

Seminar 11:

Linear fractional programming, Data envelopment analysis

Problem 1: Minimize the objective function of linear fractional program

$$f(x_1, x_2, x_3) = \frac{2x_1 + x_2 + 3x_3}{x_1 + x_2 + x_3}$$

subject to

$$x_1 - x_2 + x_3 \leq 5$$

$$x_2 \geq 3$$

$$x_1, x_3 \geq 0$$

- Find suitable substitution for linearization of the problem
- Solve linearized problem
- Check the solution with Solver

Problem 2: Consider the DEA model for 8 hospital departments whose performance is characterized by the following values:

Unit	O1	O2	O3	O4	O5	O6	O7	O8
employees	7	6	6	8	10	5	4	5
outpatients	21	24	42	16	50	45	40	60
inpatients	63	36	48	40	40	15	24	10

- Represent the problem graphically. Consider constant returns to scale and find an efficient frontier.
- Determine the efficiency of the 5th department using graphical method and find its peer units (use the output orientation).
- Formulate an input-oriented CCR model for the 5th department and find the solution in the Solver (GRG nonlinear method must be selected)
- Linearize the input-oriented CCR model for the 5th department and find the solution in the Solver (Simplex LP method)

Problem 3: (BCC model of J. and M. Zouhar) The following table shows the number of lecturers (in tens) and the number of successful graduates (in hundreds) in the last year for certain A to D colleges:

college	A	B	C	D
lecturers [10]	1	3	7	5
graduates [100]	3	5	7	4

- In the chart with the input and output axes, capture the efficient frontier for variable returns to scale. Compare the number of efficient units assuming VRS with the CRS case.
- Find input, output, and reference units for the virtual unit D_1 in the input-oriented model, calculate the efficiency of D for this model.
- Find input, output, and reference units for the virtual unit D_1 in the output-oriented model, calculate the efficiency of D for this model.

Problem 4: (Problem of J. Kalčevová)

Consider DEA with 2 inputs, 3 outputs and 10 units:

X (inputs)									
3	2,5	4	2,3	4	7	3	5	5	2
5	4,5	6	3,5	6,5	10	5	7	7	4
Y (outputs)									
40	45	55	28	48	80	45	70	45	45
55	50	45	50	20	65	64	65	65	40
30	40	30	25	65	57	42	48	40	44

Find all efficient units, if you consider

- input-oriented CCR model
- output-oriented CCR model
- input-oriented BCC model
- output-oriented BCC model

Find the peer units for the 3rd unit if you consider

a) input-oriented CCR model

b) output-oriented BCC model

Use DEA Add-in for the calculations (download from <https://webhosting.vse.cz/jablon/>)