Introduciotion to the course

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Content

Portfolio Theory

Course requirements

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2. On Average 60% of total score from 2 tests

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- Literature: Elton, E.: *Modern Portfolio Theory and Investment Analysis*

Portfolio Theory

Earned points and valuation

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- A: [27,30)
- B: [25,27)
- C: [23,25)
- D: [21,23)
- E: [18,21)
- F: [0,18)

Portfolio Theory

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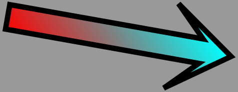

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- **Ross, S.**: *The Arbitrage Theory of Capital Asset Pricing (1976)*

⋮

Investment Portfolio

$$V_p = \sum_{i=1}^{n} A_i * w_i$$

Assets assumptions:

- Identifiability
- Measurability *(Price)*

Investment → function of → $\mathbf{r}$, σ, l

The basic premise of creating a portfolio is based on the rationality of an investor.

Thus, the aim of portfolio creation is to find a composition of assets that corresponds its/her needs.

Return

Return as Random Variable

$$\mathbf{X} \rightarrow \mathbf{r}$$

$$X(\mu, \sigma)$$

Mean (μ) :

- $\mu = \frac{1}{n}\sum_{i=1}^{n} Xi \quad \rightarrow \quad \mathrm{E}(\mathrm{X}_i)$ or $\bar{r}$

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otherwise,

- $\sigma^2(X + Y) = \sigma^2(X) + \sigma^2(Y) + 2 * cov(X, Y)$

Risk of assets

Change in expected return

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- $cov(X, X) = var(X)$

Range of cov $(-\infty; \infty)$ $\rightarrow$ *standardization*

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- Coefficient of determination OLS ... ρ^2