## 6. Optimization, monotonicity, and convexity

**Příklad 1:**
Find the intervals where the following functions are increasing:

(a) $f(x) = 3x^2 - 12x + 13$ (b) $f(x) = \frac{1}{4}\left(x^4 - 6x^2\right)$

(c) $f(x) = \frac{2x}{x^2+2}$ (d) $f(x) = \frac{x^2-x^3}{2(x+1)}$

**Příklad 2:**
Find the intervals where the following functions are increasing:

(a) $y = (\ln x)^2 - 4$ (b) $y = ln(e^x + e^{-x})$ (c) $y = x - \frac{3}{2}\ln\left(x^2 + 2\right)$
(d) $y = \frac{e^x}{x}$ (e) $y = x^4 e^{-2x}$ (f) $y = xe^{-\sqrt{x}}$

**Příklad 3:**
Find the extreme points and inflection points of the function:

(a) $y = (x+2)e^x$ (b) $y = \ln x + 1/x$ (c) $y = x^3 e^{-x}$
(d) $y = \frac{\ln x}{x^2}$ (e) $y = e^{2x} - 2e^x$ (f) $y = (x^2 + 2x)e^{-x}$

**Příklad 4:**
Find the inflection points of the function a determine the intervals of convexity:

(a) $y = x^2 - 2x + 2$ (b) $f(x) = ax^2 + bx + c$

**Příklad 5:** EMEA 272, cv. 2
Find global extreme points for the functions and given intervals

(a) $f(x) = -2x - 1 \quad \langle 0; 3\rangle$ (b) $f(x) = x^3 - 3x + 8 \quad \langle -1; 2\rangle$

(c) $f(x) = \frac{x^2+1}{x} \quad \langle \frac{1}{2}; 2\rangle$ (d) $f(x) = x^5 - 5x^3 \quad \langle -1; \sqrt{5}\rangle$

(e) $f(x) = x^3 - 4500x^2 + 6\cdot 10^6 x \quad \langle 0; 3000\rangle$

**Příklad 6:** EMEA 289 2 a *
A firm's production function is $Q(L) = 12L^2 - L^3/20$ where $L$ denotes the number of workers, with $L \in \langle 0, 200\rangle$.

a) What size of the work force $L^*$ maximizes output $Q(L)$?

b) What size of the work force $L^{**}$ maximizes output per worker, $Q(L)/L$?

**Příklad 7:** EMEA 243 4 a-d

Find the oblique asymptote for the following functions:

(a) $f(x) = \frac{x^2}{x+1}$ (b) $f(x) = \frac{2x^3-3x^2+3x-6}{x^2+1}$

(c) $f(x) = \frac{3x^2+2x}{x-1}$ (d) $f(x) = \frac{5x^4-3x^2+1}{\left(x^3-1\right)}$