Portfolio Theory

Lecture 7

Luděk Benada

Department of Finance - 402, benada.esf@gmail.com

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Structure

Testing of the model

- Model was many times testing, but with ambiguous results
- Sharpe, Lintner, Miller, Jensen, Fama, French
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Background for testing

- The model is based on expectations
- The variables are expressed in future value
- ...but the calculation is on observed values
- Prices of assets will be variing around equilibrium
- The equilibrium return of an individual asset:
- $r_i^e = r_f + (r_M - r_f) * \beta_i$
- ...but the real return:
- $r_i - r_f = (r_M - r_f) * \beta + \varepsilon_i$

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Random component of the model

- Properties of random error:
- $E(\varepsilon_i) = 0;\text{for } \forall_i = 1, 23, \ldots$
- $Cov(\varepsilon_i, r_i) = 0;\text{for } \forall_i = 1, 23, \ldots$
- $Cov(\varepsilon_i, \varepsilon_j) = 0;\text{for } \forall_i = 1, 23, \ldots \wedge i \neq j$
- $E\left[\varepsilon_i\left(r_M - \bar{r}_M\right)\right] = 0;\text{for } \forall_i = 1, 23, \ldots$

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Parametr estimates

- Parameters could be estimated:
- The set of this points is described by empirical regresion function:
- $\hat{y}_i = f(a, b, x) = a + b * x$
- Residue... $\varepsilon_i = y_i - \hat{y}_i$
- The methodology of OLS minimalize the errors
- Thus the objective function:
- $S_r = \sum_{i=1}^{N} e_i^2 \Rightarrow min$

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Solution

- Partial derivatives with respect to a, b
- $\Rightarrow$system of equiations:
  - $na + b\sum X_i = \sum Y_i$
  - $a\sum X_i + b\sum X_i^2 = \sum X_i Y_i$
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Variance of security

- Variance of excess return:
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Market and unique risk

- ε_i^2 ...concerns only an individual company or industry, could be diversified!
- $\beta_i^2 * \sigma_M^2$...undiversified part of risk, concerns all securities on the market
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Unequilibrium in the model

- Investors are looking for investment opportunities ...securities in unequilibrium
- A security is undervalueted if the return is higher then the equilibrium return
- A security is overvalueted if the return is under expected return (security is expansive)
- The equilibrium return lies on the SML
- $\delta_i = r_i - \bar{r}_i$

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Investment decision

- If $\delta_i > 0$ $\Rightarrow$the security is over the SML ...undervalueted
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