Seminar 10: Matrix operations, determinants

Problem 1: EMEA 544, ex. 3
Let:
$\mathbf{A} = \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$; $\mathbf{B} = \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$; $\mathbf{C} = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix}$
Find the matrices $\mathbf{A}+\mathbf{B}$, $\mathbf{A}-\mathbf{B}$, $\mathbf{AB}$, $\mathbf{BA}$, $\mathbf{A}(\mathbf{BC})$, $(\mathbf{AB})\mathbf{C}$

Problem 2: EMEA 550, ex. 1
Verify the distributive law $\mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{AB}+\mathbf{AC}$ when
$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$; $\mathbf{B} = \begin{pmatrix} 2 & -1 & 1 & 0 \\ 3 & -1 & 2 & 1 \end{pmatrix}$; $\mathbf{C} = \begin{pmatrix} -1 & 1 & 1 & 2 \\ -2 & 2 & 0 & -1 \end{pmatrix}$

Problem 3: EMEA 550, ex. 4
Compute:
a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 5 & 3 & 1 \\ 2 & 0 & 9 \\ 1 & 3 & 3 \end{pmatrix}$; b) $\begin{pmatrix} 1 & 2 & -3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Problem 4: EMEA 553, ex. 1
Find the transposes of
$\mathbf{A} = \begin{pmatrix} 3 & 5 & 8 & 3 \\ -1 & 2 & 6 & 2 \end{pmatrix}$; $\mathbf{B} = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 2 \end{pmatrix}$; $\mathbf{C} = \begin{pmatrix} 1 & 5 & 0 & -1 \end{pmatrix}$

Problem 5: EMEA 553, ex. 2
Let:
$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix}$; $\mathbf{B} = \begin{pmatrix} 0 & 2 \\ 2 & 2 \end{pmatrix}$, and $\alpha = -2$
Find $\mathbf{A}^\top$, $\mathbf{B}^\top$, $(\mathbf{A}+\mathbf{B})^\top$, $(\alpha\mathbf{A})^\top$, $\mathbf{AB}$, $(\mathbf{AB})^\top$, $\mathbf{B}^\top\mathbf{A}^\top$ a $\mathbf{A}^\top\mathbf{B}^\top$

Problem 6: EMEA 553, ex. 4
For what values of a is matric $\mathbf{A}$ symmetric?

$$\mathbf{A} = \begin{pmatrix} a & a^2-1 & -3 \\ a+1 & 2 & a^2+4 \\ -3 & 4a & -1 \end{pmatrix}$$

Problem 7: EMEA 545, ex. 4
Write out matrix equations corresponding to the following systems:

$$(a)\ \begin{array}{rcrcl} x_1 & + & x_2 & = & 3 \\ 3x_1 & + & 5x_2 & = & 5 \end{array};\qquad (b)\ \begin{array}{rcrcrcl} x_1 & + & 2x_2 & + & x_3 & = & 4 \\ x_1 & - & x_2 & + & x_3 & = & 5 \\ 2x_1 & + & 3x_2 & - & x_3 & = & 1 \end{array};$$

$$(c)\ \begin{array}{rcrcrcl} 2x_1 & - & 3x_2 & + & x_3 & = & 0 \\ x_1 & + & x_2 & - & x_3 & = & 0 \end{array}.$$

Problem 8: EMEA 579, cv. 1 a
Use Sarrus rule to find the determinant $\begin{vmatrix} 1 & -1 & 0 \\ 1 & 3 & 2 \\ 1 & 0 & 0 \end{vmatrix}$

Problem 9: EMEA 586, cv. 2
Let $\mathbf{A} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 1 & 2 & 5 \end{pmatrix}$. Find the transpose $\mathbf{A}^\top$ and show that $|\mathbf{A}| = |\mathbf{A}^\top|$

Problem 10: EMEA 587, cv. 5
Let $\mathbf{A} = \begin{pmatrix} a & 1 & 4 \\ 2 & 1 & a^2 \\ 1 & 0 & -3 \end{pmatrix}$. Find $\mathbf{A}^2$, and the determinant $|\mathbf{A}|$

Problem 11: EMEA 590, cv. 1
Find following determinants:

$$\begin{vmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{vmatrix},\qquad \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 0 & 11 \\ 2 & -1 & 0 & 3 \\ -2 & 0 & -1 & 3 \end{vmatrix},\qquad \begin{vmatrix} 2 & 1 & 3 & 3 \\ 3 & 2 & 1 & 6 \\ 1 & 3 & 0 & 9 \\ 2 & 4 & 1 & 12 \end{vmatrix}$$