Seminar 9:

Multi-criteria programming

Problem 1: Use graphical method to find partial optimal solutions of the multi-criteria linear program:

$z_1 = 2x_1 - x_2 \to \max,$

$z_2 = x_2 \to \max,$

on the feasible set X given by the constraints

$2x_1 + x_2 \leq 18,$

$x_1 + 2x_2 \leq 12,$

$-x_1 + x_2 \leq 3,$

$x_1,\ x_2 \geq 0$

Consider aggregated objective function $z = v_1 \cdot z_1 + v_2 \cdot z_2$ using weights

$$v_1,\ v_2 \geq 0,\ v_1 + v_2 = 1.$$

Describe the solution of $z \to max_{x \in X}$ with respect to the values of parameters $v_1,\ v_2$.

Problem 2:

Consider multi-criteria linear program:

$z_1 = 3x_1 + 2x_2 + 10 \to \max,$

$z_2 = x_1 - 4x_2 + 10 \to \max$

subject to

$4x_1 + 2x_2 \leq 20$

$x_1 + 3x_2 \leq 12$

$x_j \geq 0;\ j = 1, 2$

Show that there is no ideal solution. Find a compromise solution using minimal component method. Do all the calculations by hand in the simplex table.

Problem 3:

Consider the nutritional problem of planning a daily diet for 100 people, with

9 kinds of basic foods available. Composition of foods in terms of important nutritional components and their prices (all converted to 100g of food) is in the table:

	energy [kJ]	proteins [g]	Fe [mg]	vit. A [units]	vit. C [mg]	chol. [mg]	price [CZK]
pork meat	1200	18.4	3.1	20	0	83	12
butter	3000	0.6	0.2	2500	0	120	11.2
bread	1160	7.2	0.8	0	0	1	1.5
potatoes	300	1.6	0.6	40	10	0	1.2
apples	240	0	0.5	60	2	0	1.5
eidam cheese	1260	31.2	0.6	1100	0	71	10.6
chicken	650	20.2	1.5	0	0	57	6
yoghurt	450	7	0.2	260	0	11	4.5
strawberries	150	0	0.8	60	60	0	12

Nutritional experts have determined that the daily portion of adult nutrition should contain at least 80 g of protein, 15 mg of Fe, 6000 units of vitamin A and 200 mg of vitamin C. To plan a daily catering for 100 people, we need to determine an optimal composition of the diet while respecting the nutrition experts' recommendations. We try to obtain solution with the highest energy value, the lowest cholesterol content and the lowest price. We have a maximum of 40 kg of each food.

a) Find partial optima.

b) Convert criteria "cholesterol and energy" to constraints sequentially allowing for 10 % deviation from ideal solution.

c) Convert criteria "cholesterol and energy" to constraints simultaneously allowing for 10 % deviation from ideal solution