Portfolio Theory

Lecture 3

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Structure

Forms of admissible portfolios

- The model of Markowitz
  - The wealth is defined
  - Time period
  - Problem of portfolio selection
- There are two extrems for a portfolio construction:
  - Wealth (assets) can not be devided
  - Wealth (assets) can be arbitrarily devided

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Basic concept - two-components portfolio

- Return of portfolio: $r_p = \sum_{i=1}^{N} w_i * r_i$
- The weights in portfolio: $w_1 + w_2 = 1$
- Expected return of portfolio: $\bar{r}_p = w_1 * \bar{r}_1 + w_2 * \bar{r}_2$
- Covariance of two assets: $\sigma_{12} = \sigma_1 * \sigma_2 * \rho_{12}$
- Risk of portfolio: $\sigma_p = \sqrt{w_1^2 * \sigma_1^2 + (1 - w_1)^2 * \sigma_2^2 + 2 * w_1 * (1 - w_1) * \sigma_{12}}$

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Indifference curves of investor

- Map of investor´s ICs
- An IC represents all desirable combinations of portfolio for an investor
- Properties of ICs:
  - All portfolios on an IC are equally desirable
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  - Unsaturation (return)
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