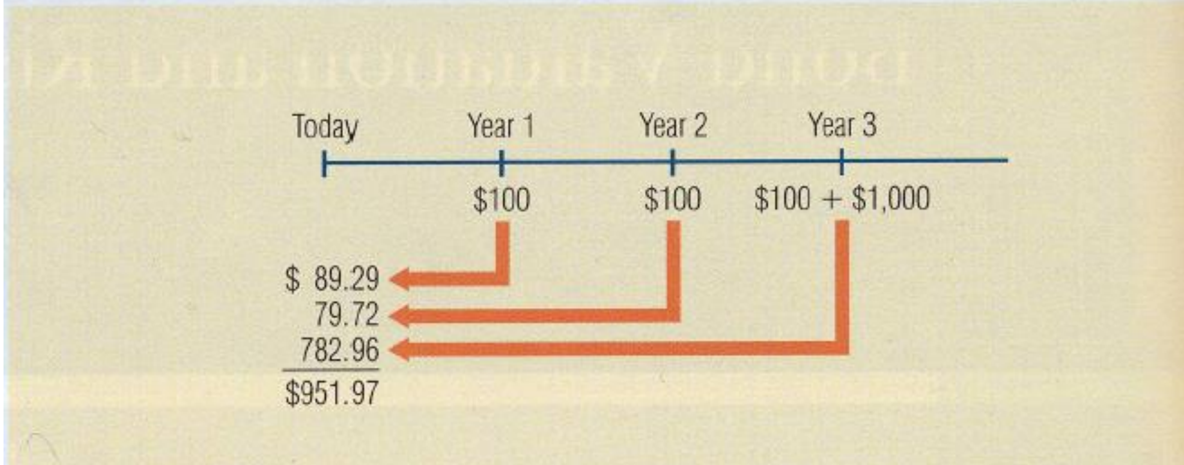
Sensitivity of Bond Prices to Interest Rate Movements

Bond Valuation Process

$$PV \text{ of bond} = \frac{C}{(1+k)^1} + \frac{C}{(1+k)^2} + \cdots + \frac{C + \text{Par}}{(1+k)^n}$$

where
- C = coupon payment provided in each period
- Par = par value
- k = required rate of return per period used to discount the bond
- n = number of periods to maturity

Exhibit 8.1 Valuation of a Three-Year Bond



- *Source: Madura, J.: Financial Markets and Institutions, 9th Edition*

Relations between Coupon Rate, Required Return and Bond Price

- 1.Discount bonds: Bonds Selling below Par (1000)
  - Coupon rate is below required rate, the price of the bond is below par (P < 1000)
- 2. Par Bonds: Bonds Selling at Par (1000)
  - Coupon rate equals the required rate, the price of the bond is equal to par value (P = 1000)
- Premium Bonds: Bonds Selling above Par (1000)
  - Coupon rate is above the required rate, the price of the bond is above the par (P > 1000)

Excel Sheet

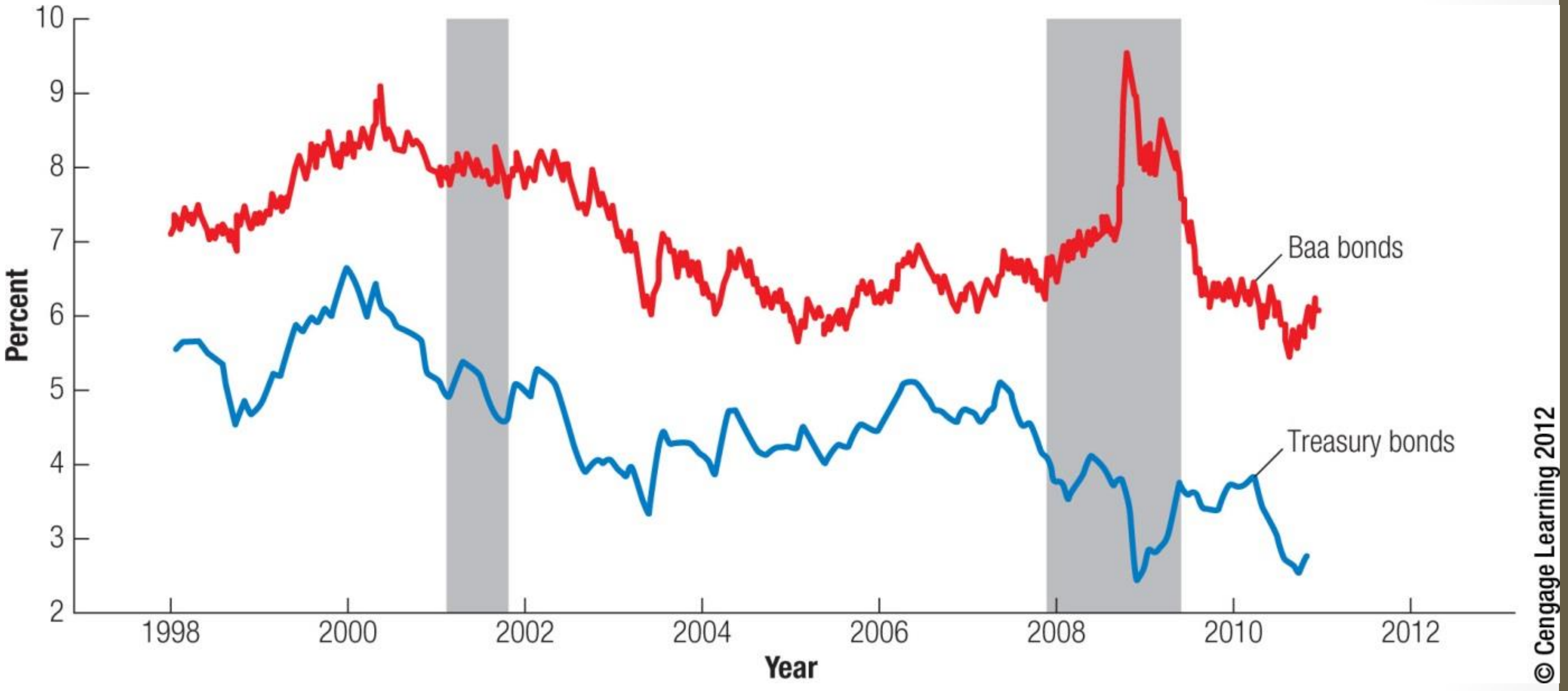
- Sheet 1
- Sheet 2

Sensitivity of Bond Prices to Interest Rate Movements

- Depends on the bond’s characteristics
- Indicates the potential to bond holdings in response to and increase in interest rates
- Measures:

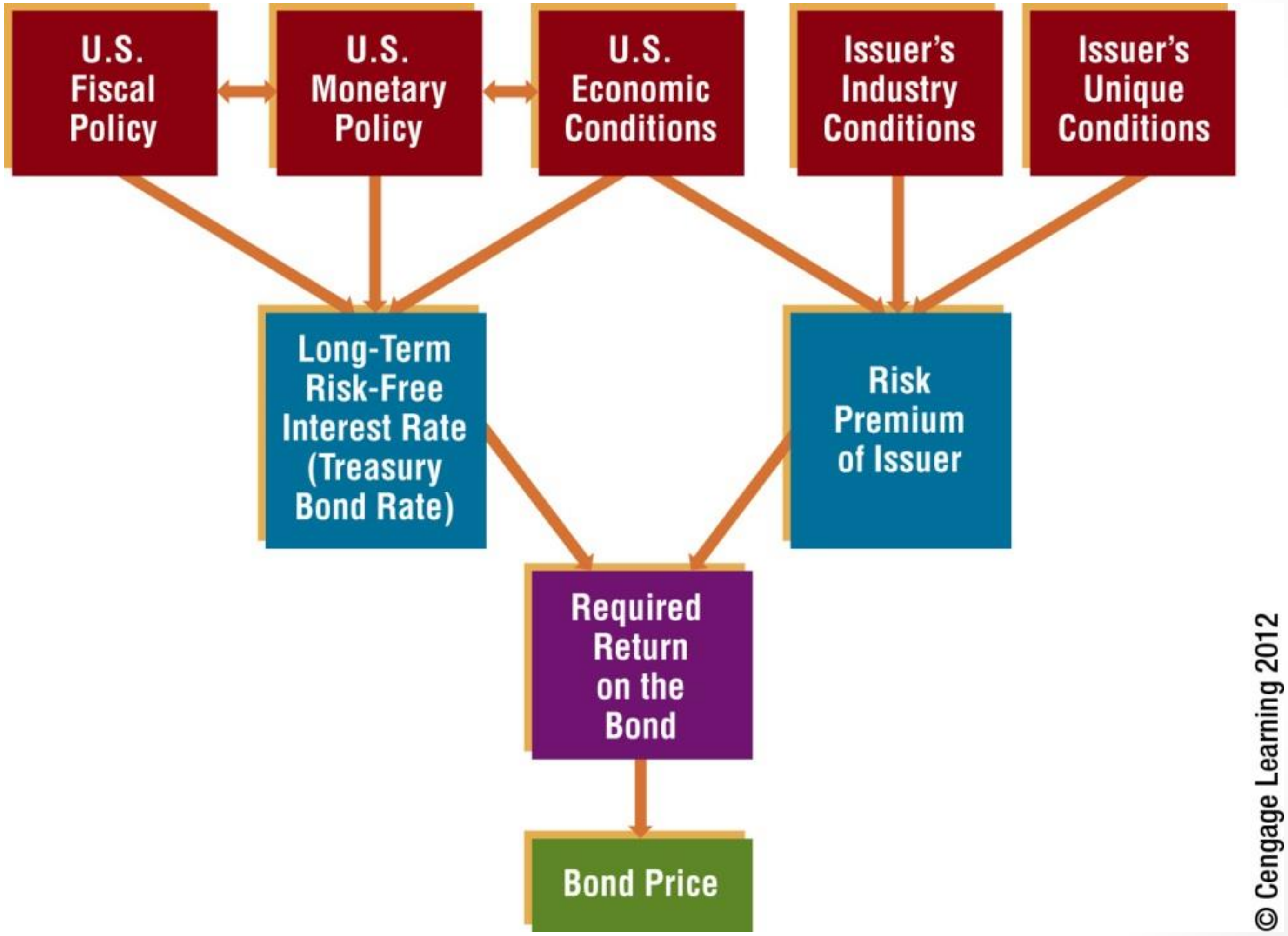
  - BOND PRICE ELASTICITY
  - DURATION
    - CONVEXITY

Bond Risk Premium over Time



- *Source: Madura, J.: Financial Markets and Institutions, 9th Edition*

Framework for Explaining Changes in Bond Prices over Time



Bond Price Elasticity

- Bond Price Elasticity = Bond price sensitivity for any % change in market interest rates
- Bond Price Elasticity =

  (% Change In Price)/(% Change In Interest Rates)
- Increased elasticity means greater price risk
- Compute for two points represented by yields (r)

$$P_b^e = \frac{\text{percentage change in } P_b}{\text{percentage change in r}}$$

Sensitivity of Bonds with Different Coupon Rates to Interest Rate Changes

EFFECTS OF A DECLINE IN THE REQUIRED RATE OF RETURN					
(1) BONDS WITH A COUPON RATE OF:	(2) INITIAL PRICE OF BONDS WHEN k = 10%	(3) PRICE OF BONDS WHEN k = 8%	(4) = [(3) − (2)]/(2) PERCENTAGE CHANGE IN BOND PRICE	(5) PERCENTAGE CHANGE IN k	(6) BOND PRICE ELASTICITY (P_b^e)
0%	$ 386	$ 463	+19.9%	−20.0%	−.995
5	693	799	+15.3	−20.0	−.765
10	1,000	1,134	+13.4	−20.0	−.670
15	1,307	1,470	+12.5	−20.0	−.625

EFFECTS OF A DECLINE IN THE REQUIRED RATE OF RETURN					
(1) BONDS WITH A COUPON RATE OF:	(2) INITIAL PRICE OF BONDS WHEN k = 10%	(3) PRICE OF BONDS WHEN k = 12%	(4) = [(3) − (2)]/(2) PERCENTAGE CHANGE IN BOND PRICE	(5) PERCENTAGE CHANGE IN k	(6) BOND PRICE ELASTICITY (P_b^e)
0%	$ 386	$ 322	−16.6%	+20.0%	−.830
5	693	605	−12.7	+20.0	−.635
10	1,000	887	−11.3	+20.0	−.565
15	1,307	1,170	−10.5	+20.0	−.525

- *Source: Madura, J.: Financial Markets and Institutions, 9th Edition*

Sensitivity of Bond Prices to Interest Rate Movements

a. **Influence of Coupon Rate on Bond Price Sensitivity**
   i. A zero-coupon bond is most sensitive to changes in the required rate of return.
   ii. The price of a bond that pays all of its yield in the form of coupon payments is less sensitive to changes in the required rate of return.
b. **Influence of Maturity on Bond Price Sensitivity** - As interest rates decrease, long-term bond prices increase by a greater degree than short-term bond prices.

Excel Sheet

- Sheet 3

Duration (Macaulay Duration)

- Measure of bond price sensitivity
- Measures the life of bond on a PV/FV basis
- Duration = Sum of discounted, time-weighted cash flows divided by price
- The longer a bond's duration, the greater its sensitivity to interest rate changes

- **The duration of a zero-coupon bond = bond's term to maturity**
- **The duration of any coupon bond is always less than the bond's term to maturity**

Duration in years

$$DUR = \frac{\sum_{t=1}^{n} \frac{C_t (t)}{(1+k)^t}}{\sum_{t=1}^{n} \frac{C_t}{(1+k)^t}}$$

where

C_t = coupon or principal payment generated by the bond

t = time at which the payments are provided

k = bond's yield to maturity (reflects investors' required rate of return

Calculating the macaulay duration

Example: A 6% annual payment bond (FV = 100) matures in 6 years. The YTM is 4%. Calculate the bond's Macaulay duration (actual/actual convention):

Period	CF (cash flow)	PV of CF	Time-Weighted PV of CF
1	6	6/(1 + 0.04)^1 = 5.76	1 × 5.76 = 5,76
2	6	5.58	11.16
3	6	5.54	16.62
4	6	5.33	21.32
5	6	4.93	24.65
6	106	83.77	502.62
		110.91	582.13
D **= 582.13/110.91 = 5.25 years**			

Properties of bond duration

Bond duration is the basic measure of interest rate risk on a fixed-rate bond.

The duration for a fixed-rate bond is a function of these input variables.

- Coupon rate or payment per period
- Yield-to-maturity per period
- Time-to-maturity (as of the beginning of the period)

Time-to-maturity and fraction of the period relation to macaulay duration

Time-to-maturity is typically directly related to the Macaulay duration.

Coupon rate and yield-to-maturity relation to macaulay duration

The coupon rate is inversely related to the Macaulay duration.

- A lower-coupon bond has a higher duration and more interest rate risk than a higher-coupon bond.
- The Macaulay duration of a zero-coupon bond is equal to its time-to-maturity.

The yield-to-maturity is inversely related to the Macaulay duration.

- A higher yield-to-maturity reduces the weighted average of the time to receipt of cash flow.

Modified duration in %

- Modified Duration (DUR*): Can be used to estimate the percentage change in the bond's price in response to a 1 percentage point change in bond yields

$$DUR^* = \frac{DUR}{1+k}$$

where ***k*** is the yield per period

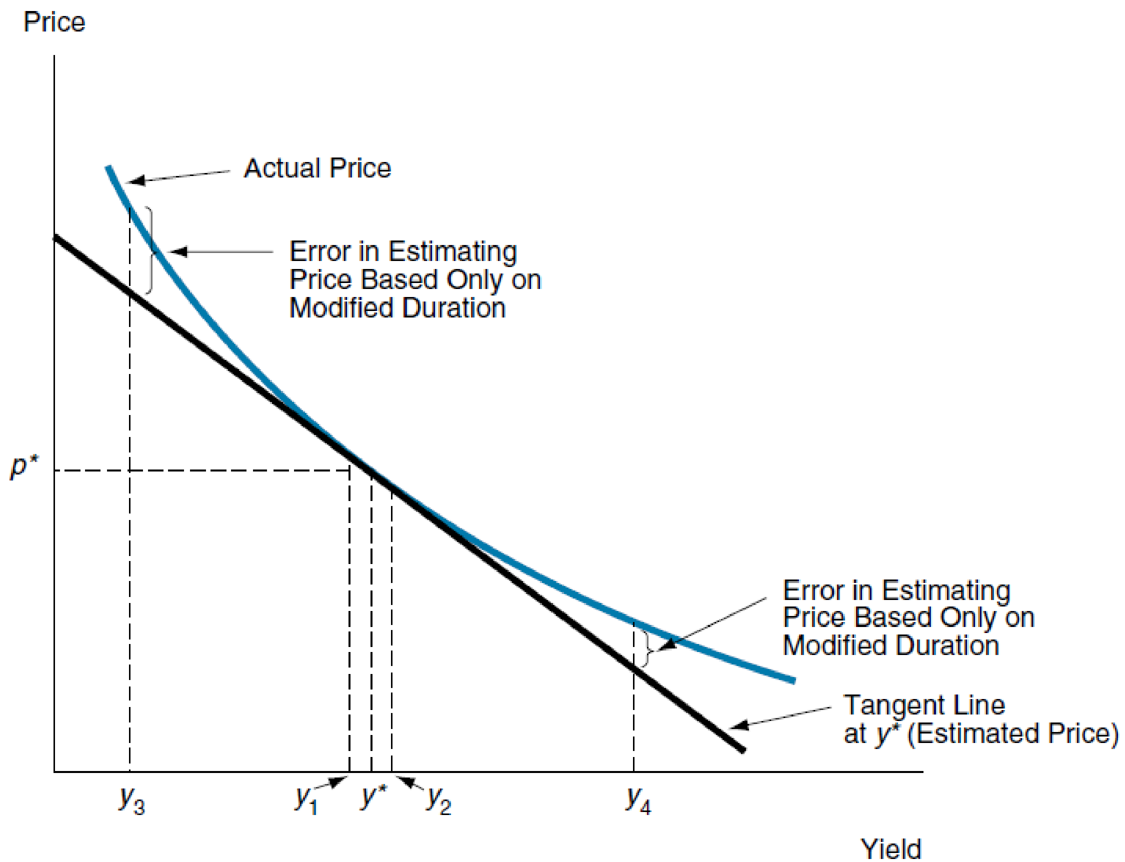
$$DUR^* = \frac{5{,}25}{1+\ 0.04}$$

$$DUR^* = \ 5.048$$

$$\%\Delta PV \quad \approx -MD \times \Delta Yield(\%)$$

Modified duration

PRICE APPROXIMATION USING MODIFIED DURATION



Source: Frank J. Fabozzi, Gerald Buetow, and Robert R. Johnson, “Measuring Interest Rate Risk” in the *Handbook of Fixed-Income Securities,* 6th ed. (New York: McGraw-Hill, 2001). Reproduced with permission from The McGraw-Hill Companies.

Convexity statistic

- The true relationship between the bond price and the yield-to-maturity is the curved (convex) line, which shows the actual bond price given its market discount rate.
- The **convexity statistic** for the bond is used to improve the estimate of the percentage price change provided by modified duration alone. There are various ways to estimate convexity:

$$\text{Conv} = \frac{1}{(1+k)^2} \times \frac{\sum_{t=1}^{T} \frac{\text{CF}_t}{(1+k)^t} \times (t^2 + t)}{P_0}$$

Convexity calculation

Example: 3-Year Bond, 12% Coupon, 9% YTM

(1) Year	(2) CF_T	(3) PV @ 9%	(4) PV CF	(5) $t^2 + t$	(4) × (5)
1	120	0.9174	$ 110.09	2	$ 220.18
2	120	0.8417	101.00	6	606.00
3	120	0.7722	92.66	12	1,111.92
3	1,000	0.7722	772.20	12	9,266.40
			Price = $1,075.95		$11,204.50

$$\text{Conv} = \frac{1}{(1+k)^2} \times \frac{\sum_{t=1}^{T} \frac{\text{CF}_t}{(1+k)^t} \times (t^2+t)}{P_0}$$

$$\text{Convexity} = \frac{9,411.78}{1,075.95} = 8.75$$

- Effect of Convexity = ½*Convexity*(Δ yield)^2
- Price Change = +/- Effect of Duration (Modified Duration)+Effect of Convexity

Excel Sheet

- Sheet 4