7. Functions of more variables

Problem 1: EMEA 373, cv. 6
For which pairs of numbers (x, y) are the functions given by the following formulas defined?, Sketch the domain in the x, y-plane.

(a) $\frac{x^2+y^2}{x-y+2}$; (b) $\sqrt{2-(x^2+y^2)}$; (c) $\sqrt{(4-x^2-y^2)(x^2+y^2-1)}$

Problem 2: EMEA 378, cv. 2
Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the following functions.

(a) $z = x^2 + 3y^2$; (b) $z = xy$; (c) $z = 5x^4y^2 - 2xy^5$; (d) $z = e^{x+y}$

(e) $z = e^{xy}$; (f) $z = \frac{e^x}{y}$; (g) $z = \ln(x+y)$; (h) $z = \ln(xy)$

Problem 3: EMEA 378, cv. 3
Find all first- and second-order partials of:

(a) $f(x, y) = x^7 - y^7$; (b) $f(x, y) = x^5 \ln y$; (c) $f(x, y) = \left(x^2 - 2y^2\right)^5$

Problem 4: EMEA 378, cv. 4
Find all first- and second-order partials of:

(a) $z = 3x + 4y$; (b) $z = x^3y^2$; (c) $z = x^5 - 3x^2y + y^6$

(d) $z = \frac{x}{y}$; (e) $z = \frac{(x-y)}{(x+y)}$; (f) $z = \sqrt{x^2 + y^2}$

Problem 5: EMEA 457, cv. 1
The function $f(x, y)$ defined as $f(x, y) = -2x^2 - y^2 + 4x + 4y - 3$ has a maximum. Find the corresponding values of x and y.

Problem 6: EMEA 457, cv. 2
The function $f(x, y)$ defined as $f(x, y) = x^2 + y^2 - 6x + 8y + 35$ has an extreme point.

a) Find it.

b) Show that $f(x, y)$ can be written in the form $f(x, y) = (x-3)^2 + (y+4)^2 + 10$ Decide what type of the extreme is obtained in a) and explain why.

Problem 7: EMEA 457, cv. 3 *
The company produces one type of product, the quantity of which is determined by the production function

$$Q = F(K, L) = 80 - (K-3)^2 - 2(L-6)^2 - (K-3)(L-6),$$

where the price of the capital is $r = 0.65$ EUR and the price of labour is 1.2 EUR. The product can be sold on the market at a unit price $p = 1$ EUR. Find the only possible values of K and L that maximize profits.

Problem 8: EMEA 466, cv. 1
The function $f(x,y)$ is defined by the formula $f(x,y) = 5 - x^2 + 6x - 2y^2 + 8y$.

a) Find all its first- and second-order partial derivatives.

b) Find the only stationary point and classify it by using the second-derivative test..

Problem 9: EMEA 466, cv. 2
The function $f(x,y)$ is defined by the formula $f(x,y) = x^2 + 2xy^2 + 2y^2$.

a) Find the first- and second-order partial derivatives of $f(x,y)$.

b) Show that the stationary points are $[0,0], [-1,1], [-1,-1]$ and classify them.