Probabilistic Scenario & Basic of Investment Portfolio

Luděk Benada

Department of Finance, office - 402

e-mail: *benada@econ.muni.cz*

Content

Content

Estimation of the probability

- The current price of the asset under consideration is known

Estimation of the probability

- The current price of the asset under consideration is known
- The dividend payments are not considered

Estimation of the probability

- The current price of the asset under consideration is known
- The dividend payments are not considered
- The potential market prices in the future are estimated

Estimation of the probability

- The current price of the asset under consideration is known
- The dividend payments are not considered
- The potential market prices in the future are estimated

Estimation of the probability

- The current price of the asset under consideration is known
- The dividend payments are not considered
- The potential market prices in the future are estimated

⇒ Price scenarios ($P_{t,i}$)

Estimation of the probability

- The current price of the asset under consideration is known
- The dividend payments are not considered
- The potential market prices in the future are estimated

→ Price scenarios ($P_{t,i}$)

where,

$$r_i = ln(\frac{P_{t,i}}{P_{t-k}})$$

Return and risk of an asset

If the future price of a considered asset is known, then could be determined:

- Expected **return**:

Return and risk of an asset

If the future price of a considered asset is known, then could be determined:

- Expected **return**:

$$E(r_i) = \sum_{i=1}^{n} r_i * p_i$$

Return and risk of an asset

If the future price of a considered asset is known, then could be determined:

- Expected **return**:

$$E(r_i) = \sum_{i=1}^{n} r_i * p_i$$

- **Risk** of the asset:

Return and risk of an asset

If the future price of a considered asset is known, then could be determined:

- Expected **return**:

$$E(r_i) = \sum_{i=1}^{n} r_i * p_i$$

- **Risk** of the asset:

$$\sigma_i = \sqrt{(r_i - E(r))^2 * p_i}$$

Calculation procedure

- A larger and more plausible probability distribution for the future prices can lead to better estimation results

Calculation procedure

- A larger and more plausible probability distribution for the future prices can lead to better estimation results
- More probabilistic scenarios need to be condensed, so the **probability** range **condition** would be ensured:

Calculation procedure

- A larger and more plausible probability distribution for the future prices can lead to better estimation results
- More probabilistic scenarios need to be condensed, so the **probability** range **condition** would be ensured:

Only one price scenario

$$\sum_{i=1}^{n} p_i = 100\%$$

Calculation procedure

- A larger and more plausible probability distribution for the future prices can lead to better estimation results
- More probabilistic scenarios need to be condensed, so the **probability** range **condition** would be ensured:

Only one price scenario

$$\sum_{i=1}^{n} p_i = 100\%$$

N prices scenarios

$$\frac{1}{N}\sum_{i=1}^{n} p_i = 100\%$$

The sequence of the calculation

- Unification of prices

The sequence of the calculation

- Unification of prices
- Linking with their probabilities

The sequence of the calculation

- Unification of prices
- Linking with their probabilities
- Sum and standardization of probabilities

The sequence of the calculation

- Unification of prices
- Linking with their probabilities
- Sum and standardization of probabilities
- Calculation of returns and their linking with probabilities

The sequence of the calculation

- Unification of prices
- Linking with their probabilities
- Sum and standardization of probabilities
- Calculation of returns and their linking with probabilities
- Calculation of $E(r_i)$ & σ_i

The sequence of the calculation

Concrete example

Content

Portfolio return

- Portfolio consists of n-assets

Portfolio return

- Portfolio consists of n-assets
- The proportion of i-th assets in the portfolio is w_i

Portfolio return

- Portfolio consists of n-assets
- The proportion of i-th assets in the portfolio is w_i
- Basic assumption:

Portfolio return

- Portfolio consists of n-assets
- The proportion of i-th assets in the portfolio is w_i
- Basic assumption:

$$\sum_{i=1}^{n} w_i = 1$$

Portfolio return

- Portfolio consists of n-assets
- The proportion of i-th assets in the portfolio is w_i
- Basic assumption:

$$\sum_{i=1}^{n} w_i = 1, also \;\; w_i < 0$$

thus,

$$R_p = \sum_{i=1}^{n} w_i * r_i$$

Portfolio return

- Portfolio consists of n-assets
- The proportion of i-th assets in the portfolio is w_i
- Basic assumption:

$$\sum_{i=1}^{n} w_i = 1, also \;\; w_i < 0$$

thus,

$$R_p = \sum_{i=1}^{n} w_i * r_i$$

or

Portfolio return

- Portfolio consists of n-assets
- The proportion of i-th assets in the portfolio is w_i
- Basic assumption:

$$\sum_{i=1}^{n} w_i = 1, also \;\; w_i < 0$$

thus,

$$R_p = \sum_{i=1}^{n} w_i * r_i$$

or

$$E(R_p) = \sum_{i=1}^{n} w_i * \bar{r}_i$$

Risk of portfolio

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i * w_j * \sigma_{i,j}}$$

Risk of portfolio

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i * w_j * \sigma_{i,j}}$$

thus,

$$\sigma_p = Var_p + Covar_p$$

Risk of portfolio

$$\sigma_p = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} w_i * w_j * \sigma_{i,j}}$$

thus,

$$\sigma_p = Var_p + Covar_p$$

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 * \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1|i \neq j}^{n} w_i * w_j * \sigma_{i,j}$$

Risk of portfolio

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i * w_j * \sigma_{i,j}}$$

thus,

$$\sigma_p = Var_p + Covar_p$$

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 * \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1|i\neq j}^{n} w_i * w_j * \sigma_{i,j}$$

or simply;

Risk of portfolio

$$\sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} w_i * w_j * \sigma_{i,j}}$$

thus,

$$\sigma_p = Var_p + Covar_p$$

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 * \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1|i\neq j}^{n} w_i * w_j * \sigma_{i,j}$$

or simply;

$$\sigma_p = \sqrt{w^T * C * w}$$

Special case - Naive Portfolio

$$w_i = \frac{1}{n}$$

Special case - Naive Portfolio

$$w_i = \frac{1}{n}$$

$$\sigma_p^2 = \frac{1}{n^2}\sum_{i=1}^{n}\sigma_i^2 + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1|i\neq j}^{n}\sigma_{i,j}$$

Special case - Naive Portfolio

$$w_i = \frac{1}{n}$$

$$\sigma_p^2 = \frac{1}{n^2}\sum_{i=1}^{n}\sigma_i^2 + \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1|i\neq j}^{n}\sigma_{i,j}$$

$$\sigma_p^2 = \frac{1}{n}\bar{\sigma_i}^2 + (1 - \frac{1}{n})\bar{\sigma_{i,j}}$$