

Seminar 5: l'Hôpital's rule, differential, Taylor formula

Problem 1: EMEA 253/3 a-c, EMEA 253/2 b, EMEA 253/1 b-c, EMEA 253/4

Use l'Hôpital's rule to find limits:

- (a) $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$ (b) $\lim_{x \rightarrow -2} \frac{x^3+3x^2-4}{x^3+5x^2+8x+4}$ (c) $\lim_{x \rightarrow 2} \frac{x^4-4x^3+6x^2-8x+8}{x^3-3x^2+4}$
(d) $\lim_{x \rightarrow 0} \frac{2\sqrt{1+x}-2-x}{2\sqrt{1+x+x^2}-2-x}$ (e) $\lim_{x \rightarrow 0} \frac{e^x-1-x-x^2/2}{3x^3}$ (f) $\lim_{x \rightarrow 0} \frac{e^{-3x}-e^{-2x}+x}{x^2}$
(g) $\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}$ (h) $\lim_{x \rightarrow 0^+} x \ln x$ (i) $\lim_{x \rightarrow 0^+} (xe^{1/x} - x)$.

Problem 2: EMEA 253/4

Find the error in the following:

$$\lim_{x \rightarrow 1} \frac{x^2 + 3x - 4}{2x^2 - 2x} = \lim_{x \rightarrow 1} \frac{2x + 3}{4x - 2} = \lim_{x \rightarrow 1} \frac{2}{4} = \frac{1}{2}.$$

Problem 3: EMEA 203/7 a

Find the equation of the tangent to the graph

- (a) $y = \ln x$ at the point $x =$ (i) 1 (ii) $\frac{1}{2}$ (iii) e
(b) $y = xe^x$ at the point $x =$ (i) 0 (ii) 1 (iii) -2

Problem 4: EMEA 204/5 a-c

In each case, find the slope of the tangent to the graph of f at the specified point:

- (a) $y = -3x^2$ pro $x = 1$,
(b) $y = \sqrt{x} - x^2$ pro $x = 4$,
(c) $y = \frac{x^2-x^3}{x+3}$ pro $x = 1$

Problem 5: EMEA 183/7 a-c

In each case, find the slope of the tangent to the graph of f at the specified point:

- (a) $y = 3 - x - x^2$ pro $x_0 = 1$,
(b) $y = \frac{x^2-1}{x^2+1}$ pro $x_0 = 1$,
(c) $y = \left(\frac{1}{x^2} + 1\right)(x^2 - 1)$ pro $x_0 = 2$

Problem 6: EMEA 220/3 a-c, EMEA 255/9 a-c

Find the linear approximation to the function about $x_0 = 0$

- (a) $f(x) = (1+x)^{-1}$, (b) $f(x) = (1+x)^5$, (c) $f(x) = (1-x)^{1/4}$
(d) $f(x) = \ln(2x+4)$, (e) $f(x) = \frac{1}{\sqrt{1+x}}$, (f) $f(x) = xe^{2x}$.

Problem 7: EMEA 224/1 a

Find the linear approximation to the function $f(x) = (1 + x)^5$ about $x_0 = 0$.

Problem 8: EMEA 227/1, 4 a,c

Let $g(x) = \sqrt[3]{1 + x}$.

(a) Find the Taylor polynomial of order 2 for the function $g(x)$ about $x_0 = 0$.

(b) Use the polynomial to estimate $\sqrt[3]{1003}$.

Problem 9: EMEA 224/2

Find the Taylor polynomial of order 2 for the function $f(x) = \ln(1 + x)$ about $x_0 = 0$.