can use type-comparable values. Final score for each variant is then a scalar product of normalized values of criteria and their weights.

Solution steps: Transformation formula for normalizing the maximizing criteria:

$$y_{ij}(normalized) = \frac{y_{ij}max - B_j}{I_j - B_j}$$

Resp. transformation formula for normalizing the minimizing criteria:

$$y_{ij}(normalized) = \frac{B_j - y_{ij}min}{B_j - I_j}$$

Using these transformations we get normalized matrix of values between 0 and 1 and then we multiply the values with the weights. Normalized matrix looks like this:

Model	N-Acceleration	N-Top speed	N-Fuel consum.	N-Price
Octavia	1	1	0	0
Rapid	0.5	0.5	0.6	0.625
Fabia	0	0	1	1

Solution: Octavia gets 40%, Rapid 56.75% and Fabia 60%. Best model is **Fabia**.

Scales and ranges – assigning points within a scale

(model example)

Region is deciding between projects of several water plants on different rivers. Three projects were submitted, with criteria of building costs, running costs, output and safety level (range 0-10). You as an expert should evaluate criteria of projects on a scale of 0-100 and choose the best.

River	Building costs	Running costs	Output	Safety (0-10)
Bobrava	170	73	67	9
Ponávka	132	38	45	7
Želetavka	99	41	33	5

Method of scales requires ability of the quantitative evaluation of given parameters within evaluated criteria, meaning that the evaluator assigns values based on his expert opinion. Unlike with strictly mathematical methods this allows the consideration of other factors as well, like the experience of the evaluator, preferences or other aspects. The better the value of a parameter, the more points should be assigned. Thanks to that we do not have any more issues with min/max criteria and their transformations or normalization.

On the other hand, the disadvantage of this method is the dependency on the subjective evaluation of parameters. For reducing the risk of making an incorrect decision, multiple independent expert evaluations are often used. Final score is then a result of sum of individual evaluations, or their weighted sum, if opinions of different experts are weighted differently.

Analogically we can assign different weights to different criteria based on their importance. Assigned points for each parameter are then weighted and summed together afterwards. Such method is then called scoring method.

Example of a possible solution, a subjective assignation of points to the parameters (0-100 scale):

River	Building costs	Running costs	Output	Safety	Sum
Bobrava	40	50	100	90	280
Ponávka	61	100	77	70	**308**
Želetavka	80	95	50	50	275

Lexicographic method

(model example)

Region wants to build a bridge over the river Jevišovka. Estimated parameters of variants are in the following table. For the decision use lexicographic method with criteria preferences C→B→D→A, and requirements for the criteria being A ≥ 440, B ≥ 6, C ≥ 7 a D ≤ 50.

Bridge	A) Capacity	B) Looks	C) Place	D) Costs
of victory (1)	406	7	8	50
Red (2)	444	8	8	39
of friendship (3)	505	4	9	44
of labor (4)	568	5	10	32
of proletariat (5)	541	8	6	52

Lexicographic method evaluates available variants sequentially based on the importance of individual criteria and limiting requirements. In the first step we take the most important criterion and discard the variants that do not meet the requirements for this criterion. In the second step we continue analogically with the reduced set of remaining variants. The evaluation process is finished when only one variant remains, and this variant is chosen as the best. It is not necessary to evaluate options using all available criteria, if we get to the point when there is only one option left. In case we are left with multiple variants even after the last criterion, we need to use some additional method for choosing a compromise variant (for instance selecting one with the best parameter in the last evaluated criterion).

The disadvantage of this method is that we are practically taking into account only the last evaluated criterion and do not consider previous as long as the minimal requirements were met. Moreover, the result can be notably biased by the criteria preferences selection. In cases with no non-dominated variants, with “appropriate” preference order and minimal requirements it is sometimes possible to secure any of the variants as the winning one.

Solution steps: In the first step we evaluate according to the C criterion – we discard variant 5, remaining set is {1, 2, 3, 4}. In the second step we evaluate according to the B criterion – we discard variants 3 and 4, remaining set is {1, 2}. In the third step we evaluate according to the D criterion – we do not discard any variants, the set remains {1, 2}. In the fourth step we evaluate according to the A criterion – we discard variant 1, remaining set is {2}.

Solution: Based on the lexicographic method we choose the **Red bridge (2).**

Public procurement evaluation

(model example)

You are a member of the evaluation committee choosing the company constructing a migration corridor over the highway. Assume that all applicants met all requirements like delivering the complete documentation, references, qualifications, etc. and submitted bids on time. Evaluate bids.

Bid	Price (mil. CZK)	Delivery (months)	Guarantee (months)	Technical aspects (0-10 scale)
A	15.2	25	43	9
B	16	23	69	6
C	18.9	19	86	7
weight	50%	20%	15%	15%

Process of public procurement evaluation is close to the WSA (weighted sum approach). Values of each variant are transformed into the normalized form and multiplied by given weights. The difference is in the different transformation formula. Normalized values in this case do not necessarily acquire value 0 in case of the worst parameter – they acquire a proportional value compared to the best parameter (maximizing criterion), or value calculated as an inverse proportional (minimizing criterion). Normalized values are finally multiplied by weights and summed.

In case of maximizing criteria we calculate transformed values as a proportion of the best value. In case of minimizing criteria we calculate the value as an inverse proportional (using flipped fractions).

Solution steps: Normalized matrix of parameters (below) is multiplied by given weights:

Bid	Criterion 1	Criterion 2	Criterion 3	Criterion 4
A	1.00	0.76	0.50	0.90
B	0.95	0.83	0.80	0.60
C	0.80	1.00	1.00	0.70

Solution: Bid A acquired score of **86.20%**, bid B acquired **85.06%**, and bid C acquired **85.71%**. Bid A is victorious and this applicant should be asked to deliver the realization of the public procurement.