Applied Macroeconomic Modeling

OGResearch

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About me

- Masaryk university alumni, applied mathematics / economics
- Two years PhD with prof. Vasicek, then left
- Since 2012 with OGResearch – macro forecasting, model development, ...
- Since 2013 technical assistance missions under the IMF to Africa, Asia, ...
- Ad hoc work on macroeconomic modelling

About the course

- Point is to show you how macro forecasting is done
- Showcase workhorse macro model, show what it can and cannot do
- First, we'll examine a generic model and learn the basics
- Then, we'll apply the model to Azerbaijan and recent, important period
- I'll be sending emails, out stuff into Study materials
- **ASK QUESTIONS, HAVE COMMENTS, BE ACTIVE**

To pass the course...

- The point is for you to learn, I won't force you
- I'll require three things:
  - prepare a presentation on Azerbaijan and what happened there since 2014
  - do your own forecast for Azerbaijan
  - you are active during the lectures
- Work will be in groups of 3-4 people
- Deadlines will not be tight

About the QPM models

- The Quarterly Projection Model is a workhorse in applied macro modeling
  - QPM developed by Bank of Canada for MonPol analysis and forecasting
- The key equations describe gaps (deviations from trends)
- Trends described by simpler equations
- Maleable, flexible structure to incorporate many different mechanisms
- Semi-structural: blueprint based on DSGE (micro-foundations) but amended to remain flexible and describe data
- New Keynesian tradition => nominal rigidities (sticky prices, monopolistic competition => suitable for monetary policy analysis
- Rational expectations (with modifications if necessary) and endogenous monetary policy => suitable for monetary policy analysis
- Parameters usually not estimated, but calibrated to achieve desired properties

The QPM is a simple model

- QPM building blocks are simple – that’s a virtue, we can work with the model
- Simple structure leads to uncertainty of model parameters
  - That’s why we calibrate
- Some important economic mechanisms missing
  - Balance sheet effects, revisions in perceived riskiness of bank assets
  - Supply side basically exogenous, labor supply, migration, tricky behavior of commodities
  - Fiscal risks, policies
  - Structural issues
- We should not ”believe” the model, it’s only a tool to help us

Output gap

Output gap is a measure of the demand-side inflationary pressures and is described by the IS curve:

$$\begin{aligned}\widehat{y}_t = &\beta_1 \widehat{y}_{t+1} \\ &+ \beta_2 \widehat{y}_{t-1} \\ &- \beta_3 \widehat{r}_t \\ &+ \varepsilon_t^{\widehat{y}}\end{aligned}$$

- Own lead and lag - why?
- Real interest rate gap $\widehat{r}$, which represents the stance of monetary policy
- Idiosyncratic shock $\varepsilon^{\widehat{y}}$ - shocks, but also model imperfection, misspecification, ...

Supply side – inflation

Core inflation is modelled using a New-Keynesian Phillips Curve (PC):

$$
\begin{aligned}
\pi_t = \alpha_1 E\pi_{t+1} \\
+ (1-\alpha_1)\pi_{t-1} \\
+ \alpha_2(\widehat{y}_t) \\
+ \varepsilon_t^{\pi}
\end{aligned}
$$

- PC is homogenous $\alpha_1 + (1-\alpha_1) = 1$. This is important, otherwise inflation would always go to zero.
- Output gap represents demand pressures on inflation

Monetary Policy

Central bank follows inflation-forecast-based reaction function (IFBRF):

$$\widehat{r}_t = g_1 \widehat{r}_{t-1} + (1 - g_1)(g_2(E\pi_{t+1} - \pi^{tar}) + g_3 \widehat{y}_t) + \varepsilon_t^{\widehat{r}}$$

- It’s the **real** interest rate that matters - anyone knows how this is called?
- Note that the inflation target here is a parameter, but it can be made into an equation

So we have the equations – what now?

- To work with the model, we need to have at least basic understanding of the mathematical methods in the background:
  - How is the model solved?
  - How is the model solution represented?
  - How are model parameters determined?
  - What methods are available to check the model parameterization?
  - How do we forecast with the model?
- Let's do a (very brief) primer into these topics

Model solution, state-space representation

- Using dummy variables for leads and lags, we can rewrite the model as:

$$G(X_t, X_{t-1}, X_{t+1}, \varepsilon_t) = 0$$

- X_t is a vector of endogenous variables, ε_t of exogenous shocks

$$X_t = \begin{bmatrix} \pi_t \\ \widehat{y}_t \\ \widehat{r}_t \end{bmatrix}, \quad \varepsilon_t = \begin{bmatrix} \varepsilon_t^{\pi} \\ \varepsilon_t^{\widehat{y}} \\ \varepsilon_t^{\widehat{r}} \end{bmatrix}$$

- We want to solve the model to obtain:

$$X_t = g(X_{t-1}, \varepsilon_t)$$

- For linear models, the solution can be written as

$$X_t = AX_{t-1} + B\varepsilon_t + B_2 E_t[\varepsilon_{t+1}] + B_3 E_t[\varepsilon_{t+2}] \dots$$

- Remember this, I'll refer to the model solution often

Steady-state

- A dynamic system is in steady-state if the variables do not change in time unless there are shocks

$$X_t = A \cdot X_{t-1} = X^{ss}$$

- In our case:

$$\begin{aligned} \widehat{y}^{ss} &= ? \\ \widehat{r}^{ss} &= ? \\ \pi^{ss} &= ? \end{aligned}$$

- Can be quite complicated for non-linear models, we need to use numerical solvers
- Implication: in the absence of shocks, the model does nothng more than converge to steady-state!

Impulse Response Functions

- We now see shocks are important, we need to examine how they impact the model
- To do that, we use the impulse-response functions (IRFs)
- Given the initial condition X_t, we do a forecast

$$X_k = A^k X_0 + A^{k-1} B_0 \varepsilon_0$$

- The initial condition is usually chosen to be:
  - Steady-state
  - Deviation from steady-state (= zero for all variables)
- Thourough analysis of the IRFs is always useful, let's do it

Impulse Response Functions – the code

- Go to study materials, zipfile ”closed_model”, download, unzip
- Go to study materials, zipfile ”IRIS_Tbx_20150119.zip”, download, unzip
- Start matlab, navigate to the IRIS folder, type ”irisstartup”
- Open files ”closed_model.model”, ”setparam.m”, ”run_toy_model_irf.m”
- Let’s have a look at the files
- Let’s do the IRFs and interpret them
- Note: the model should return to steady-state

Determining initial condition

$$X_k = A^k X_0 + \sum_{i=1}^{k} B^i \varepsilon_i$$

- Initial condition X_0 is very important :
  - Is the output gap now positive or negative?
  - Is the MonPol stance now tight or easy?
  - Initial condition **largely determines the forecast**
  - Central banks put enormous effort into identification of the initial condition, and we should too
- How we can determine the initial condition, given that many variables are unobservable?
  - Start from steady-state - not correct
  - Set the numbers by hand - cumbersome
  - Use unilateral filters (Hodrick-Prescott, ...)
  - Best practice: Use multivariate filter and our model => the **Kalman filter**

State Space Model Representation

State space, denoted as M:

$$\begin{aligned} X_t &= AX_{t-1} + B\varepsilon_t \\ y_t &= Z_t X_t + \eta_t \end{aligned}$$

where

- X_t is a vector of endogenous variables
- ε_t a vector of exogenous structural shocks, $\varepsilon_t \sim N(0, Q_t)$
- y_t are a vector of observations in period t:
  - usually/always smaller size than X_t – we can observe π_t, but not $\widehat{y}_t$
  - the size of the vector can be changing
- Z_t is a matrix transforming the variables into observations, and
- η_t are the measurement errors, $\eta_t \sim N(0, H_t)$ – in economics we generally set $H_t = 0$

Kalman smoother as LSQ

- Now we have observations $Y_t = [y_1, \ldots, y_T]$, all the available observations denoted as Y_T
- Kalman filter / smoother estimates are joint distributions $p(X_t, \eta_t, \varepsilon_t | M, Y_T)$
- For any given x_t and Y_T, there is linear space of realizations of shocks ε and measurement errors η
- Because shocks and measurement errors are distributed normally, there is only one realization in that space yielding the maximum likelihood, or equivalently the least square errors
- The square errors are weighted according to the variance-covariance matrices Q and H => variance decomposition matters
- Therefore, mean of $x_{t|T}$ is a constrained least square solution
  - the constraints come from the state space for each period
  - the optimization minimizes sum of square shocks and measurement errors wieghted by their respective standard errors

Kalman smoother as LSQ

- In practice we assume the following:

$$H_t = 0, \quad Q_t = \begin{bmatrix} \sigma^1 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^3 \end{bmatrix}$$

- Basically, we solve the following optimization problem:

$$\min \sum_{t=1}^{T} \left(\frac{\varepsilon_t^1}{\sigma^1}\right)^2 + \left(\frac{\varepsilon_t^2}{\sigma^2}\right)^2 + \ldots s.t. :$$

$$X_t = AX_{t-1} + B\varepsilon_t$$

$$y_t = Z_t X_t$$

- That's a least squares problem similar to fitting a OLS

Kalman filter – the code

- Open file ”run_toy_kalman.m”
- Let’s have a look at the resulting PDF files

Open economy model

- So far, we're using very simple model that doesn't have many important features:
  - No exchange rate
  - No nominal interest rates
  - No trends
- Let's fix that, we'll build the smallest open economy model possible

Exchange Rates

- We assume open capital account. The exchange rates are thus determined through standard Uncovered Interest Parity (UIP) equation.

$$i_t = (Es_{t+1} - s_t) + i_t^* + prem_t + \varepsilon_t$$
$$Es_{t+1} = \alpha s_{t+1} + (1-\alpha)(\overline{z}_t + \pi^{ss} - \pi^{*,ss})$$

- Can anyone explain the logic?
- We also define the **real** exchange rate z_t using price level p_t

$$z_t = s_t + p_t^* - p_t$$
$$z_t = \overline{z}_t + \widehat{z}_t$$

- What is real exchange rate?
- Which exchange rate is more important? Which is more stable? Which is under control of the central bank?

Exchange Rate Fundamentals

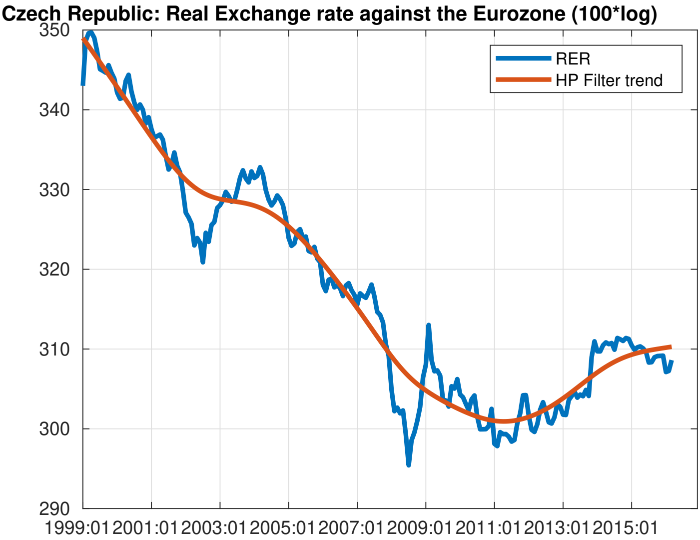
- The medium-term fundamentals of the FX rate are given by the real exchange rate trend $\overline{z}_t$.
- The RER trend accounts for a variety of factors influencing the real convergence of the block: terms of trade, productivity, ...

$$\overline{z}_t = \overline{z}_{t-1} + \Delta\overline{z}_t + \varepsilon_t^z$$
$$\Delta\overline{z}_t = \rho^z \Delta\overline{z}_{t-1} + (1 - \rho^z) \cdot \overline{z}_{ss} + \varepsilon_t^{\overline{z}}$$

- Country risk premium has similar, simple equation:

$$prem_t = \rho^{prem} prem_{t-1} + (1 - \rho^{prem}) prem^{ss} + \varepsilon_t^{prem} \quad (1)$$

RER in Czech Republic



GDP Fundamentals

- The medium-term fundamentals of the GDP are given by output potential $\overline{y}_t$.
- It's similar to RER trend: accounts for a variety of factors; terms of trade, productivity, ...

$$\overline{y}_t = \overline{y}_{t-1} + \Delta\overline{y}_t + \varepsilon_t^y$$
$$\Delta\overline{y}_t = \rho^{\overline{y}}\Delta\overline{y}_{t-1} + (1-\rho^y)\cdot\overline{y}_{ss} + \varepsilon_t^{\overline{y}}$$

- It's important to understand the interpretation:
  - Output gap is a measure of how the economic activity affects inflation
  - Output potential is everything else – non-inflationary level of output

Monetary policy in open economy

- The central bank in reality sets short nominal interest rate i_t

$$i_t = g_1 i_{t-1} + (1 - g_1)(\overline{r}_t + \pi^{tar} + g_2(E\pi_{t+1} - \pi^{tar}) + g_3\widehat{y}_t) + \varepsilon^i_t$$

- The real interest rate is then given by the Fisher equation:

$$r_t = i_t - E\pi_{t+1}$$

- The real interest then again has gap and trend components:

$$r_t = \overline{r}_t + \widehat{r}_t$$

- And the trend is given as

$$\overline{r}_t = \overline{r^*}_t + \Delta\overline{z}_t + prem_t$$

What about IS curve and Phillips Curve?

They change to

$$\begin{aligned}\widehat{y}_t = &\beta_1 \widehat{y}_{t+1} \\ &+ \beta_2 \widehat{y}_{t-1} \\ &- \beta_3 \widehat{r}_t \\ &+\beta_4 \widehat{z}_t \\ &+\beta_5 \widehat{y^*}_t \\ &+ \varepsilon_t^{\widehat{y}}\end{aligned}$$

and

$$\begin{aligned}\pi_t = &\alpha_1 E\pi_{t+1} \\ &+ (1 - \alpha_1 - \alpha_4)\pi_{t-1} \\ &+ \alpha_2(\widehat{y}_t) \\ &+\alpha_3 \widehat{z}_t \\ &+\alpha_4(\Delta s_t + \pi_t^* - \pi_t) \\ &+ \varepsilon_t^{\pi}\end{aligned}$$

Open model IRFs

- Let’s have a look
- Open ”open_model.zip”, and run ”run_open_model_irf.m”

Presentations for the next block

- Groups of 3-4 people
- Approx. 10 slides describing the economy of Azerbaijan in 2014:
  - What were the key sources of growth?
  - What were the sources of inflation volatility?
  - What happened to real exchange rate in 2005-2014? Why?
  - Who were the key trading partners?
  - What was the monetary policy regime?
  - Do not forget to have a look at balance of payments
- We want to understand if the standard QPM model describes the Azeri economy well ...
- ...and if not, what we should change

Conditional forecasts

Recall model solution:

$$X_t = AX_{t-1} + B\varepsilon_t$$

Consider the following problem:

- We have an information that something is going to happen
- We know that the default model behavior is not right for the current situation, and we need to "rewrite" something
- We condition on initial condition X_0
- We condition on future "expert" information $(\varepsilon_t, \varepsilon_{t+1}, \ldots)$
- And also, we condition on the model as well (matrices A, B)

Basic Set-up

Consider the following model:

$$\begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix} = A \cdot \begin{bmatrix} x_{t-1}^1 \\ x_{t-1}^2 \end{bmatrix} + B \cdot \begin{bmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{bmatrix}$$

We have endogenous (calculated by the model) variables and exogenous (given from outside) shocks.
The simple, unconditioned forecast is then

	Period	x^1	x^2	ϵ^1	ϵ^2
Init. Cond.	$t-1$	x_{t-1}^1	x_{t-1}^2	ϵ_{t-1}^1	ϵ_{t-1}^2
Forecast	t	x_t^1	x_t^2	0	0
	$t+1$	x_{t+1}^1	x_{t+1}^2	0	0
	$t+2$	x_{t+2}^1	x_{t+2}^2	0	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Soft Tunes

- **Soft tunes** means that we impose non-zero value of the shock
- The shock is still treated as exogenous input, and the model calculates the values of X_t
- Suitable to represent for example:
  - Impact of VAT hike – we know what will be the (approximate) impact on inflation, but we do not know the resulting inflation outcome.
  - Model imperfection – we know the model fails to capture something in recent quarters and therefore filters shocks. We want to extend these shocks on the forecast.

	Period	x^1	x^2	ϵ^1	ϵ^2
Init. Cond.	$t-1$	x^1_{t-1}	x^2_{t-1}	ϵ^1_{t-1}	ϵ^2_{t-1}
Forecast	t	x^1_t	x^2_t	0	0
	$t+1$	x^1_{t+1}	x^2_{t+1}	0	0
	$t+2$	x^1_{t+2}	x^2_{t+2}	**1.3**	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Hard Tunes

- **Hard tunes** means that we directly impose the value of the variable
- We exogenize one variable in one or more periods. For the equations to have a solution, we need to endogenize one shock in the corresponding periods:

$$\begin{bmatrix} x_t^1 \\ x_t^2 \end{bmatrix} = A \cdot \begin{bmatrix} x_{t-1}^1 \\ x_{t-1}^2 \end{bmatrix} + B \cdot \begin{bmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{bmatrix}$$

- Suitable to represent for example near-term forecast – we know the value of the variable, but we do not know how big the shock should be
- But, we need to choose the shock ourselves. The choice of the shock matters!
- General rule: hard tunes belong on short horizons, soft tunes on long horizons

Hard Tunes cont.

Assume we want to tune $x_t^1 = 3$:

	Period	x^1	x^2	ϵ^1	ϵ^2
Init. Cond.	$t-1$	x_{t-1}^1	x_{t-1}^2	ϵ_{t-1}^1	ϵ_{t-1}^2
Forecast	t	3	x_t^2	?	0
	$t+1$	x_{t+1}^1	x_{t+1}^2	0	0
	$t+2$	x_{t+2}^1	x_{t+2}^2	0	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Or we can choose the other shock:

	Period	x^1	x^2	ϵ^1	ϵ^2
Init. Cond.	$t-1$	x_{t-1}^1	x_{t-1}^2	ϵ_{t-1}^1	ϵ_{t-1}^2
Forecast	t	3	x_t^2	0	?
	$t+1$	x_{t+1}^1	x_{t+1}^2	0	0
	$t+2$	x_{t+2}^1	x_{t+2}^2	0	0
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Combining tunes

We can combine both methods, as long as we do not "overtune" = impose unsolvable conditions:

- We will hard tune x_t^1, explained by ϵ_t^1
- We will also impose soft tunes on ϵ^2

	Period	x^1	x^2	ϵ^1	ϵ^2
Init. Cond.	$t-1$	x_{t-1}^1	x_{t-1}^2	ϵ_{t-1}^1	ϵ_{t-1}^2
Forecast	t	3	x_t^2	?	**0.75**
	$t+1$	x_{t+1}^1	x_{t+1}^2	0	**0.5**
	$t+2$	x_{t+2}^1	x_{t+2}^2	0	**0.25**
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

The system is flexible, but we need to know what we want to achieve.