Seminar 4: Derivatives

Problem 1: EMEA 177/3 g-h

Find the derivatives of the following:

(a) $y = \frac{x^{12}}{12}$ (b) $y = -\frac{2}{x^2}$ (c) $y = \frac{3}{\sqrt[3]{x}}$ (d) $y = -\frac{2}{x\sqrt{x}}$

Problem 2: EMEA 183/1 e

Differentiate the function $y = \frac{1}{2}x - \frac{3}{2}x^2 + 5x^3$

Problem 3: EMEA 183/2 a-c, EMEA 183/3 b,c,h, EMEA 183/4 a

Differentiate the functions:

(a) $y = \frac{3}{5}x^2 - 2x^7 + \frac{1}{8} - \sqrt{x}$ (b) $y = \left(2x^2 - 1\right)\left(x^4 - 1\right)$ (c) $y = \left(x^5 + \frac{1}{x}\right)\left(x^5 + 1\right)$

(d) $y = x^{-1}\left(x^2 + 1\right)\sqrt{x}$ (e) $y = \frac{1}{\sqrt{x^3}}$ (f) $y = \frac{3x-1}{x^2+x+1}$

(g) $y = \frac{\sqrt{x}-2}{\sqrt{x}+1}$.

Problem 4: EMEA 188/1a

Use the chain rule to find dy/dx for the following:

(a) $y = 5u^4$, where $u = 1 + x^2$ (b) $y = u - u^6$, where $u = 1 + \frac{1}{x}$

Problem 5: EMEA 188/10 a,b

Differentiate each of the following in two different ways:

(a) $y = \left(x^4\right)^5 = x^{20}$ (b) $y = (1-x)^3 = 1 - 3x + 3x^2 - x^3$

Problem 6: EMEA 188/3 a-c

Find the derivatives of the following functions, where $p,\ q,\ a,\ b$ are constants:

(a) $y = \frac{1}{\left(x^2+x+1\right)^5}$ (b) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$ (c) $y = x^a(px + q)^b$

Problem 7: EMEA 188/6 *

Compute $\frac{dx}{dp}$ for the demand function $x = b - \sqrt{ap - c}$, where a, b, c are positive constants, while x is the number of units demanded, and p is the price per unit, with $p > \frac{c}{a}$.

Problem 8: EMEA 188/9

Let $C = 20q - 4q\left(25 - \frac{1}{2}x\right)^{\frac{1}{2}}$, where q is a constant, and $x < 50$. Find $\frac{dC}{dx}$.

Problem 9: EMEA 197/1 d-g EMEA 203/3 a-c, e

Find the first-order derivatives of:

(a) $y = \frac{x+x^2}{e^x+1}$ (b) $y = -x - 5 - e^x$ (c) $y = x^3 e^x$ (d) $y = e^x x^{-2}$

(e) $y = \ln(\ln x)$ (f) $y = \ln\sqrt{1-x^2}$ (g) $y = e^x \ln x$ (h) $y = \ln(e^x + 1)$.

Problem 10: EMEA 188/7

Find a formula for $h'(x)$, if

a) $h(x) = f(x^2)$

b) $h(x) = f(x^n g(x))$

Problem 11: EMEA 193/1,2, EMEA 197/3

Find the second-order derivatives of:

(a) $y = x^5 - 3x^4 + 2$ (b) $\sqrt{x}$ (c) $y = (1+x^2)^{10}$ (d) $y = \sqrt{1+x^2}$

(e) $y = e^{-3x}$ (f) $y = 2e^{x^3}$ (g) $y = \frac{e^{\frac{1}{x}}}{x^2}$ (h) $y = 5e^{2x^2-3x+1}$.