Seminar 11: Linear systems of equations

Problem 1: EMEA 603, ex. 1

Use Cramer's rule to solve the following systems of equations.

a)
$$\begin{array}{rcrcrcr} x & + & 2y & - & z & = & -5 \\ 2x & - & y & + & z & = & 6 \\ x & - & y & - & 3z & = & -3 \end{array}$$

b)
$$\begin{array}{rcrcrcrcr} x & + & y & & & & & = & 3 \\ x & & & + & z & & & = & 2 \\ & & y & + & z & + & u & = & 6 \\ & & y & & & + & u & = & 1 \end{array}$$

Problem 2: EMEA 580, ex. 3 b,c

Use Cramer's rule to solve the following systems of equations. Test the answers by substitution.

a)
$$\begin{array}{rcrcrcr} x_1 & - & x_2 & & & = & 0 \\ x_1 & + & 3x_2 & + & 2x_3 & = & 0 \\ x_1 & + & 2x_2 & + & x_3 & = & 0 \end{array}$$

b)
$$\begin{array}{rcrcrcr} x & + & 3y & - & 2z & = & 1 \\ 3x & - & 2y & + & 5z & = & 14 \\ 2x & - & 5y & + & 3z & = & 1 \end{array}$$

Problem 3: EMEA 608, ex. 11

For what values of a does the system of equations a) one solution b) no solution c)infinitely many solutions

$$\begin{array}{rcrcrcr} ax & + & y & + & 4z & = & 2 \\ 2x & + & y & + & a^2 z & = & 2 \\ x & & & - & 3z & = & a \end{array}$$

Next, replace the right-hand sides by general numbers b_1, b_2, and b_3. Find a necessary and sufficient condition for the new system of equations to have infinitely many solutions.

Problem 4: EMEA 576, ex. 6*

Use Cramer's rule to find Y (national product) and C (private consumption) when

$$Y = C + I_0 + G_0, \qquad C = a + bY,$$

where symbols I_0 (private investment), G_0 (public consumption and investment), a and $b < 1$ all represent constants.

Problem 5: EMEA 555, ex. 2

Use Gaussian elimination to find all solutions of the linear system.

$$\begin{array}{rcrcrcr} x_1 & + & 3x_2 & - & x_3 & = & 4 \\ 2x_1 & + & x_2 & + & x_3 & = & 7 \\ 2x_1 & - & 4x_2 & + & 4x_3 & = & 6 \\ 3x_1 & + & 4x_2 & & & = & 11 \end{array}$$

Problem 6: EMEA 558, ex. 1, b,c

Solve the following equation systems by Gaussian elimination.

a) $$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & x_3 & = & 4 \\ x_1 & - & x_2 & + & x_3 & = & 5 \\ 2x_1 & + & 3x_2 & - & x_3 & = & 1 \end{array}$$

b) $$\begin{array}{rcrcrcr} 2x_1 & - & 3x_2 & + & x_3 & = & 0 \\ x_1 & + & x_2 & - & x_3 & = & 0 \end{array}$$