

Probabilistic Scenario & Basic of Investment Portfolio

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Content

- 1 Calculations by available probabilities
- 2 Investment portfolio

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2 Investment portfolio

Estimation of the probability

- The current price of the asset under consideration is known

Estimation of the probability

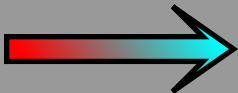
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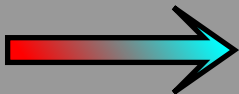
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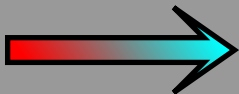
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Price scenarios ($P_{t,i}$)

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where,

$$r_i = \ln\left(\frac{P_{t,i}}{P_{t-k}}\right)$$

Return and risk of an asset

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$$\sigma_i = \sqrt{(r_i - E(r))^2 * p_i}$$

Calculation procedure

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N prices scenarios

$$\frac{1}{N} \sum_{i=1}^n p_i = 100\%$$

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- Calculation of $E(r_i)$ & σ_i

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C o n c r e t e e x a m p l e

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$$E(R_p) = \sum_{i=1}^n w_i * \bar{r}_i$$

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$$\sigma_p = \sqrt{w^T * C * w}$$

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$$\sigma_p^2 = \frac{1}{n} \bar{\sigma}_i^2 + \left(1 - \frac{1}{n}\right) \bar{\sigma}_{i,j}$$