

# Financial Mathematic Seminar

Luděk Benada, Dagmar Linnertova

Department of Finance - 402, [benada.esf@gmail.com](mailto:benada.esf@gmail.com)

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*The Study materials prepared by*

**Mikhail Dmitrievich Balyka**

## Exercise 1

You put **333\$** into a bank account 4 times per year (**at the beginning** of each quarter) for 25 years at **4,5% p.a. Interest** is calculated **monthly**.

!!BUT, to avoid inflation, every quarter you deposit 0,5% more (i.e  $\text{annuity}_2/\text{annuity}_1=1,005$ ).

$S_{25\text{yrs}} - ?$

## Exercise 1 cont'd

Inputs:

$a=333$       ahead       $PP=4t/y$        $IP=1m$

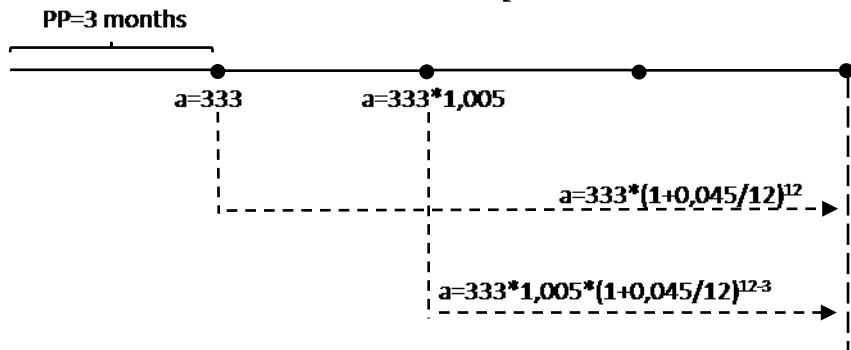
$T=25yrs$        $r=4,5\%p.a.$        $a_2=a_1*1,005$

$S_{25yrs} - ?$

$$PP > IP \rightarrow S = a \times q \times \frac{q^n - 1}{q - 1}$$

## Exercise 1 cont'd

$$S = a \times q \times \frac{q^n - 1}{q - 1}$$



## Exercise 1 cont'd

$$S = a \times q \times \frac{q^n - 1}{q - 1}$$

$$q = \frac{333 \cdot (1 + 0,045/12)^{12}}{333 \cdot 1,005 \cdot (1 + 0,045/12)^{12-3}} = \frac{(1 + \frac{0,045}{12})^3}{1,005}$$

$$S = 333 \times \frac{(1 + \frac{0,045}{12})^3}{1,005} \times \frac{[\frac{(1 + \frac{0,045}{12})^3}{1,005}]^{4 \times 25} - 1}{\frac{(1 + \frac{0,045}{12})^3}{1,005} - 1} = 46382,69$$

## Exercise 2

You put **200 000\$** into a bank account at **2% p.q.** You pay **5 000\$** to open this account and also **at the end of each month** you pay a bank fee **200\$**. **Interest** is calculated **quarterly**.

Find the average annual return on this investment

## Exercise 2 cont'd

### Inputs:

PV=200 000      Initial costs=5000      Monthly costs (i.e. annuities)=200  
PP=1month      IP=3months      r=2%p.q. T=5yrs      After payment

Here we just need to respect time value of money and to calculate FVs of all cash flows:

$$FV = 200000 \times (1 + 0,02)^{20} - 5000 \times (1 + 0,02)^{20} - 200 \times 3 \times \left(1 + \frac{4}{6} \times 0,02\right) \times \frac{(1+0,02)^{20}-1}{0,02} = 275084,1$$

## Exercise 2 cont'd

$$FV = 200000 \times (1 + 0,02)^{20} - 5000 \times (1 + 0,02)^{20} - 200 \times 3 \times \left(1 + \frac{4}{6} \times 0,02\right) \times \frac{(1+0,02)^{20} - 1}{0,02} = 275084,1$$

$$FV = PV \times (1 + r)^n$$

$$r = \left(\frac{FV}{PV}\right)^{\frac{1}{n}} - 1$$

$$r = \left(\frac{275084,1}{200000}\right)^{\frac{1}{5}} - 1 = 6,58\%$$



## Exercise 3

You put **15 000 \$** into a bank account **at the end** of each **quarter** at **3,7% p.a. for 10 years**. **Interest** is calculated **2 times per year**. **Tax rate is 15%** and it's calculated at the end of investment's period

$S_{\text{tax}} - ?$

## Exercise 3 cont'd

Inputs:

$a=15\ 000$      $PP=3m(\text{after})$      $IP=6m$      $r=3,7\% \text{ p.a.}$

$T=10 \text{ yrs}$      $\text{tax}=15\%$      $TP=10\text{yrs}$

$S_{\text{tax}} - ?$

$$S = a \times m \times \left(1 + \frac{m-1}{2m} \times r\right) \times \frac{(1+r)^n}{r}$$

To calculate the after tax's amount of money we need to deduct from our FV a sum of annuities made in one TP

## Exercise 3 cont'd

$$S = a \times m \times \left(1 + \frac{m-1}{2m} \times r\right) \times \frac{(1+r)^n - 1}{r}$$

To calculate the after tax's amount of money we need to deduct from our FV a sum of annuities made in one TP

we deduct our annu-s  
(# of 'a' in one TP)

$$S = 15000 \times \left\{ 3 \times \left(1 + \frac{3-1}{2 \times 3} \times \frac{0,037}{2}\right) \times \frac{\left(1 + \frac{0,037}{2}\right)^{20} - 1}{\frac{0,037}{2}} - 40 \right\} \times 0,85 + 40$$

mult-ing by the % retained after tax and adding back our 'a' we will get 'S' after tax

## Exercise 4

$a=40\,000$        $PP=3m$  ahead       $IP=1m$        $TP=1m$   
 $r=3,9\%$  p.a.       $tax=15\%$        $T=10$ years

$S_{tax} - ?$

$$PP > IP \rightarrow S = a \times q \times \frac{q^n - 1}{q - 1}$$

Now we just need to define  $q$

## Exercise 4 cont'd

$$S = a \times q \times \frac{q^n - 1}{q - 1}$$

As we know tax is calculated from interest

How we can find it:

$$I = PV \times [(1 + r)^n - 1]$$

But in our case  $n=1$ , so we can rearrange the formula and find our interest after tax:

$$I = PV \times r \times (1 - tax)$$

Since, our  $q = (1 + \frac{0,039}{12} \times 0,85)^3$

## Exercise 5 cont'd

$$S = a \times q \times \frac{q^n - 1}{q - 1}$$

$$S = 40000 \times \left(1 + \frac{0,039}{12} \times 0,85\right)^3 \times \frac{[(1 + \frac{0,039}{12} \times 0,85)^3]^{4 \times 10} - 1}{(1 + \frac{0,039}{12} \times 0,85)^3 - 1}$$

## Exercise 5

**a=40 000**

**PP=3m ahead**

**IP=1m**

**TP=1year**

**r=3,9% p.a.**

**tax=15%**

**T=10 years**

**$S_{\text{tax}}$  -?**

$$\text{PP} > \text{IP} \rightarrow S = a \times q \times \frac{q^n - 1}{q - 1}$$

## Exercise 5 cont'd

$$S = a \times q \times \frac{q^n - 1}{q - 1}$$

$$S = 40000 \times \left[ \left( \left( 1 + \frac{0,039}{12} \right)^3 \times \frac{\left( 1 + \frac{0,039}{12} \right)^{3 \times 4} - 1}{\left( 1 + \frac{0,039}{12} \right)^3 - 1} - 4 \right) \times 0,85 + 4 \right] \\ \times \frac{\left( \left( 1 + \frac{0,039}{12} \right)^{12} - 1 \right) \times 0,85 + 1)^{10} - 1}{\left( 1 + \frac{0,039}{12} \right)^{12} - 1) \times 0,85}$$



## Exercise 6

$a=500$        $PP=4m$  ahead       $IP=2m$        $TP=1year$   
 $r=4,7\%$  p.a.       $tax=10\%$        $T=7$  years

$S_{tax}$  for continuous interest -?

$$q = e^{ft}$$

$f = \ln\left(1 + \frac{0,047}{6}\right)^2 = 0,0156$  - interest intensity for 4 months

## Exercise 6

a=500      PP=4m ahead      IP=2m      TP=1year  
r=4,7% p.a.      tax=10%      T=7 years

$$q = e^{ft}$$

$f = \ln\left(1 + \frac{0,047}{6}\right)^2 = 0,0156$  - interest intensity for 4 months

$$S = 500 \times \left[ \left( \frac{e^{0,0156 \times 3} - 1}{e^{0,0156} - 1} - 3 \right) \times 0,9 + 3 \right] \times \frac{((e^{0,0156 \times 3} - 1) \times 0,9 + 1)^7}{(e^{0,0156 \times 3} - 1) \times 0,9}$$