

1. Predictiong Bond Values. A bond you are interested in pays an annual coupon of 4, FV 1000 percent, has a yield to maturity of 6 percent and has 13 years to maturity. If interest rates remain unchanges, at what price would you expect this bond to be selling 8 years from now? Ten years from now?

$N = 13 \text{ years}$
 $FV = 1,000$
 $C = 0,04 \text{ p.a.}$
 $I = 0,06 \text{ p.a.}$

Price = PV of all future CFs

Pattern of CFs

| | | | | | | |
|------|---------|--------|---------|--------------|--------|-----------|
| time | 0 | 1 year | 2 year | 3 year | 8 year | 9 year |
| | | 40 | 40 | 40 | 40 | 40 |
| | 10 year | | 11 year | 12 year | | 13 year |
| | 40 | | 40 | 40 | | 40 + 1000 |

CF P=?

$$P_8 = \sum_{i=1}^5 \frac{CF_i}{(1+i)^i} = ???$$

$$P_{10} = \sum_{i=1}^3 \frac{CF_i}{(1+i)^i} = ???$$

2. Calculate the duration of an 8 percent, \$1,000 par bond that matures in three years if the bond's YTM is 10 percent and interest is paid semiannually.
- Calculate this bond's modified duration.
 - Assuming the bond's YTM goes from 10 percent to 9.5 percent, calculate an estimate of the price change.

$N = 3Y - 6 \text{ periods}$
 $C = 0,08 \text{ p.a.} - 0,04 \text{ p.s.}$
 $FV = 1000$
 $I = 0,1 \text{ p.a.} - 0,05 \text{ p.s.}$

Duration = time-weighted CFs/ bond price

$$Duration = \frac{1 * \frac{40}{(1+0,05)^1} + 2 * \frac{40}{(1+0,05)^2} + 3 * \frac{40}{(1+0,05)^3} + 4 * \frac{40}{(1+0,05)^4} + 5 * \frac{40}{(1+0,05)^5} + 6 * \frac{40}{(1+0,05)^6}}{P_0 = \frac{40}{(1+0,05)^1} + \frac{40}{(1+0,05)^2} + \frac{40}{(1+0,05)^3} + \frac{40}{(1+0,05)^4} + \frac{40}{(1+0,05)^5} + \frac{40}{(1+0,05)^6}}$$

$$Duration = \frac{5159,03898}{949,2431}$$

$$\begin{aligned}
 \text{Duration} &= 5,4349 \text{ semi - annual periods} \\
 \text{Duration} &= 2,71745 \text{ years} \\
 \text{Modified duration} &= \frac{\text{Duration}}{(1 + \text{yield in period})}
 \end{aligned}$$

$$\begin{aligned}
 \text{Modified duration} &= \frac{2,71745}{(1 + 0,05)} \\
 \text{Modified duration} &= 2,5880
 \end{aligned}$$

$$\text{Effect in bond price in \%} = -\text{Modified duration} * (\Delta\text{yield in \%})$$

$$\text{Effect of duration in a bond price in \%} = -2,5880 * (-0,5\%) = 1,2940 \%$$

$$\text{Approx. Price (for yield} = 0,095) = 949,2431 * (1 + 0,01294) = 961,5263$$

3. A semiannual bond for the Webster Corporation has the following characteristics:

Maturity—12 years

Coupon—10%

Yield to maturity—9.50%

Macaulay duration—5.7 years

Convexity—48

Noncallable

- a. Calculate the approximate price change for this bond using only its duration assuming its yield to maturity increased by 150 basis points.

$$\begin{aligned}
 \Delta\text{yield in \%} &= 1,5 \% \\
 \text{Modified duration} &= \frac{\text{Duration}}{(1 + \text{yield in period})}
 \end{aligned}$$

$$\begin{aligned}
 \text{Modified duration} &= \frac{5,7}{(1 + 0,0475)} \\
 \text{Modified duration} &= 5,4415
 \end{aligned}$$

$$\text{Effect in bond price in \%} = -5,4415 * (1,5\%) = -8,1623 \%$$

- b. Calculate the approximate price change for this bond (using only its duration) if its yield to maturity declined by 300 basis points.

$$\text{Effect of duration in a bond price in \%} = -5,4415 * (-3\%) = 16,3246 \%$$

4. Philip Morris has issued bonds that pay semiannually with the following characteristics:

| |
|--|
| Coupon/ Yield to Maturity/ Maturity/ Macaulay Duration |
| 8%/ 8%/ 15 years/ 10 years |

- a. Calculate modified duration using the preceding information.

$$\text{Modified duration} = \frac{\text{Duration}}{(1 + \text{yield in period})}$$

$$\text{Modified duration} = \frac{10}{(1 + 0,04)}$$

$$\text{Modified duration} = 9,6154$$

- b. Identify the direction of change in modified duration if

(1) the coupon of the bond were 4 percent, not 8 percent.

If coupon ↓ => duration ↑

(2) the maturity of the bond were 7 years, not 15 years.

If maturity ↓ => duration ↓

5. Using the information in the following table, calculate the projected price change for Bond B if the yield to maturity for this bond falls by 75 basis points.

Bond A (callable)/ Bond B (noncallable)

Maturity 2016/ 2016

Coupon 11.50% /7.25%

Current price 125.75/ 100.00

Yield to maturity 7.70%/ 7.25%

Modified duration to maturity 6.20/ 6.80

Convexity to maturity 0.50/ 0.60

Call date 2010/ —

Call price 105/ —

Yield to call 5.10%/ —

Modified duration to call 3.10/ —

Effect of duration in a bond price in % = $-6,8 * (-0,75\%) = 5,1\%$

**Effect of convexity in a bond price = $0,5 * 0,60(-0,0075)^2 = 0,000016875$
=> **0,0016875 %****

Both effects = $5,1\% + 0,0016875\% = 5,1016875\%$ which is the projected price change.

New Price = $100 * (1 + 0,051016875) = 105.102$

