Econometrics

Panel Data Methods

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Lecture 9

Data used in Econometrics

Cross-sectional

- Data for different entities,
- No time dimension,
- Order of data does not matter

Time series

- Data for a single entity collected at multiple time periods.
- Order of data is important
- Observations are typically not independent over time

Panel data

- Data for multiple entities in which outcomes and characteristics of each entity are observed at multiple points in time.
- Combine cross-sectional and time series issues
- Present several advantages with respect to cross-sectional and time series data

Pooled Cross Sectional and Panel Data

An ***independently pooled cross section*** (or ***repeated cross sectional***) is obtained by sampling randomly from a large population at different points in time (for example, annual labor force surveys)

A ***panel dataset*** contains observations on multiple entities (individuals, states, companies…), where each entity is observed at two or more points in time.

Hypothetical examples:

- Data on 420 California school districts in 2010 *and again* in 2012, for 840 obs.
- Data on 50 U.S. states, each state is observed in 3 years, for a total of 150 obs.
- Data on 1000 individuals, in four different months, for 4000 obs total.

Panel Data

A double subscript distinguishes **entities** (individual units) and **time** periods (years)

i = entity (state), n = number of entities, so $i = 1,\ldots,n$

t = time period (year), T = number of time periods, so $t = 1,\ldots,T$

Data: Suppose we have 1 regressor.

The data are:

$$(X_{it}, Y_{it}),\ i = 1,\ldots,n,\ t = 1,\ldots,T$$

Panel Data

Panel data with k regressors:

$$(X_{1it}, X_{2it}, \ldots, X_{kit}, Y_{it}),\ i = 1, \ldots, n,\ t = 1, \ldots, T$$

n = number of entities (states)

T = number of time periods (years)

Another term for panel data is ***longitudinal data***

balanced panel: no missing observations, that is, all variables are observed for all entities (states) and all time periods (years)

Why are Panel Data Useful?

With panel data we can control for factors that:

Vary across entities but do not vary over time

- These could cause omitted variable bias if they are omitted

Are unobserved or unmeasured – and therefore cannot be included in the regression using multiple regression

Here's the key idea:

- If an omitted variable does not change over time, then any *changes* in Y over time cannot be caused by the omitted variable.

Panel Data: Example of a Dataset

Observational unit: a year in a U.S. state

- 48 U.S. states, so n = # of entities = 48
- 7 years (2002,…, 2008), so T = # of time periods = 7
- Balanced panel, so total # observations = $7 \times 48 = 336$

Variables:

- Traffic fatality rate (# traffic deaths in that state in that year, per 10,000 state residents)
- Tax on a case of beer
- Other (legal driving age, drunk driving laws, etc.)

Policy Analysis with Pooled Cross Sections

Two or more independently sampled cross sections can be used to evaluate the impact of a certain event or policy change

- **Example: Effect of new garbage incinerator on housing prices (Kiel and McClain (1995))**
  - Examine the effect of the location of a house on its price before and after the garbage incinerator was built:

$$\widehat{rprice} = \underset{(3,093.0)}{101,307.5} \underset{(5,827.71)}{- 30,688.27}\, nearinc$$

After incinerator was built (1981) – no causality!

$$\widehat{rprice} = \underset{(2,653.79)}{82,517.23} \underset{(4,744.59)}{- 18,824.37}\, nearinc$$

Before incinerator was built (1978)

Policy Analysis with Pooled Cross Sections

- **Example: Garbage incinerator and housing prices (cont.)**
  - It would be wrong to conclude from the regression after the incinerator is there that being near the incinerator depresses prices so strongly
  - One has to compare with the situation before the incinerator was built:

Incinerator depresses prices but location was one with lower prices anyway

$$\hat{\delta}_1 = -30,688.27 - (-18,824.37) = -11,863.9$$

  - In the given case, this is equivalent to

$$\hat{\delta}_1 = (\overline{rprice}_{1,nr} - \overline{rprice}_{1,fr}) - (\overline{rprice}_{0,nr} - \overline{rprice}_{0,fr})$$

- **This is the so called difference-in-differences estimator (DiD)**

Policy Analysis with Pooled Cross Sections

- **Difference-in-differences in a regression framework**

$$rprice = \beta_0 + \delta_0\, after + \beta_1\, nearinc + \delta_1\, after \cdot nearinc + u$$

Differential effect of being in the location and after the incinerator was built

  - In this way standard errors for the DiD-effect can be obtained
  - If houses sold before and after the incinerator was built were systematically different, further explanatory variables should be included
  - This will also reduce the error variance and thus standard errors

- **Before/After comparisons in „natural experiments“**
  - DiD can be used to evaluate policy changes or other exogenous events

Policy Analysis with Pooled Cross Sections

- **Policy evaluation using difference-in-differences**

$$y = \beta_0 + \delta_0 after + \beta_1 treated + \delta_1 after \cdot treated + other factors$$

$$\hat{\delta}_1 = (\overline{y}_{1,T} - \overline{y}_{1,C}) - (\overline{y}_{0,T} - \overline{y}_{0,C})$$ ← Compare outcomes of the two groups before and after the policy change

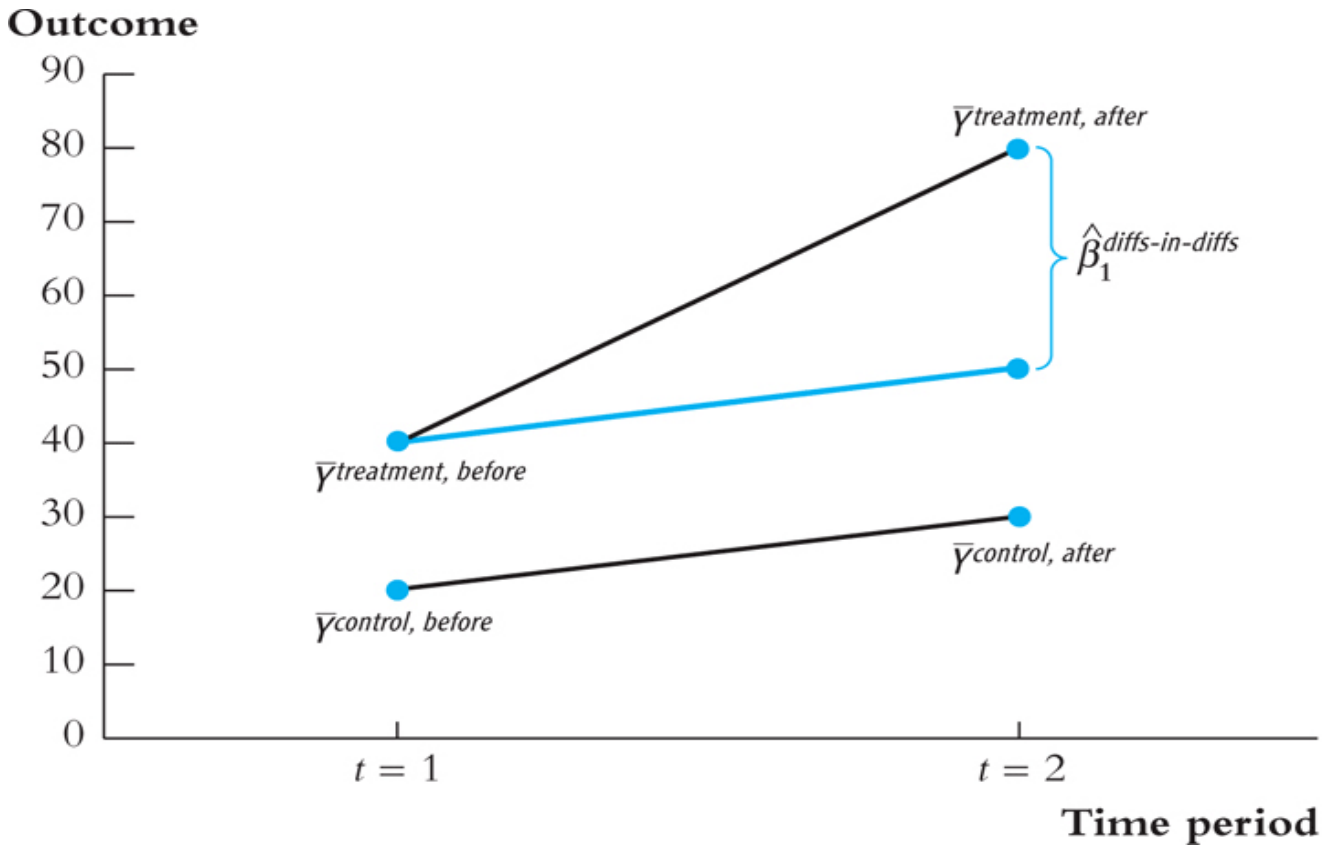
Compare the difference in outcomes of the units that are affected by the policy change (= treatment group) and those who are not affected (= control group) before and after the policy was enacted.

For example, the level of unemployment benefits is cut but only for group A (= treatment group). Group A normally has longer unemployment durations than group B (= control group). If the difference in unemployment durations between group A and group B becomes smaller after the reform, reducing unemployment benefits reduces unemployment duration for those affected.

Caution: Difference-in-differences only works if the difference in outcomes between the two groups is not changed by other factors than the policy change (e.g. there must be no differential trends).

Diff-in-Diff Estimator (DID)

$$\hat{\beta}_1^{DiD} = \left(\bar{Y}^{treat,after} - \bar{Y}^{treat,before}\right) - \left(\bar{Y}^{control,after} - \bar{Y}^{control,before}\right)$$



Two-Period Panel Data Analysis

- **Example: Effect of unemployment on city crime rate**
  - Assume that no other explanatory variables are available. Will it be possible to estimate the causal effect of unemployment on crime?
  - Yes, if cities are observed for at least two periods and other factors affecting crime stay approximately constant over those periods:

$$crmrte_{it} = \beta_0 + \delta_0 d87_{it} + \beta_1 unem_{it} + a_i + u_{it}, \quad t = 1982, 1987$$

Time dummy for the second period

Unobserved time-constant factors (= fixed effect)

Other unobserved factors (= idiosyncratic error)

Two-Period Panel Data Analysis

- **Example: Effect of unemployment on city crime rate (cont.)**

$$crmrte_{i1987} = \beta_0 + \delta_0 \cdot 1 + \beta_1 \, unem_{i1987} + a_i + u_{i1987}$$

$$crmrte_{i1982} = \beta_0 + \delta_0 \cdot 0 + \beta_1 \, unem_{i1982} + a_i + u_{i1982}$$

Subtract: $\Rightarrow \Delta crmrte_i = \delta_0 + \beta_1 \Delta unem_i + \Delta u_i$

Fixed effect drops out!

- **Estimate differenced equation by OLS:**

$$\widehat{\Delta crmrte} = \underset{(4.70)}{15.40} + \underset{(.88)}{2.22} \, \Delta unem$$

Secular increase in crime

+ 1 percentage point unemployment rate leads to 2.22 more crimes per 1,000 people

$$n = 46, R^2 = .127$$

Two-Period Panel Data Analysis

- **Discussion of first-differenced panel estimator**
  - Further explanatory variables may be included in the original equation
  - Note that there may be arbitrary correlation between the unobserved time-invariant characteristics and the included explanatory variables
  - OLS in the original equation would therefore be inconsistent
  - The first-differenced panel estimator is thus a way to consistently estimate causal effects in the presence of time-invariant endogeneity
  - For consistency, strict exogeneity has to hold in the original equation
  - First-differenced estimates will be imprecise if explanatory variables vary only little over time (no estimate possible if time-invariant)

Fixed Effects Estimation

Consider the panel data model,

$$FatalityRate_{it} = \beta_0 + \beta_1 BeerTax_{it} + \beta_2 Z_i + u_{it}$$

Z_i is a factor that does not change over time, at least during the years on which we have data *(examples: "culture" around drinking and driving; density of cars on the road;).*

- Suppose Z_i is not observed, so its omission could result in omitted variable bias.
- The effect of Z_i can be eliminated using $T = 2$ years by method described above.

Fixed Effects Estimation

What if you have more than 2 time periods ($T > 2$)?

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 Z_i + u_{it},\ i = 1,\ldots,n,\ T = 1,\ldots,T$$

We can rewrite this in two useful ways:

1. "***n*-1 binary regressor**" regression model
2. "**Fixed Effects**" regression model

We first rewrite this in "**fixed effects**" form. Suppose we have $n = 3$ states: **California**, **Texas**, and **Massachusetts** and we want to estimate the effect of **tax on a case of beer** on the **traffic fatality rate**.

Fixed Effects Estimation

Population regression for California (that is, i = CA):

$$Y_{CA,t} = \beta_0 + \beta_1 X_{CA,t} + \beta_2 Z_{CA} + u_{CA,t}$$
$$= (\beta_0 + \beta_2 Z_{CA}) + \beta_1 X_{CA,t} + u_{CA,t}$$

Or

$$Y_{CA,t} = a_{CA} + \beta_1 X_{CA,t} + u_{CA,t}$$

- $a_{CA} = \beta_0 + \beta_2 Z_{CA}$ **doesn't change over time**
- a_{CA} is the intercept for CA, and β_1 is the slope
- The intercept is unique to CA, but the slope is the same in all the states: parallel lines.

Fixed Effects Estimation

$$Y_{TX,t} = \beta_0 + \beta_1 X_{TX,t} + \beta_2 Z_{TX} + u_{TX,t}$$
$$= (\beta_0 + \beta_2 Z_{TX}) + \beta_1 X_{TX,t} + u_{TX,t}$$

(*population regression for Texas*)

or

$$Y_{TX,t} = a_{TX} + \beta_1 X_{TX,t} + u_{TX,t}, \text{ where } a_{TX} = \beta_0 + \beta_2 Z_{TX}$$

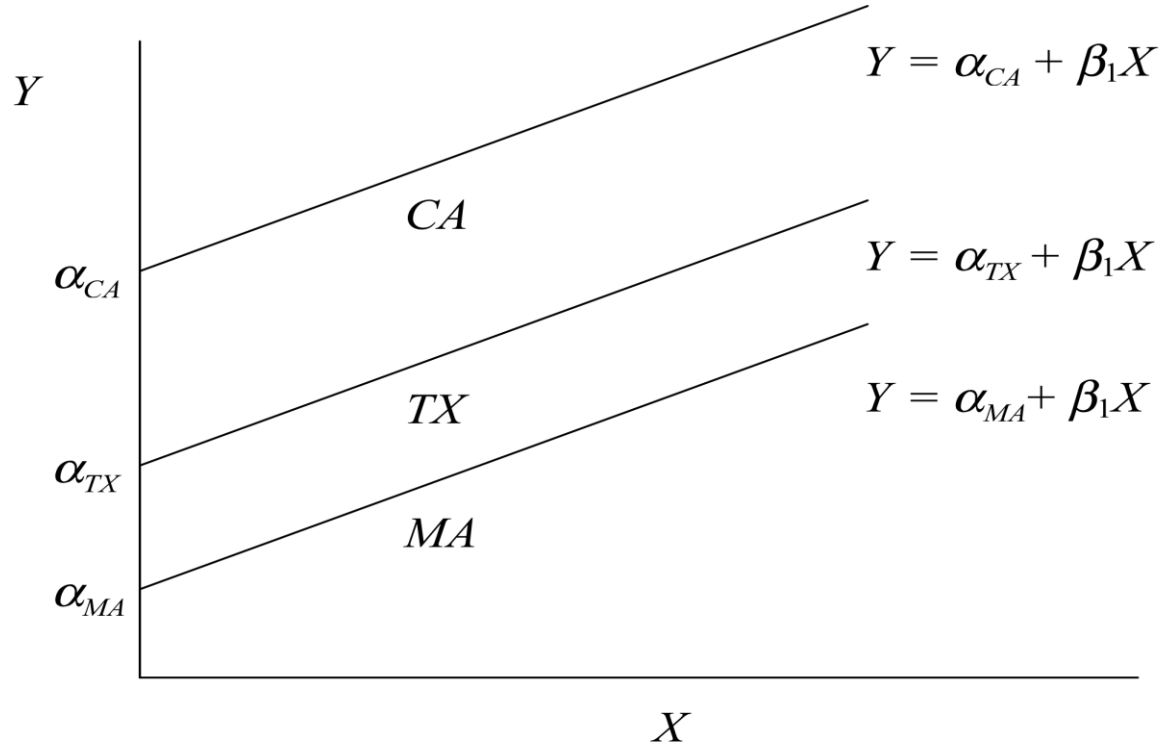
Collecting the lines for all three states:

$$Y_{CA,t} = a_{CA} + \beta_1 X_{CA,t} + u_{CA,t}$$
$$Y_{TX,t} = a_{TX} + \beta_1 X_{TX,t} + u_{TX,t}$$
$$Y_{MA,t} = a_{MA} + \beta_1 X_{MA,t} + u_{MA,t}$$

or

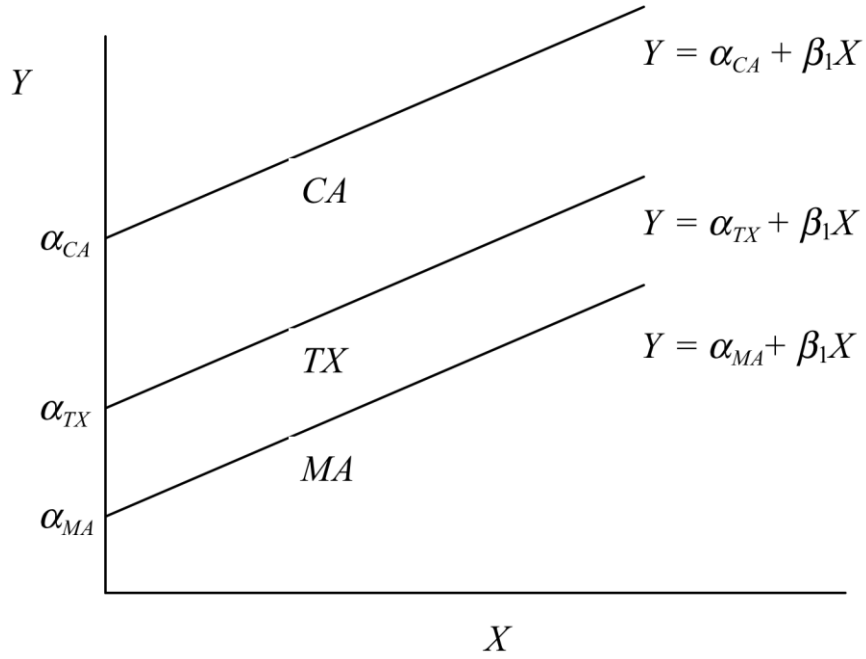
$$Y_{it} = a_i + \beta_1 X_{it} + u_{it}, \; i = \text{CA, TX, MA}, \; T = 1,\ldots,T$$

Fixed Effects Estimation



Recall that shifts in the intercept can be represented using binary regressors…

Fixed Effects Estimation



In binary regressor form:

$$Y_{it} = \beta_0 + \gamma_{CA} DCA_i + \gamma_{TX} DTX_i + \beta_1 X_{it} + u_{it}$$

DCA_i = 1 if state is *CA*, = 0 otherwise

DTX_t = 1 if state is *TX*, = 0 otherwise

leave out DMA_i (*why*?)

Fixed Effects Estimation

1. "*n*-1 binary regressor" form

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \gamma_2 D2_i + \ldots + \gamma_n Dn_i + u_{it}$$

where $D2_i = \begin{cases} 1 \text{ for } i=2 \text{ (state \#2)} \\ 0 \text{ otherwise} \end{cases}$, etc.

2. "Fixed effects" form:

$$Y_{it} = \beta_1 X_{it} + a_i + u_{it}$$

- a_i is called a "state fixed effect" or "state effect" – it is the constant (fixed) effect of being in state i

Fixed Effects Estimation

Three estimation methods:

1. "n-1 binary regressors" OLS regression
2. "Entity-demeaned" OLS regression
3. "Changes" specification, without an intercept

These three methods produce identical estimates of the regression coefficients, and identical standard errors.

We already did the "changes" specification – but this works well for T = 2 years

Methods #1 and #2 work for general T

Method #1 is only practical when n isn't too big

Fixed Effects Estimation

- **Fixed effects estimation**

Fixed effect, potentially correlated with explanatory variables

$$y_{it} = \beta_1 x_{it1} + \cdots + \beta_k x_{itk} + a_i + u_{it}, \; i = 1, \ldots, N, t = 1, \ldots, T$$

$$\bar{y}_i = \beta_1 \bar{x}_{i1} + \cdots + \beta_k \bar{x}_{ik} + \bar{a}_i + \bar{u}_i$$

Form time-averages for each individual

$$\Rightarrow [y_{it} - \bar{y}_i] = \beta_1 [x_{it1} - \bar{x}_{i1}] + \cdots + \beta_k [x_{itk} - \bar{x}_{ik}] + [u_{it} - \bar{u}_i]$$

Because $a_i - \bar{a}_i = 0$ (the fixed effect is removed)

- **Estimate time-demeaned equation by OLS**
  - Uses time variation within cross-sectional units (= within-estimator)

Fixed Effects Estimation

- **Example: Effect of training grants on firm scrap rate**

$$scrap_{it} = \beta_1 d88_{it} + \beta_2 d89_{it} + \beta_3 grant_{it} + \beta_4 grant_{it-1} + a_i + u_{it}$$

Time-invariant reasons why one firm is more productive than another are controlled for. The important point is that these may be correlated with the other explanatory variables.

Fixed-effects estimation using the years 1987, 1988, 1989:

Stars denote time-demeaning

$$\widehat{scrap}^*_{it} = -\underset{(.109)}{.080}\, d88^*_{it} - \underset{(.133)}{.247}\, d89^*_{it} - \underset{(.151)}{.252}\, grant^*_{it} - \underset{(.210)}{.422}\, grant^*_{it-1}$$

$$n = 162, R^2 = .201$$

Training grants significantly improve productivity (with a time lag)

Fixed Effects Estimation with Time Fixed Effects

An omitted variable might vary **over time** but **not across states**:

- Safer cars (air bags, etc.); changes in national laws
- These produce **intercepts that change over time**
- Let S_t denote the combined effect of variables which changes over time but not states ("safer cars").
- The resulting population regression model is:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 Z_i + \beta_3 S_t + u_{it}$$

Fixed Effects Estimation with Time Fixed Effects

This model can be recast as having an intercept that varies from one year to the next:

$$\begin{aligned} Y_{i,1982} &= \beta_0 + \beta_1 X_{i,1982} + \beta_3 S_{1982} + u_{i,1982} \\ &= (\beta_0 + \beta_3 S_{1982}) + \beta_1 X_{i,1982} + u_{i,1982} \\ &= \lambda_{1982} + \beta_1 X_{i,1982} + u_{i,1982}, \end{aligned}$$

where $\lambda_{1982} = \beta_0 + \beta_3 S_{1982}$ Similarly,

$$Y_{i,1983} = \lambda_{1983} + \beta_1 X_{i,1983} + u_{i,1983},$$

where $\lambda_{1983} = \beta_0 + \beta_3 S_{1983}$, etc.

Fixed Effects Estimation with Time Fixed Effects

1. "T-1 binary regressor" formulation:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \delta_2 B2_t + \ldots \delta_T BT_t + u_{it}$$

where $B2_t = \begin{cases} 1 \text{ when } t=2 \text{ (year \#2)} \\ 0 \text{ otherwise} \end{cases}$, etc.

2. "Time effects" formulation:

$$Y_{it} = \beta_1 X_{it} + \lambda_t + u_{it}$$

Fixed Effects Estimation with Time Fixed Effects

1. "T-1 binary regressor" OLS regression

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \delta_2 B2_{it} + \ldots \delta_T BT_{it} + u_{it}$$

   - Create binary variables $B2,\ldots,BT$
   - $B2 = 1$ if t = year #2, = 0 otherwise
   - Regress Y on X, $B2,\ldots,BT$ using OLS
   - **Where's *B1*?**

2. "Year-demeaned" OLS regression
   - Deviate Y_{it}, X_{it} from ***year*** (not state) averages
   - Estimate by OLS using "year-demeaned" data

Fixed Effects Estimation

- **Discussion of fixed effects estimator**
  - Strict exogeneity in the original model has to be assumed
  - The *R-squared* of the demeaned equation is inappropriate
  - The effect of time-invariant variables cannot be estimated
  - But ***the effect of interactions with time-invariant variables can be estimated*** (e.g. the interaction of education with time dummies)
  - If a full set of time dummies are included, the effect of variables whose change over time is constant cannot be estimated (e.g. experience)

Fixed Effects Estimation

- **Fixed effects or first differencing?**
  - In the case T = 2, fixed effects and first differencing are identical
  - For T > 2, fixed effects is more efficient if classical assumptions hold
  - First differencing may be better in the case of severe serial correlation in the errors, for example if the errors follow a random walk
  - If T is very large (and N not so large), the panel has a pronounced time series character and problems such as strong dependence arise (unit root process – spurious regression)
  - In these cases, it is probably better to use first differencing
  - Otherwise, it is a good idea to compute both and check robustness