Application of discounted cash flows

Course Information

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- Application of discounted CF
- Statistical concepts and market returns
- Probability concepts
- Common Probability distributions

1. Introduction

- **Capital budgeting** is the allocation of funds to long-lived capital projects.
- A **capital project** is a long-term investment (in tangible assets).

Conventional and nonconventional cash flows

Conventional Cash Flow (CF) Patterns

Today	1	2	3	4	5
–CF	+CF	+CF	+CF	+CF	+CF
–CF	–CF	+CF	+CF	+CF	+CF
–CF		+CF	+CF	+CF	+CF

Conventional and nonconventional cash flows

Nonconventional Cash Flow Patterns

Today	1	2	3	4	5
–CF	+CF	+CF	+CF	+CF	–CF
–CF	+CF	–CF	+CF	+CF	+CF
–CF	–CF	+CF	+CF	+CF	–CF

Independent vs. mutually exclusive projects

- When evaluating more than one project at a time, it is important to identify whether the projects are independent or mutually exclusive
  - This makes a difference when selecting the tools to evaluate the projects.
- **Independent projects** are projects in which the acceptance of one project does not preclude the acceptance of the other(s).
- **Mutually exclusive projects** are projects in which the acceptance of one project precludes the acceptance of another or others.

Investment decision criteria

- Net Present Value (NPV)
- Internal Rate of Return (IRR)
- Payback Period
- Discounted Payback Period
- Average Accounting Rate of Return (AAR)
- Profitability Index (PI)

Net present Value

The **net present value** is the present value of all incremental cash flows, discounted to the present, less the initial outlay:

$$\text{NPV} = \sum_{t=1}^{n} \frac{\text{CF}_t}{(1+r)^t} - \text{Outlay} \qquad (2\text{-}1)$$

Or, reflecting the outlay as CF_0,

$$\text{NPV} = \sum_{t=0}^{n} \frac{\text{CF}_t}{(1+r)^t} \qquad (2\text{-}2)$$

where

CF_t	= After-tax cash flow at time t
r	= Required rate of return for the investment
Outlay	= Investment cash flow at time zero

If NPV > 0:

- Invest: Capital project adds value

If NPV < 0:

- Do not invest: Capital project destroys value

Example: NPV

Consider the Hoofdstad Project, which requires an investment of $1 billion initially, with subsequent cash flows of $200 million, $300 million, $400 million, and $500 million. We can characterize the project with the following end-of-year cash flows:

Period	Cash Flow (millions)
0	–$1,000
1	200
2	300
3	400
4	500

What is the net present value of the Hoofdstad Project if the required rate of return of this project is 5%?

Example: NPV

Time Line

0	1	2	3	4
–$1,000	$200	$300	$400	$500

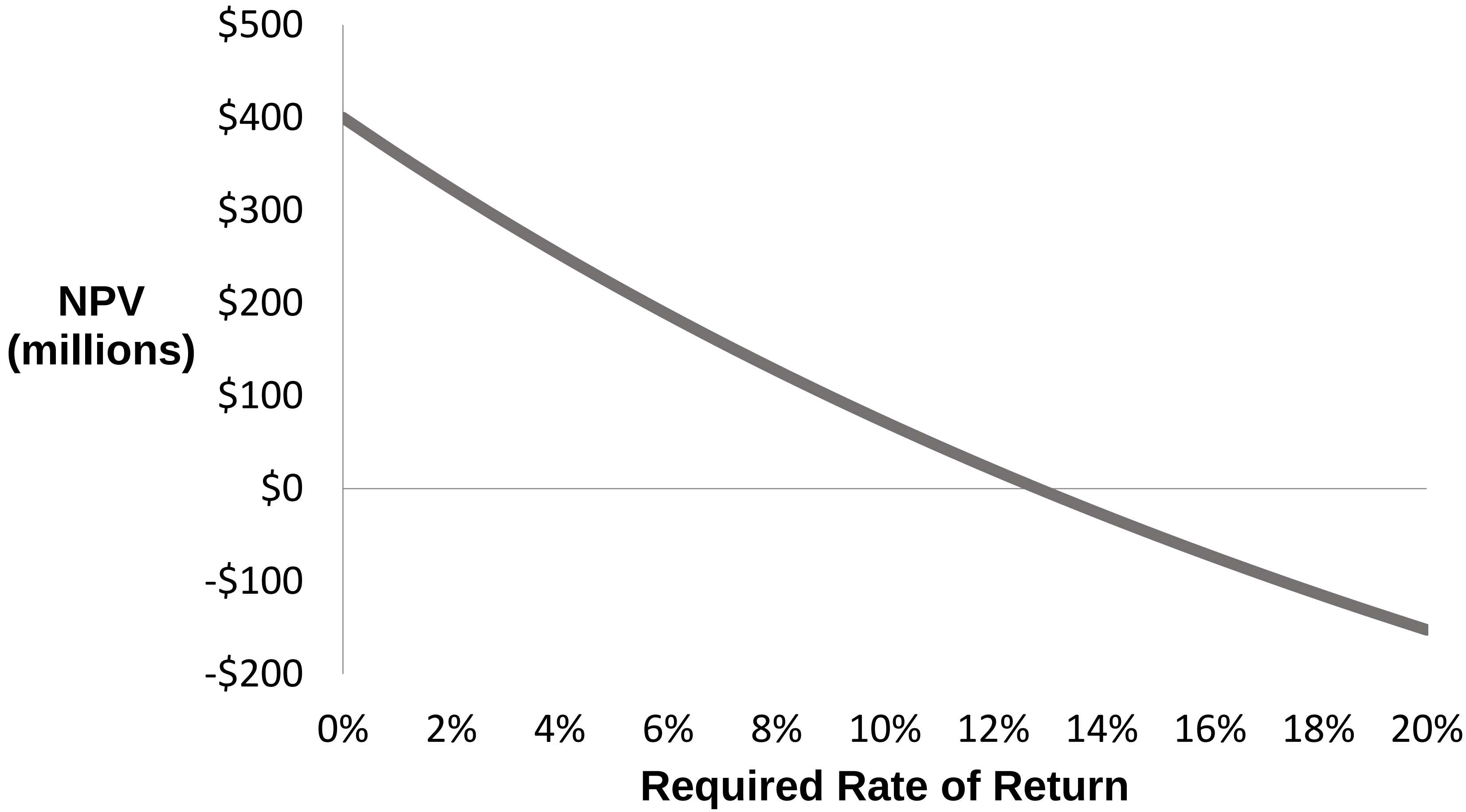
Solving for the NPV:

$$NPV = -\$1{,}000 + \frac{\$200}{(1+0.05)^1} + \frac{\$300}{(1+0.05)^2} + \frac{\$400}{(1+0.05)^3} + \frac{\$500}{(1+0.05)^4}$$

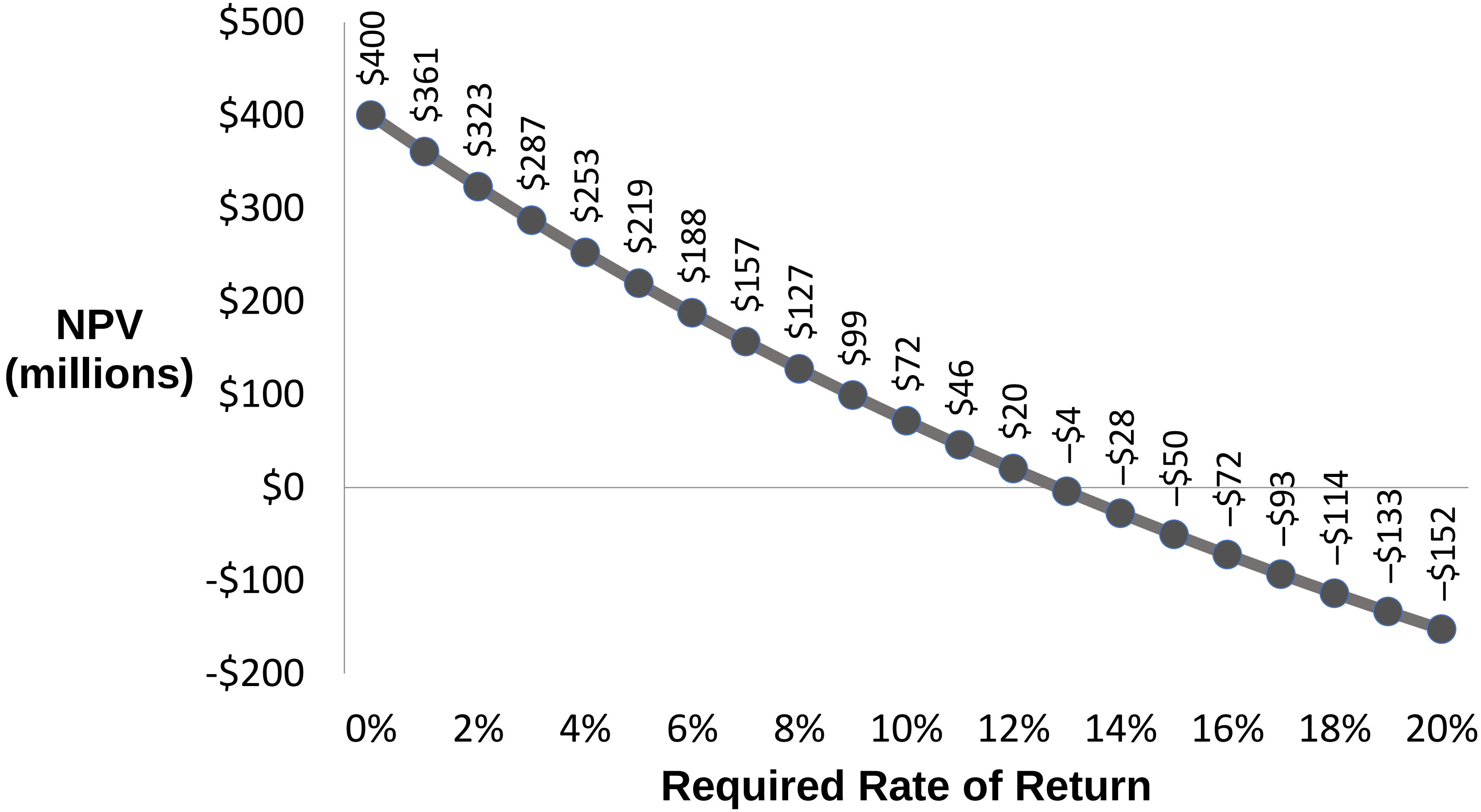
NPV = −$1,000 + $190.48 + $272.11 + $345.54 + $411.35

NPV = $219.47 million

NPV Profile: Hoofdstad Capital project



NPV Profile: Hoofdstad Capital project



Internal rate of return

The **internal rate of return** is the rate of return on a project.

- The internal rate of return is the rate of return that results in NPV = 0.

$$\sum_{t=1}^{n} \frac{\text{CF}_t}{(1 + \text{IRR})^t} - \text{Outlay} = 0 \qquad (2\text{-}3)$$

Or, reflecting the outlay as CF_0,

$$\sum_{t=0}^{n} \frac{\text{CF}_t}{(1 + \text{IRR})^t} = 0 \qquad (2\text{-}4)$$

If IRR > *r (required rate of return):*

- Invest: Capital project adds value

If IRR < *r:*

- Do not invest: Capital project destroys value

Example: IRR

Consider the Hoofdstad Project that we used to demonstrate the NPV calculation:

Period	Cash Flow (millions)
0	–$1,000
1	200
2	300
3	400
4	500

The IRR is the rate that solves the following:

$$\$0 = -\$1{,}000 + \frac{\$200}{(1 + \text{IRR})^1} + \frac{\$300}{(1 + \text{IRR})^2} + \frac{\$400}{(1 + \text{IRR})^3} + \frac{\$500}{(1 + \text{IRR})^4}$$

A note on solving for IRR

- The IRR is the rate that causes the NPV to be equal to zero.
- The problem is that we cannot solve directly for IRR, but rather must either iterate (trying different values of IRR until the NPV is zero) or use a financial calculator or spreadsheet program to solve for IRR.
- In this example, IRR = 12.826%:

$$\$0 = -\$1{,}000 + \frac{\$200}{(1+0.12826)^1} + \frac{\$300}{(1+0.12826)^2} + \frac{\$400}{(1+0.12826)^3} + \frac{\$500}{(1+0.12826)^4}$$

- Or linear interpolation

Example: IRR

- Initial investment 1.000.000.000
- Perpetual CFs 100.000.000
- Using IRR accept the project or not if your required rate of return is 8% p.a.

- Using IRR accept the project or not if your required rate of return is 15% p.a.

Payback Period

- The **payback period** is the length of time it takes to recover the initial cash outlay of a project from future incremental cash flows.
- In the Hoofdstad Project example, the payback occurs in the last year, Year 4:

Period	Cash Flow (millions)	Accumulated Cash flows
0	–$1,000	–$1,000
1	200	–$800
2	300	–$500
3	400	–$100
4	500	+400

Payback Period: Ignoring Cash Flows

For example, the payback period for both Project X and Project Y is three years, even through Project X provides more value through its Year 4 cash flow:

Year	Project X Cash Flows	Project Y Cash Flows
0	–£100	–£100
1	£20	£20
2	£50	£50
3	£45	£45
4	£60	£0

Discounted Payback Period

- The **discounted payback period** is the length of time it takes for the cumulative discounted cash flows to equal the initial outlay.
  - In other words, it is the length of time for the project to reach NPV = 0.

Example: Discounted Payback Period

Consider the example of Projects X and Y. Both projects have a discounted payback period close to three years. Project X actually adds more value but is not distinguished from Project Y using this approach.

	Cash Flows		Discounted Cash Flows		Accumulated Discounted Cash Flows	
Year	Project X	Project Y	Project X	Project Y	Project X	Project Y
0	–£100.00	–£100.00	–£100.00	–£100.00	–£100.00	–£100.00
1	20.00	20.00	19.05	19.05	–80.95	–80.95
2	50.00	50.00	45.35	45.35	–35.60	–35.60
3	45.00	45.00	38.87	38.87	3.27	3.27
4	60.00	0.00	49.36	0.00	52.63	3.27

Profitability index

The **profitability index** (**PI**) is the ratio of the present value of future cash flows to the initial outlay:

$$\text{PI} = \frac{\text{Present value of future cash flows}}{\text{Initial investment}} = 1 + \frac{\text{NPV}}{\text{Initial investment}} \quad \text{(2-5)}$$

If PI > 1.0:

- Invest
- Capital project adds value

If PI < 0:

- Do not invest
- Capital project destroys value

Example: PI

In the Hoofdstad Project, with a required rate of return of 5%,

Period	Cash Flow (millions)
0	-$1,000
1	200
2	300
3	400
4	500

the present value of the future cash flows is $1,219.47. Therefore, the PI is:

$$PI = \frac{\$1,219.47}{\$1,000.00} = 1.219$$

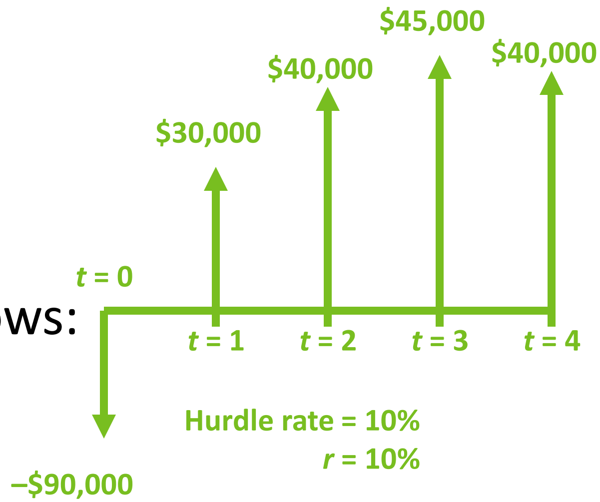
NPV vs. IRR

- If projects are **independent**, the decision to invest in one does not preclude investment in the other.
  - NPV and IRR will yield the same investment decisions.
- Projects are **mutually exclusive** if the selection of one project precludes the selection of another project → project selection is determined by rank.
  - NPV and IRR may give different ranks when
    - The projects have different scales (sizes)
    - The timing of the cash flows differs
  - If projects have different ranks → use NPV.

NPV vs. IRR

Focus On: Calculations

- Consider Project C with the following cash flows:
  - The NPV is $28,600.26.
  - The IRR is 24.42%.



	Project A	Project B	Project C
NPV	$29,872.52	$27,783.12	$28,600.26
IRR	21.84%	25.62%	24.42%
Decision	Accept	Accept	Accept

- If the projects are independent, you accept all three.
- If the projects are mutually exclusive, you accept Project A even though it has the smallest IRR.
- If Projects B and C are mutually exclusive, you accept Project C.

IRR Challenges

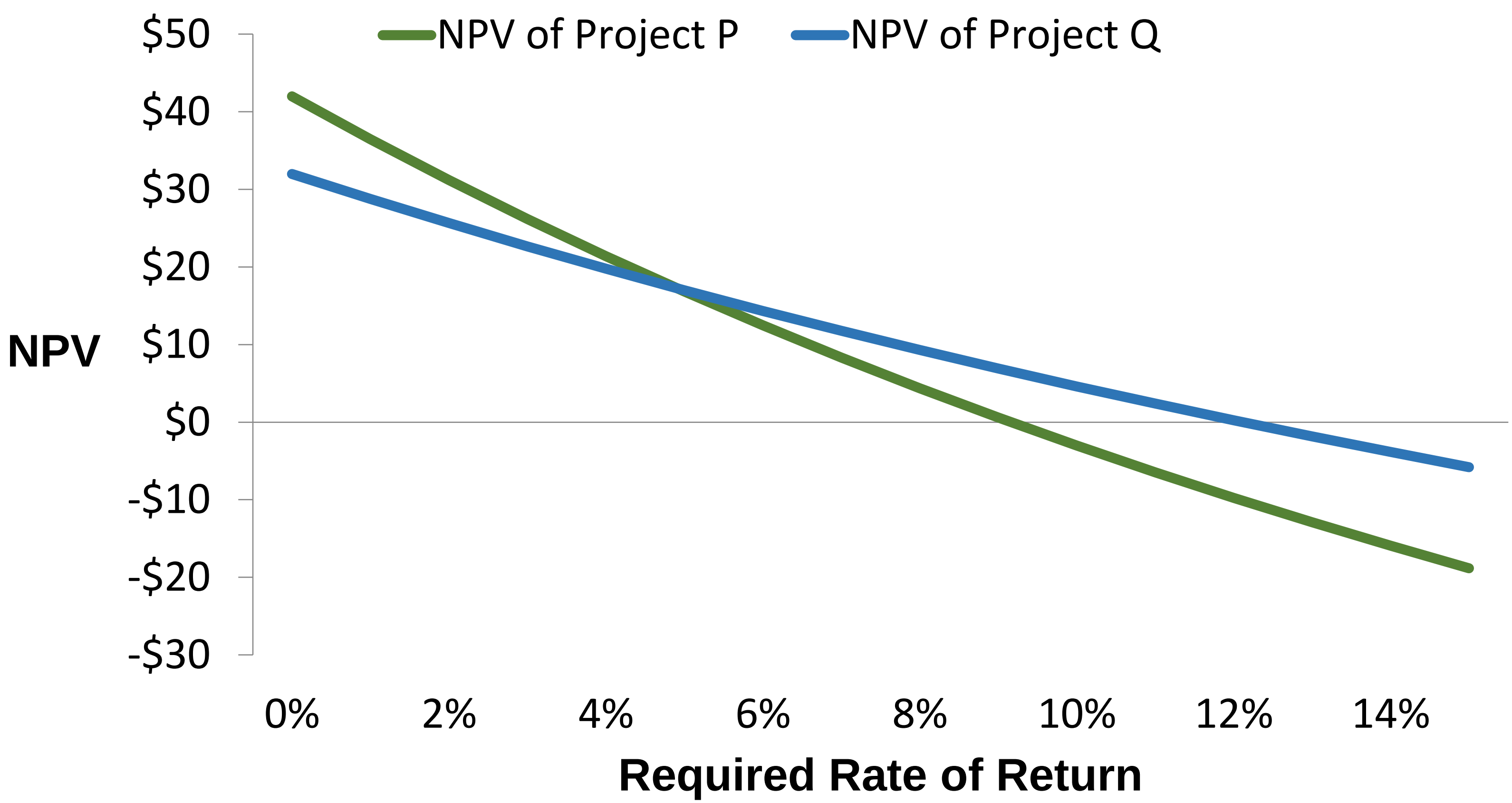
IRR is a very appealing measure because it is intuitive; we all understand (or think we do) rates of return.

- Unfortunately, IRR has several shortcomings.
  - We will only realize the IRR as calculated if we
    a) can reinvest all the project cash flows at that IRR, and
    b) hold the investment to maturity.
  - IRR and NPV *can* give different rankings when
    - The scale of the projects being compared is different
    - The timing of the cash flows is different
- **Conclusion**: NPV should be preferred to IRR.

Decision at various required rates of return

	Project P	Project Q	Decision
NPV @ 0%	$42	$32	Accept P, Reject Q
NPV @ 4%	$21	$20	Accept P, Reject Q
NPV @ 6%	$12	$14	Reject P, Accept Q
NPV @ 10%	–$3	$5	Reject P, Accept Q
NPV @ 14%	–$16	–$4	Reject P, Reject Q
IRR	9.16%	12.11%	

NPV Profiles: Project P and Project Q



Example NPV and IRR

5. Westcott–Smith is a privately held investment management company. Two other investment counseling companies, which want to be acquired, have contacted Westcott–Smith about purchasing their business. Company A's price is £2 million. Company B's price is £3 million. After analysis, Westcott–Smith estimates that Company A's profitability is consistent with a perpetuity of £300,000 a year. Company B's prospects are consistent with a perpetuity of £435,000 a year. Westcott–Smith has a budget that limits acquisitions to a maximum purchase cost of £4 million. Its opportunity cost of capital relative to undertaking either project is 12 percent.

A. Determine which company or companies (if any) Westcott–Smith should purchase according to the NPV rule.

B. Determine which company or companies (if any) Westcott–Smith should purchase according to the IRR rule.

C. State which company or companies (if any) Westcott–Smith should purchase. Justify your answer.

The multiple IRR problem

- If cash flows change sign more than once during the life of the project, there may be more than one rate that can force the present value of the cash flows to be equal to zero.
  - This scenario is called the "multiple IRR problem."
  - In other words, there is no unique IRR if the cash flows are nonconventional.

Example: The multiple IRR problem

Consider the fluctuating capital project with the following end of year cash flows, in millions:

Year	Cash Flow
0	–€550
1	€490
2	€490
3	€490
4	–€940

What is the IRR of this project?

Example: The Multiple IRR Problem