Valuation of Contingent Claims

Pricing and Valuation Methodology

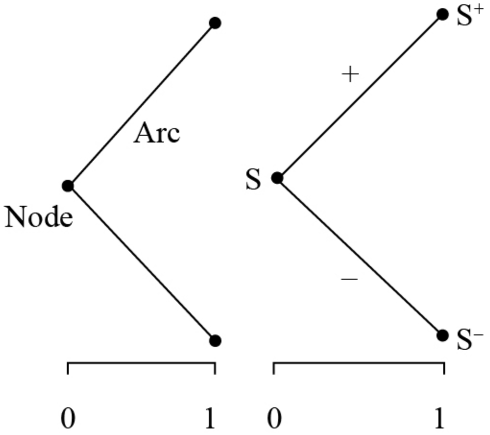
Our approaches to pricing and valuation are based on the assumption that prices adjust to not allow arbitrage profits.

The arbitrageur abides by two fundamental rules:

- Rule #1: Do not use your own money.
- Rule #2: Do not take any price risk.

Law of one price: two investments with the same or equivalent future cash flows, regardless of what will happen in the future, will sell for the same current price.

One-Period Binomial Option Valuation Model



At time 1, there are only two possible outcomes and two resulting values of the underlying, S^+ (up occurs) and S^- (down occurs). The return implied by the up jump (the up factor) $u = S^+/S$. The down factor is $d = S^-/S$.

Recall, the value of the **call option** at expiration is the greater of zero or the value of the underlying minus the exercise price (X). If the underlying asset value jumps up at expiration to a value S^+, the call will have a value of:

$$c^+ = Max(0, S^+ - X)$$

The down jump value of the call at expiration will be:

$$c^- = Max(0, S^- - X)$$

The value of the **put option** at expiration is the greater of zero or the value of the exercise price minus the underlying.

If the underlying asset value jumps up at expiration to a value S^+, the put will have a value of:

$$p^+ = Max(0, X - S^+)$$

The down jump value of the put at expiration will be:

$$p^- = Max(0, X - S^-)$$

One-Period Binomial Option Valuation I

A hedge portfolio can be created by taking a long position in h units of the underlying asset and a short position in the call so that the initial value of the portfolio is given by:

$$V = hS - c$$

If the asset price jumps up at time 1, the hedge portfolio will be worth:

$$V^+ = hS^+ - c^+$$

If the asset price jumps down at time 1, the hedge portfolio will be worth:

$$V^- = hS^- - c^-$$

If we set the up and down portfolios equal, we can solve for the hedge ratio:

$$h = \frac{c^+ - c^-}{S^+ - S^-}$$

One-Period Binomial Option Valuation II

Using the idea that a hedged portfolio will return the risk-free rate, we can solve for the initial value of the call or put. The expectations approach computes the option values as the present value of the expected terminal option payoffs:

$$c = \frac{\pi c^{+} + (1-\pi)c^{-}}{1+r}$$

$$p = \frac{\pi p^{+} + (1-\pi)p^{-}}{1+r}$$

where the probability of an up move, π, is given by:

$$\pi = \frac{1+r-d}{u-d}$$

One-Period Binomial Option Valuation Example

Assume an initial stock price of £60 and a risk-free rate of 5%. Assume the asset price can move up 15% or down 10%, so the up jump and down jump factors are u = 1.15 and d = 0.90. Price a European call and put option with X = £60 using a single period binomial model.

For the call:

$$c^+ = Max(0, £69 - £60) = £9 \text{ and } c^- = Max(0, £54 - £60) = £0$$

For the put:

$$p^+ = Max(0, £60 - £69) = £0 \text{ and } p^- = Max(0, £60 - £54) = £6$$

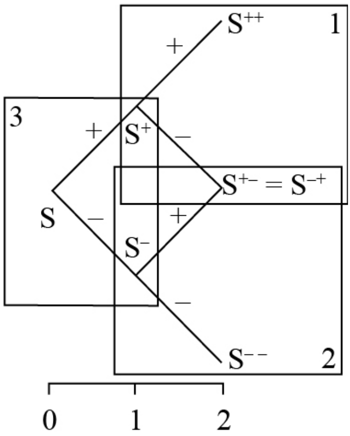
The call and put values are:

$$\pi = \frac{1 + r - d}{u - d} = \frac{1 + 0.05 - 0.90}{1.15 - 0.90} = \frac{0.15}{0.25} = 0.60$$

$$c = \frac{\pi c^+ + (1 - \pi)c^-}{1 + r} = \frac{0.6(£9) + 0.4(£0)}{1.05} = £5.143$$

$$p = \frac{\pi p^+ + (1 - \pi)p^-}{1 + r} = \frac{0.6(£0) + 0.4(£0)}{6} = £2.286$$

Two-Period Binomial Option Valuation Model



At time 2, there are only three possible resulting values of the underlying: S^{++} (up jump occurs twice), S^{+-} (resulting from an up move then a down move, or a down move then an up move), or S^{--} (down move occurs twice).

Two-Period Binomial Values

At time 2, the **call** will have three possible payoffs depending on the number of up and down moves in S, as follows:

$$c^{++} = Max(0, S^{++} - X) = Max(0, u^2 S - X)$$

$$c^{+-} = Max(0, S^{+-} - X) = Max(0, udS - X)$$

$$c^{--} = Max(0, S^{--} - X) = Max(0, d^2 S - X)$$

The **put** will have 3 possible payoffs:

$$p^{++} = Max(0, X - S^{++} = Max(0, X - u^2 S)$$

$$p^{+-} = Max(0, X - S^{+-} = Max(0, X - udS)$$

$$p^{--} = Max(0, X - S^{--} = Max(0, X - d^2 S)$$

Two-Period Binomial Option Valuation

The two-period binomial model can also be represented as the present value of an expectation of future cash flows.

Again is the probability of an up move, so the price of a **European call** option using the two-period binomial model is:

$$c = \frac{\pi^2 c^{++} + 2\pi(1-\pi)c^{+-} + (1-\pi)^2 c^{--}}{(1+r)^2}$$

And the two-period **European put** value is:

$$p = \frac{\pi^2 p^{++} + 2\pi(1-\pi)p^{+-} + (1-\pi)^2 p^{--}}{(1+r)^2}$$

Two-Period Binomial Valuation Example I

Assume an initial stock price of £100 and a risk-free rate of 2% per year. The asset price can jump up 20% or down 20%, so the up jump and down jump factors are $u = 1.20$ and $d = 0.80$. Price a call and put option with two years to maturity and $X = £95$ using a two-period binomial model.

First, we must solve for the probability of an up move.

Based on the RN probability equation, we have:

$$\pi = \frac{FV(1) - d}{u - d} = \frac{1 + r - d}{u - d} = \frac{1 + 0.02 - 0.80}{1.20 - 0.80} = \frac{0.22}{0.4} = 0.55$$

Two-Period Binomial Valuation Example II

Next, we must solve for the call and put payoffs at the terminal nodes:

$$c^{++} = Max(0, u^2S - X) = Max(0, 1.2^2 100 - 95) = £40$$

$$c^{+-} = Max(0, udS - X) = Max(0, 1.2(0.8)100 - 95) = £1$$

$$c^{--} = Max(0, d^2S - X) = Max(0, 0.8^2 100 - 95) = £0$$

The put will also have 3 possible payoffs:

$$p^{++} = Max(0, X - u^2S) = Max(0, 0.95 - 1.2^2 100) = £0$$

$$p^{+-} = Max(0, X - udS) = Max(0, 0.95 - 1.2(0.8)100) = £0$$

$$p^{--} = Max(0, X - d^2S) = Max(0, 0.95 - 0.8^2 100) = £31$$

The price of a European call option using the two-period binomial model is:

$$c = \frac{0.55^2(49) + 2(0.55)(0.45)(1) + 0.45^2(0)}{1.02^2} = £14.723$$

And the two-period European put value is:

$$p = \frac{0.55^2(0) + 2(0.55)(0.45)(0) + 0.45^2(31)}{1.02^2} = £6.034$$

Two-Period Valuation for an American Option

American-style options may be more valuable than similar European options due to the early exercise feature.

- The difference in value is called the early exercise premium.
- Early exercise premium = American option value - European option value
- American puts may have an early exercise premium and must be checked for early exercise.
- American calls will not have an early exercise premium unless the underlying asset pays a dividend.

Two-Period Binomial Valuation for American Options

American puts must be checked for early exercise. American calls may be exercised early if the underlying asset pays a dividend.

American-style put options cannot be valued simply as the present value of the expected future option payouts. We must work backward through the binomial tree and address whether early exercise is optimal at each step.

At time 1, the unexercised values will be:

$$p_1^+ = \frac{\pi p^{++} + (1-\pi)p^{+-}}{(1+r)}$$

$$p_1^- = \frac{\pi p^{+-} + (1-\pi)p^{--}}{(1+r)}$$

Next, each node must be checked for early exercise. For example, the value at the down jump node will be the maximum of the unexercised and exercised values:

$$p^- = Max(p_1^-, X - dS)$$

Two-Period American Put Example I

Consider the previous example, with
$S = £100, X = £95, r = 2\%, u = 1.20,$ and $d = 0.80.$

$$p^{++} = Max(0, X - u^2S) = Max(0, 0.95 - 1.2^2 100) = £0$$

$$p^{+-} = Max(0, X - udS) = Max(0, 0.95 - 1.2(0.8)100) = £0$$

$$p^{--} = Max(0, X - d^2S) = Max(0, 0.95 - 0.8^2 100) = £31$$

At time 1, the unexercised values will be:

$$p_1^+ = \frac{\pi p^{++} + (1-\pi)p^{+-}}{(1+r)} = \frac{0+0}{1.02} = 0$$

$$p_1^- = \frac{\pi p^{+-} + (1-\pi)p^{--}}{(1+r)} = \frac{0 + (0.45)31}{1.02} = 13.677$$

Two-Period American Put Example II

Next, each node must be checked for early exercise.

$$p^- = Max(p_1^-, X - dS) = Max(13.68, 95 - 80) = 15$$

Recall, the European put value was £6.034. The American put value is:

$$p = \frac{\pi p^+ + (1 - \pi)p^-}{(1 + r)} = \frac{(0.55)0 + (0.45)15}{1.02} = £6.618$$

In this example, the American put is worth more than its European counterpart and has an early exercise premium of
$£6.618 - £6.034 = £0.584$

Black-Scholes-Merton Assumptions

The Black-Scholes-Merton (BSM) option valuation model presents a formula to price European puts and calls. Assumptions of the BSM model include:

- The underlying follows a geometric Brownian motion statistical process.
- Geometric Brownian motion implies continuous prices (no jumps).
- The underlying instrument is liquid (it can be easily bought and sold).
- Continuous trading is available (one must be able to trade at every instant).
- Short selling of the underlying instrument is permitted.
- There are no market frictions: no transaction costs, regulatory constraints, or taxes.
- No arbitrage opportunities are available in the marketplace.
- The options are European-style (early exercise is not allowed).
- The continuously compounded risk-free interest rate is known and constant.
- The volatility of the return on the underlying is known and constant.
- If the underlying instrument pays a yield, it is expressed as a continuous, known, and constant yield at an annualized rate.

Black-Scholes-Merton Option Valuation Model

The BSM model for options on a stock is as follows:

European call: $c = SN(d_1) - e^{-rT}XN(d_2)$

European put: $p = e^{-rT}XN(-d^2) - SN(-d_1)$, where

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

- $N(x)$ denotes the standard normal cumulative distribution function.
- r: annualized continuously compounded risk-free rate
- σ: annualized constant volatility
- S: stock price
- X: strike price
- e: constant that is the base of the natural logarithm (≈ 2.718)
- T: time to expiration for the option (in years)

Interpretation of the BSM Model I

The BSM model for the European call is:

$$c = SN(d_1) - e^{-rT}XN(d_2)$$

- The BSM model can be described as the present value of the expected option payoff at expiration. Specifically, we can express the BSM model for calls as:

$$c = PV_r[E(c_T)] \text{ where } E(c_T) = Se^{rT}N(d_1) - XN(d_2)$$

- The BSM model can be described as having two components: a stock component and a bond component.
- For call options, the stock component is $SN(d_1)$, and the bond component is $e^{-rT}XN(d_2)$. The BSM model call value is the stock component minus the bond component.

Interpretation of the BSM Model II

The BSM model can be interpreted as a dynamically managed portfolio of the stock and zero-coupon bonds.

For both call and put options, we can represent the initial cost of this replicating strategy as:

$$\text{Replicating strategy cost} = n_S S + n_B B$$

Here, the equivalent number of underlying shares is:

$$n_S = N(d_1) > 0 \text{ for calls and } n_S = -N(-d_1) < 0 \text{ for puts}$$

The equivalent number of bonds is:

$$n_B = -N(d_2) < 0 \text{ for calls and } n_B = N(-d_2) > 0 \text{ for puts}$$

BSM Model Component Interpretation for a Call

Suppose we are given the following information on call and put options on a stock: $S = 52, X = 50, r = 3\%$, $T = 1.0$, and $\sigma = 21\%$.

Thus, based on the BSM model, it can be demonstrated that:

$$PV(X) = 48.522, d_1 = 0.435, d_2 = 0.225, N(d_1) = 0.668, N(d_2) = 0.589$$

These inputs result in a call price of $c = 6.167$.

The no-arbitrage approach to replicating the call option involves: Purchasing $n_S = N(d_1) = 0.668$ shares of stock partially financed with $n_B = -N(d_2) = -0.589$ shares of zero-coupon bonds priced at $B = Xe^{-r}T = 48.522$ per bond.

Note that by definition the cost of this replicating strategy is the BSM call model value.

Replicating Strategy cost is:

$$n_S S + n_B B = 0.668(52) + (-0.589)48.522 = 6.156$$

BSM Valuation With Carry Benefits

Carry benefits include dividends for stock options, foreign interest rates for currency options, and coupon payments for bond options. We assume the carry benefit pays a continuous yield, γ.

The BSM model adjusted for carry benefits is:

- European call: $c = Se^{-\gamma T}N(d_1) - e^{-rT}XN(d_2)$
- European put: $p = e^{-rT}XN(-d_2) - Se^{-\gamma T}N(-d_1)$ where

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - \gamma + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

For an underlying equity paying a dividend, the BSM model adjusts the stock price by $e^{-\gamma T}$ to reflect the continuous payout.

BSM Valuation for Currencies

The BSM model can be used to value currency options. The carry benefit for a foreign exchange option ($\gamma = r^f$) is the continuously compounded foreign risk-free interest rate.

- European call: $c = Se^{-r^f T} N(d_1) - e^{-rT} XN(d_2)$
- European put: $p = e^{-rT} XN(-d_2) - Se^{-r^f T} N(-d_1)$ where

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r - r^f + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

Input example: A US auto exporter is planning to receive GBP for his cars. To protect against a fall in the GBP exchange rate, the exporter purchases a 3-month put struck at $X = 1.20\$/£$. The current exchange rate, $1.25\$/£$, is the underlying ($S$, the value of the domestic currency per unit of the foreign currency). The annualized US risk-free rate is (r), and the British rate is (r^f). The time to expiration (T) is 0.25 years. The standard deviation of the log return of the spot exchange rate (σ) is the last input required to value the put.

Black introduced a modified version of the BSM model approach that is applicable to options on underlying instruments that are costless to carry, such as options on futures contracts.

- European call: $c = e^{-rT}[F_0(T)N(d_1) - XN(d_2)]$
- European put: $p = e^{-rT}XN(-d_2) - F_0(T)N(-d_1)$ where

$$d_1 = \frac{\ln\left(\frac{F_0 T}{X}\right) + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \text{ and } d_2 = d_1 - \sigma\sqrt{T}$$

Note that $F_0(T)$ denotes the futures price at time 0 that expires at time T, and σ denotes the volatility related to the futures price. The other terms are as previously defined.

Black Option Valuation Example

The S&P 500 Index is at 2300, and a futures contract on it trades at 2293.11. The exercise price is 2250, the continuously compounded risk-free rate is 0.7%, time to expiration is 0.2 years for the option and futures contract, volatility is 20%, and the S&P dividend yield is 2.2%. Based on this information, the following results are obtained for options on the futures contract:

Calls	Puts
$N(d_1) = 0.601$	$N(-d_1) = 0.399$
$N(d_2) = 0.566$	$N(-d_2) = 0.434$
c = $104.253	p = $61.203

The underlying asset is the current futures price of 2293.11. So, the value of the European call option on the futures contract will be:

$$c = e^{-rT}[F_0(T)N(d_1) - XN(d_2)]$$

$$c = e^{-0.007(0.2)}[2293.11(0.601) - 2250(0.566) = 104.45$$

The European put option value will be:

$$p = e^{-0.007(0.2)}[2250(0.434) - 2293.11(0.399)] = 61.46$$

The option Greeks tell us how much the BSM option value will change for a small change in a particular parameter, *holding everything else constant*.

The Option Greeks (for stock options) include:

Delta: The change in an option value for a given small change in the value of the underlying stock.

Gamma: The change in a given option delta for a given small change in the underlying stock's value.

Theta: The change in a option value for a given small change in calendar *time*.

Vega: The change in an option value for a given small change in the *volatility* of the underlying stock.

Rho: The change in a given option value for a given small change in the risk-free interest *rate*.

Delta Hedging I

Delta hedging an option is the process of establishing a position in the underlying stock of a quantity that is prescribed by the option delta, so as to have no exposure to very small moves up or down in the stock price.

The delta of the hedging instrument is denoted as $Delta_H$.

The optimal number of hedging units, N_H, is

$$N_H = -(\text{Portfolio delta}/Delta_H)$$

A delta-neutral portfolio will not change in value for small changes in the stock instrument. Delta neutral implies the portfolio delta plus $N_H Delta_H$ is equal to 0.

- Stock Delta $= +1.0$
- Call Delta: $Delta_c = e^{-\delta T} N(d_1)$. So, $0 \leq$ delta of a call ≤ 1.
- Put Delta: $Delta_p = -e^{\delta T} N(-d_1)$. So, -1 $\leq$ delta of a put ≤ 0.

Delta Hedging II

Example: If a portfolio consists of 1,000 shares of stock, then the portfolio delta $= +1{,}000$.

If a call option with $Delta_H = +0.40$ is used as the hedging instrument for the portfolio, then to achieve a delta neutrality, we must sell 2,500 calls: $N_H = -(1,000/0.40) = -2,500$

Delta Hedging Example

Suppose we know $S = 50, X = 52, r = 3\%$, $T = 1.0, \sigma = 30\%$, and the underlying stock does not pay a dividend. We have a short position of 10,000 shares of stock.

Based on this information, we note $Delta_c = 0.668$, and $Delta_p =$ -0.332.

The optimal number of hedging units, N_H, is

$$N_H = -(\text{Portfolio delta}/Delta_H)$$

We further note the portfolio delta = -10,000.

Now, if we hedge using call options with $Delta_H = +0.668$ as the hedging instrument:

We must buy 14,970 calls, since $N_H = -(-10,000/+0.668) = +14,970$

But, if we hedge using put options with $Delta_H = -0.332$ as the hedging instrument:

We must sell 30,120 puts, since $N_H = -(-10,000/-0.332) = -30,120$

Gamma Risk I

Gamma: The change in a given option delta for a given small change in the underlying stock's value, holding everything else constant.

Option gamma is a measure of the curvature in the option price in relationship to the stock price.

Gamma of a long or short position in one share of stock is zero because the delta of a share of stock never changes.

Gamma for a call and put option are the same and can be expressed as:

$$Gamma_c = Gamma_p = \frac{e^{-\delta T}}{S\sigma\sqrt{T}} n(d_1)$$

where $n(d_1)$ is the standard normal probability density function.

Gamma is always non-negative and takes on its largest value when an option is near at the money.

Gamma measures the non-linearity risk or the risk that remains once the portfolio is delta neutral.

Gamma Risk II

For very small changes in the stock, the delta approximation works well:

$$\hat{c} = c + Delta_c(\hat{S} - S)$$

A portfolio that is delta neutral may be hedged for small changes in the stock price.

For fairly large changes in the stock price, the delta-plus-gamma approximation is more accurate:

$$\hat{c} = c + Delta_c(\hat{S} - S) + \frac{Gamma_c}{2}(\hat{S} - S)^2$$

Stock prices often jump rather than move continuously and smoothly, which creates "gamma risk."

Gamma risk is so-called because gamma measures the risk of stock prices jumping when hedging an option position and thus leaving a previously hedged option position suddenly unhedged.

Theta, Vega, and Rho

Theta: The change in an option value for a given small change in calendar time. Theta measures time decay, the rate at which option time value declines as time passes (and expiration approaches). Theta is negative for options; as calendar time passes, time to expiration declines and the option loses value. Note that stocks have no expiration date, so stock theta is zero.

Vega: The change in an option value for a given small change in the volatility of the underlying stock. An increase in volatility results in an increase in option value. Vega is always positive, and vega of a call equals vega of a similar put. Note that vega is based on an unobservable parameter, future volatility. Of the five BSM variables, an option's value is most sensitive to volatility changes.

Rho: The change in a given option value for a given small change in the risk-free interest rate. Rho is positive (negative) for a call (put). This is because buying a call avoids the financing costs involved with purchasing the stock, while buying a put delays the chance to earn interest on proceeds of the stock sale.

Implied Volatility

For options, their value is particularly sensitive to volatility.

Unlike the price of the underlying, however, volatility is not an observable value in the marketplace.

Volatility can be, and often is, estimated based on a sample of historical data.

Volatility can also be inferred from option prices. This inferred volatility is called the **implied volatility**.

The key advantage in using implied volatility is that it provides information regarding the market's perception of uncertainty going forward.

Implied Volatility in Option Trading

Implied volatility has several uses in options trading.

- Implied volatility can be interpreted as the market's view of option value.
- In the option markets, participants use volatility as the medium in which to quote options. Rather than quote a call price as $10, traders may quote the implied volatility as 25%, which prices the option at $10.
- Implied volatility can be used to assess the relative value of different options, neutralizing the effects of moneyness and time to expiration.
- A trader can decide which options are cheap (lower implied volatility) or expensive (higher implied volatility) using implied volatility rather than price.
- Implied volatility is useful for revaluing existing positions over time.
- Regulators, banks, compliance officers, and most option traders use implied volatilities to communicate information related to options portfolios.

Arbitrage Summary

The arbitrageur would rather have more money than less and abides by two fundamental rules: Do not use your own money and do not take any price risk.

The no-arbitrage approach is used for option valuation and is built on the key concept of the law of one price, which says that if two investments have the same future cash flows regardless of what happens in the future, then these two investments should have the same current price.

The following key assumptions are made:

- Replicating instruments are identifiable and investable.
- There are no market frictions.
- Short selling is allowed with full use of proceeds.
- The underlying instrument price follows a known distribution.
- Borrowing and lending is available at a known risk-free rate.

Valuation Summary

- The two-period binomial model can be viewed as three one-period binomial models, one positioned at time 0, and two positioned at time 1.
- In general, European-style options can be valued based on the expectations approach in which the option value is determined as the present value of the expected future option payouts, where the discount rate is the risk-free rate and the expectation is taken based on the risk-neutral probability measure.
- Both American-style options and European-style options can be valued based on the no-arbitrage approach, which provides clear interpretations of the component terms; the option value is determined by working backward through the binomial tree to arrive at the correct current value.
- Interest rate option valuation requires the specification of an entire term structure of interest rates, so valuation is often estimated via a binomial tree.

Black-Scholes-Merton Model Summary

A key assumption of the Black-Scholes-Merton option valuation model is that the return of the underlying instrument follows geometric Brownian motion, implying a lognormal distribution of the return.

The BSM model can be interpreted as a dynamically managed portfolio of the underlying instrument and zero-coupon bonds.

BSM model interpretations related to $N(d_1)$ are that it is the basis for the number of units of underlying instrument required to replicate an option, that it is the primary determinant of delta, and that it answers the question of how much the option value will change for a small change in the underlying.

BSM model interpretations related to $N(d_2)$ are that it is the basis for the number of zero-coupon bonds required to replicate an option, and for estimating the risk-neutral probability of an option expiring in the money.

Black Model Summary

The Black futures option model assumes the underlying is a futures or a forward contract.

Interest rate options can be valued based on a modified Black futures option model in which the underlying is an FRA, there is an accrual period adjustment as well as an underlying notional amount, and that care must be given to day-count conventions.

An interest rate cap is a portfolio of interest rate call options termed caplets, each with the same exercise rate and with sequential maturities. An interest rate cap can be used to hedge a set of floating rate loan payments.

An interest rate floor is a portfolio of interest rate put options termed floorlets, each with the same exercise rate and with sequential maturities. An interest rate floor can be used to hedge a floating rate bond investment or a floating rate loan.

A swaption is an option on a swap. A payer swaption is an option on a swap to pay fixed and receive floating. A receiver swaption is an option on a swap to receive fixed and pay floating.

Delta Summary

Delta is a static risk measure defined as the change in a given portfolio for a given small change in the value of the underlying instrument, holding everything else constant.

Delta hedging refers to managing the portfolio delta by entering additional positions into the portfolio.

A delta-neutral portfolio is one in which the portfolio delta is set and maintained at zero.

A change in an option price can be estimated with a delta approximation.

Because delta is used to make a linear approximation of the non-linear relationship that exists between the option price and the underlying price, there is an error that can be estimated by gamma.

Other Greeks

Gamma is a static risk measure defined as the change in a given portfolio delta for a given small change in the value of the underlying instrument, holding everything else constant.

Gamma captures the non-linearity risk or the risk—via exposure to the underlying—that remains once the portfolio is delta neutral. A gamma neutral portfolio is one in which the portfolio gamma is maintained at zero.

The change in an option price can be better estimated by a delta-plus-gamma approximation compared with just a delta approximation.

Theta is a static risk measure defined as the change in the value of a portfolio given a small change in calendar time, holding everything else constant.

Vega is a static risk measure defined as the change in a given portfolio for a given small change in volatility, holding everything else constant.

Rho is a static risk measure defined as the change in a given portfolio for a given small change in the risk-free interest rate, holding everything else constant.

Implied Volatility Summary

Although historical volatility can be estimated, there is no objective measure of future volatility.

Implied volatility is the BSM model volatility that yields the market option price.

Implied volatility is a measure of future volatility, whereas historical volatility is a measure of past volatility.

Option prices reflect the beliefs of option market participants about the future volatility of the underlying.

The volatility smile is a two dimensional plot of the implied volatility with respect to the exercise price.

The volatility surface is a three-dimensional plot of the implied volatility with respect to both expiration time and exercise prices.

If the BSM model assumptions were true, then one would expect to find the volatility surface flat, but in practice, the volatility surface is not flat.